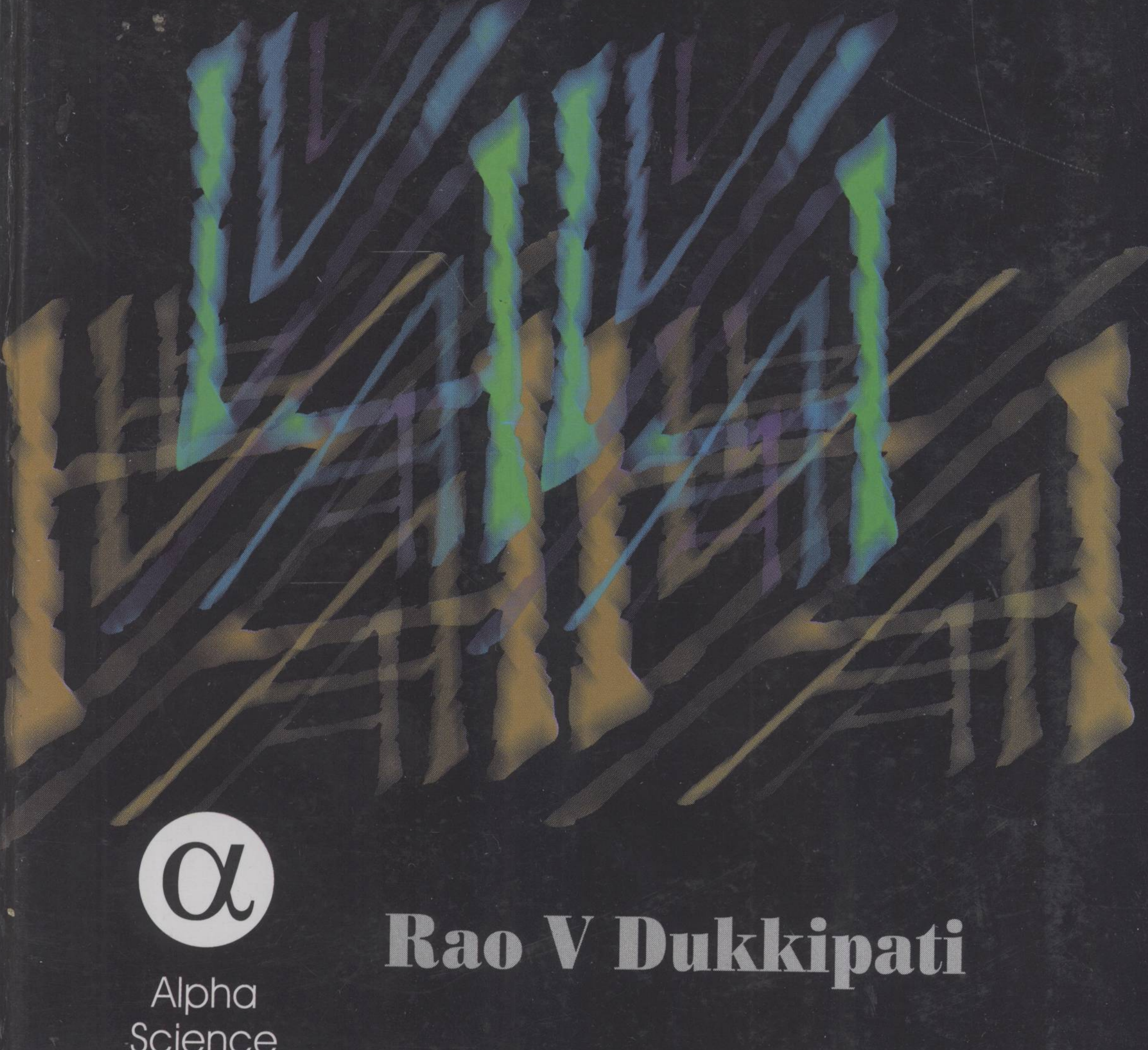
Vibration Analysis



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Alpha Science

VIBRATION ANALYSIS

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VIBRATION ANALYSIS

Dedicated to my mother: Annapurnamma Dukkipati

the memory of my father: Nagabhushanam Dukkipati [1896-1990]

and

my dearest loving son: Raghu Nag Dukkipati [1975-1992]

Preface

Introduction

Engineering is basically an application of mathematics and sciences to the solution of real world problems. The purpose of this book is to impart a basic understanding, both physical and mathematical, of the fundamentals of the theory of vibration and its applications. In this book, an attempt is made to present in a simple and systematic manner techniques that can be applied to the analysis of vibration of mechanical or structural systems. An emphasis is placed on analytical developments and computational solutions. Certain related but necessary material was included in order to render the book self-contained, and hence suitable for self-study. Abundant number of examples and homework problems, as well as programs written in MATLAB is provided.

The material of this book which is an outgrowth of class notes which I have used for the past thirty years of teaching undergraduate and graduate courses in mechanical engineering at Concordia University, Montreal, McGill University, Montreal, The University of Windsor, Windsor, Algonquin College, Ottawa, Carleton University, Ottawa, The University of Toledo, Toledo, Ohio, and Fairfield University, Fairfield, Connecticut. Since I intended this book as a first course on the theory of vibration, the concepts have been presented in simple terms and the solution procedures have been explained in detail.

Audience

This book is a comprehensive text on mechanical vibrations analysis and control systems. The analyses of vibrations of systems and the dynamics of control systems are closely linked; there is a lot to be gained by studying these two topics together in a single text. It is self-contained and the subject matter is presented in an organized and systematic manner. No previous knowledge of dynamics and vibrations is assumed. This book is quite appropriate for several groups of people including:

- Senior undergraduate and graduate students taking the course mechanical vibrations.
- The book can be adapted for a short professional course on the subject matter.
- Design and research engineers will be able to draw upon the book in selecting and developing mathematical models for analytical and design purposes.
- Practicing engineers and managers who want to learn about the basic principles and concepts involved in vibration analysis and how it can be applied at their own work place concerns.

Because the book is aimed at a wider audience, the level of mathematics is kept intentionally low. All the principles presented in the book are illustrated by numerous worked examples. The book draws a balance between theory and practice.

Contents

Books differ in content and organization. I have striven hard in the organization and presentation of the material in order to introduce the student gradually the concepts and in their use to problems in vibrations. The subject of mechanical vibrations deals with the methods and means of formulation of mathematical models of physical systems and discusses the methods of solution. In this book, I have concentrated on both of these aspects: the tools for formulating the mathematical equations and also the methods of solving them.

The study of vibrations is a formidable task. Each chapter in this book consists of a concise but thorough fundamental statement of the theory; principles and methods, followed by a selected number of illustrative worked examples. A number of sample unsolved exercise problems for student's practice, to amplify and extend the theory are also included. Bibliography provided at the end of the book serves as helpful sources for further study and research by interested readers.

Chapter 1 gives a review of the basics of Newtonian dynamics starting with the kinematics and dynamics of a particle and rigid body in plane motion. The abstract concepts of analytical dynamics including the degrees of freedom, generalized coordinates, constraints, principle of virtual work and D'Alembert's principle for formulating the equations of motion for systems are introduced. Energy and momentum from both the Newtonian and analytical point of view are presented. Dynamