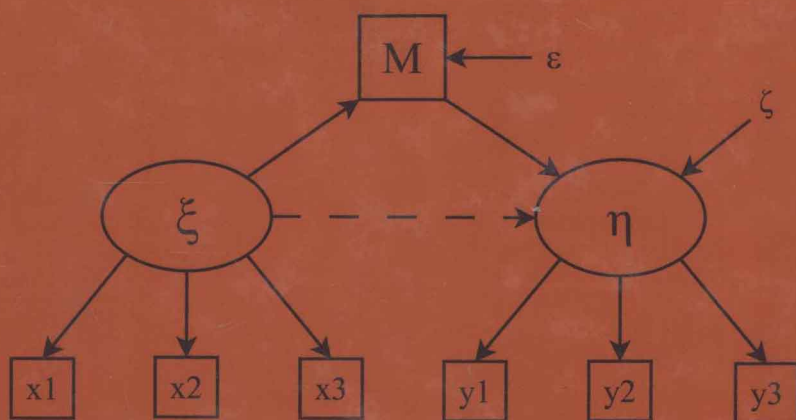


STATISTICS IN THE SOCIAL SCIENCES

CURRENT METHODOLOGICAL DEVELOPMENTS



Edited by

STANISLAV KOLENIKOV / DOUGLAS STEINLEY / LORI THOMBS

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Stanislav Kolenikov

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STATISTICS IN THE SOCIAL SCIENCES

PREFACE

This book is aimed at a wide spectrum of researchers and students who are concerned with current statistical methodology applied in the social sciences. In short, all social scientists working in fields including (but not limited to) psychology, sociology, test theory, market research, and many more would benefit from being kept abreast of the cutting-edge research contained herein.

The impetus for the book was *The Sixth Annual Winemiller Conference: Methodological Developments of Statistics in the Social Sciences* held at the University of Missouri in Columbia, Missouri from October 11 to 14 in 2006. The aim of the conference was to foster collaboration among mathematical statisticians and quantitatively oriented social science researchers. This interdisciplinary conference brought top researchers from major social sciences disciplines, highlighting the interface between recent developments in each area. The idea was to gather experts in the field and to assemble and edit a text that encompassed many of the current methodologies applied by researchers in different social sciences disciplines.

Chapter 1's devoted to structural equation modeling is by Peter Bentler and Victoria Savalei. Their focus is, however, on correlation structures. They describe the typical problems that arise in analysis of correlations and correlation matrices. From the field of structural equation and covariance structure modeling, they borrow the estimation methods and test statistics commonly used in SEM, as well as corrections commonly used to improve the finite sample behavior of those methods, and draw

parallels in correlation structure analysis. Bentler and Savelle begin by providing different parameterizations of correlation matrices from structural equation models in Jöreskog's LISREL formulation and in the Bentler–Weeks formulation. Then they highlight the differences between covariance (or mean-and-covariance) structure and correlation structure analyses, show where each is best applied, and demonstrate why treating correlation matrices as covariance matrices may lead to erroneous inference. Asymptotic theory under correct specification is derived for estimation of structural correlation models based on the general concept of discrepancy functions. Quadratic forms in parameter estimates are shown to be the generic asymptotic form of such discrepancy functions, and further analysis proceeds in terms of those forms leading to (asymptotically distribution free, ADF) M -estimates of the structural parameters. Asymptotic normality of those estimates is demonstrated through the standard arguments, and considerations of asymptotic efficiency via the optimal choice of the weighting matrix are given. To obtain asymptotic efficiency, the weight matrix in the quadratic form minimization problem should be chosen to be the asymptotic variance of the sample correlations, the general form of which is given. Misspecification of the model structure and weight matrix is then considered, leading to more complicated distributions of the goodness-of-fit statistics (noncentral χ^2 and mixtures of χ^2_1 's). Other limitations of the ADF estimation methods, such as indomitable sample-size requirements, are discussed and some remedies proposed that might improve the finite sample performance of the goodness-of-fit tests, similar to the way that those corrections operate in classical covariance structure SEMs.

When the weight matrix is misspecified, or computation of the optimal matrix is not feasible (e.g., in large models with dozens of variables), the distribution of the goodness-of-fit test statistics becomes a mixture of χ^2_1 's with potentially unequal weights. Bentler and Savelle provide several ways of approaching that distribution. The weights can be estimated from the existing parameter estimates, leading to a complicated problem of finding the quantiles of the resulting distribution. Satterthwaite-type corrections (known as Satorra–Bentler scaled and adjusted statistics in SEM) can be applied. Or, alternatively, an entirely different statistic with asymptotic χ^2 distribution can be constructed from correlation residuals rather than from the discrepancy functions. Some additional simplification of the analysis is feasible when specific distributions of the data can be assumed. Under normality, more structure can be found in the variance of the sample correlations, and hence the weight matrix of the ADF, leading to an analytically feasible inverse of the weight matrix. The normality assumption can be relaxed somewhat to the assumption of elliptically contoured distributions, and the only modification that needs to be made to the normal theory methods is scaling by the common kurtosis. A further step down from distributional assumptions might be heterogeneous kurtosis theory, which has not yet received much attention in the literature. Bentler and Savelle exemplify their ideas with a classic anthropometric example with eight physical measurement variables and a small simulation. A two-factor model with five variables per factor was used in simulations, and Vale–Maurelly transformation was used to make data nonnormal. It was found that while the ADF method that was applied to either covariance or correlation matrix was overrejecting for most sample sizes, the ADF with structured correlation matrix

mildly underrejected at moderate sample sizes, attaining its asymptotic test size with samples of size 400 and above. It was also found that the residual-based test statistic and residual-based F-statistic behaved rather poorly in unstructured ADF correlation analysis, while the Satorra–Bentler scaled statistic yielded rejection rates closer to the nominal. All statistics improved considerably, however, when a structured weight matrix was used in ADF. Thus, Bentler and Savalei have proposed a number of approaches to the analysis of correlation structures. Some of those proposals are completely new, and most of the others have received very limited attention in the social science statistics literature. It might then be expected that this chapter will provide a fertile field for research that can provide further analytical, simulation and empirical support to Bentler and Savalei's ideas.

Chapter 2 is by Kenneth Bollen, Daneil Bauer, Sharon Christ, and Michael Edwards, who review the area of structural equation modeling. They give a brief review of the history of the field, introduce the basic ideas and notation, and demonstrate how general SEMs specialize to such special cases as simultaneous equation models in econometrics, multiple regression and ANOVA, and confirmatory factor analysis. In general, structural equation modeling would proceed by specifying the model, computing the implied moment matrices, establishing identification, estimating parameters and assessing the fit of the model, with additional respecification if the model fits poorly. Bollen et al. consider those steps one by one, briefly discussing the procedures commonly used. They present the maximum likelihood estimator, the two-stage instrumental variable estimator, and the least-squares/asymptotically distribution-free estimator commonly used by applied researchers. Having outlined the general modeling steps, Bollen et al. proceed to discuss several recent extensions. One of them is the hybrid structural equation and multilevel modeling. A common way to analyze multilevel SEM is to specify a parametric model for both within- and between-group covariance matrices, which makes it possible to model the contextual and individual effects, just as in traditional linear multilevel models. Moreover, not only the parameter values, but even the factor structure, can be specified differently for the within- and between-group parts of the model. Another view of multilevel SEM is to explicitly specify the higher-level random effects as latent factors. Similar ideas have long been used in growth curve modeling, and the synthesis with SEM has been proposed recently, as reviewed in Section 2.2.

Another hybrid type of modeling arises when the structural equation models are crossed with latent class models, giving rise to structural equation mixture models. Those models are also related to the item response theory models operating on discrete outcomes, to growth mixture models, and to nonparametric maximum likelihood estimators of SEM that do not specify distribution of the latent variables but, rather, estimate it. Bollen et al. further discuss the issues of identification and sensitivity to assumptions, which become more acute in those more complicated models. A number of interesting applications are considered, from direct class modeling to semi-parametric nonlinearity modeling and semiparametric modeling of the latent variable distributions. In the next section of the chapter they review the relation of SEM to item response models, some forms of which can be cast as confirmatory factor analysis with categorical variables. Complications arising from the discrete nature of the

data are discussed, and estimation methods reviewed. The last extension of the SEM discussed by Bollen et al. is to complex samples. An overview of the basic complex sample design features, such as clustering, stratification, and unequal probabilities of selection, is given. It is shown how those features violate the model-based SEM assumption, and how estimation procedures are then affected. Sample weights are motivated through a Horvitz–Thompson estimator of a total. An applied researcher can then proceed by attempting to model the sample design with, say, random effects for clusters and categorical variables for strata; or one can use estimation procedures that correct for the complex survey design, such as weighted estimation and pseudo-maximum likelihood. Special care should be taken to estimate the variances of the parameter estimates properly, through either a sandwich-type estimator or through appropriate survey design resampling schemes.

In **Chapter 3**, Lawrence Hubert, Hans-Friedrich Köhn, and Douglas Steinley discuss strategies for the hierarchical clustering of an object set to produce a sequence of nested partitions in which object classes within each successive partition are constructed from the union of classes present at the previous level. In turn, any such sequence of nested partitions can be characterized by what is referred to as an ultrametric, and conversely, any ultrametric generates a nested collection of partitions. There are three major areas of concern in this paper: (1) the imposition of a given fixed order, or the initial identification of such a constraining order, in constructing and displaying an ultrametric; (2) extensions of the notion of an ultrametric to use alternative collections of partitions that are not necessarily nested but which do contain objects within classes consecutive with respect to some particular object ordering. A method for fitting such structures to a given proximity matrix is discussed along with an alternative strategy for graphical representation; (3) for the enhanced visualization of additive trees, the development of a rational method of selecting a root by imposing some type of order-constrained representation on the ultrametric component in a decomposition of an additive tree (nonuniquely into an ultrametric and a centroid metric). A simple numerical example will be used throughout the paper based on a data set characterizing the agreement among the Supreme Court Justices for the decade of the Rehnquist Court. All the various MATLAB M-files used to illustrate the extensions are available as open-source code from a web site given in the text.

In **Chapter 4**, Michael Brusco, Stephanie Stahl, and Dennis Cradit discuss using multidimensional scaling (MDS) in the city-block metric, an important tool for representing the psychological space associated with separable stimuli. When two or more proximity matrices are available for the same set of stimuli, the development of a city-block MDS structure that fits each of the matrices reasonably well presents a challenging problem that might not be solved by pooling the data (e.g., averaging) across matrices. These authors present a multiobjective programming approach for multidimensional city-block scaling of multiple proximity matrices. The multiobjective function of the model is composed of either weighted least-squares loss functions or, in cases where nonmetric relaxations of the proximities are desired, weighted stress functions. The multiobjective function is optimized subject to constraints on the permutation of the objects on each dimension. Because there are well-noted problems with gradient-based approaches for city-block MDS, a combinatorial heuristic proce-

ture is proposed for solving the multiobjective programming city-block model. The model is demonstrated using empirical data from the psychological literature.

In **Chapter 5**, Jeff Gill compares Bayesian and frequentist approaches to estimation and testing of social science theories. He first argues why fixing the data and conditioning on them, as is done in Bayesian statistics, is a reasonable starting point in social sciences: indeed, the repeated sampling necessary to justify the frequentist paradigm is hardly feasible with constantly changing social and human environments. Upon providing the mechanics of Bayes theorem and Bayesian inference, he considers a small example with count data, and demonstrates graphically the process of prior updating. The choice of the prior is further provided. Differences in how the models are set up, and how analysis then proceeds and inference is conducted, are highlighted between the Bayesian and the frequentist paradigms, with somewhat provocative comparisons between the two paradigms and dominant data analysis standards. Then Gill reviews the existing approaches to hypothesis testing and shows step-by-step procedures in the Fisher paradigm, Neyman–Pearson paradigm, Bayesian paradigm, and the null hypothesis significance testing paradigm. Gill’s argument against the latter is supported by several dozen references in statistics and social and behavioral sciences. He then comes back to the counts example and shows an extension of his analysis to a (rather difficult, in any paradigm) problem of change-point estimation. He shows how a Gibbs sampler can be set up for this problem by explicitly specifying the full conditional distributions, and how convergence of the resulting Markov chain can be established. He then reviews the substantial results and notes that the Bayesian estimates of the change point are the probable cause of the change. He concludes by highlighting again the critical differences between Bayesian and frequentist paradigms, and provides philosophical considerations for the former.

In **Chapter 6**, Jeff Rouder, Paul Speckman, Douglas Steinley, Michael Pratte, and Richard Morey show how the shape of a response-time distribution provides valuable clues about the underlying mental processing. If a manipulation affects the shape of an RT distribution, it is reasonable to suspect that the manipulation has done more than simply speed or slow the rate of processing. They develop a nonparametric bootstrap test of shape invariance. Simulations reveal that the test is sufficiently powered to detect small shape changes in reasonably sized experiments while maintaining appropriate type I error control. The test is simple and can be applied broadly in cognitive psychology. An application to a number priming experiment provides a demonstration of how shape changes may be detected.

In **Chapter 7**, Joseph Hilbe outlines the standard computer programs used for statistical analysis and emphasizes those that should get more use. The packages include R, SAS, SPSS, STATISTICA, Stata, StatXact/LogXact, Stat/Transfer, ePrint Professional, and nQueary Advisor.

The 2006 Winemiller Conference and this book would not have been possible without the generous support of Albert Winemiller, whom we would like to thank for his interest in cultivating the integration of mathematical statistics and social science at both the theoretical and applied levels. We would also like to thank the Department of Statistics (and its Chair, Dr. Nancy Flournoy) for sponsoring the conference and

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CONTENTS

List of Figures	ix
List of Tables	xi
Preface	xiii
1 Analysis of Correlation Structures: Current Status and Open Problems	1
1.1 Introduction	1
1.2 Correlation versus Covariance Structures	2
1.3 Estimation and Model Testing	5
1.3.1 Basic Asymptotic Theory	5
1.3.2 Distribution of T Under Model Misspecification	6
1.3.3 Distribution of T Under Weight Matrix Misspecification	7
1.3.4 Estimation and Testing with Arbitrary Distributions	8
1.3.5 Tests of Model Fit Under Distributional Misspecification	12
1.3.6 Scaled and Adjusted Statistics	14
1.3.7 Normal Theory Estimation and Testing	15
1.3.8 Elliptical Theory Estimation and Testing	17
1.3.9 Heterogeneous Kurtosis Theory Estimation and Testing	19
	v

1.3.10 Least Squares Estimation and Testing	21
1.4 Example	22
1.5 Simulations	24
1.5.1 Data	24
1.5.2 Correlation Structure with ADF Estimation and Testing	25
1.5.3 Correlation Structure with Robust Least Squares Methods	26
1.6 Discussion	27
References	28

2 Overview of Structural Equation Models and Recent Extensions 37

2.1 Model Specification and Assumptions	39
2.1.1 Illustration of Special Cases	39
2.1.2 Modeling Steps	41
2.2 Multilevel SEM	47
2.2.1 The Between-and-Within Specification	47
2.2.2 Random Effects as Factors Specification	49
2.2.3 Summary and Comparison	53
2.3 Structural Equation Mixture Models	53
2.3.1 The Model	54
2.3.2 Estimation	56
2.3.3 Sensitivity to Assumptions	56
2.3.4 Direct and Indirect Applications	58
2.3.5 Summary	59
2.4 Item Response Models	59
2.4.1 Categorical CFA	60
2.4.2 CCFA Estimation	61
2.4.3 Item Response Theory	62
2.4.4 CCFA and IRT	63
2.4.5 Advantages and Disadvantages	64
2.5 Complex Samples and Sampling Weights	65
2.5.1 Complex Samples and Their Features	65
2.5.2 Probability (Sampling) Weights.	67
2.5.3 Violations of SEM Assumptions	68
2.5.4 SEM Analysis Using Complex Samples with Unequal Probabilities of Selection	69
2.5.5 Future Research	72
2.6 Conclusion	73
References	73

3	Order-Constrained Proximity Matrix Representations	81
3.1	Introduction	81
3.1.1	Proximity Matrix for Illustration: Agreement Among Supreme Court Justices	83
3.2	Order-Constrained Ultrametrics	84
3.2.1	The M-file <code>ultrafnd.confim.m</code>	85
3.2.2	The M-file <code>ultrafnd.confnd.m</code>	87
3.2.3	Representing an (Order-Constrained) Ultrametric	88
3.2.4	Alternative (and Generalizable) Graphical Representation for an Ultrametric	91
3.2.5	Alternative View of Ultrametric Matrix Decomposition	93
3.3	Ultrametric Extensions by Fitting Partitions Containing Contiguous Subsets	95
3.3.1	Ordered Partition Generalizations	104
3.4	Extensions to Additive Trees: Incorporating Centroid Metrics	106
	References	111
4	Multiobjective Multidimensional (City-Block) Scaling	113
4.1	Introduction	113
4.2	City-Block MDS	115
4.3	Multiobjective City-Block MDS	116
4.3.1	The Metric Multiobjective City-Block MDS Model	116
4.3.2	The Nonmetric Multiobjective City-Block MDS Model	118
4.4	Combinatorial Heuristic	119
4.5	Numerical Examples	121
4.5.1	Example 1	121
4.5.2	Example 2	124
4.6	Summary and Conclusions	128
	References	130
5	Critical Differences in Bayesian and Non-Bayesian Inference	135
5.1	Introduction	135
5.2	The Mechanics of Bayesian Inference	137
5.2.1	Example with Count Data	139
5.2.2	Comments on Prior Distributions	141
5.3	Specific Differences Between Bayesians and non-Bayesians	142
5.4	Paradigms For Testing	143
5.5	Change-point Analysis of Thermonuclear Testing Data	148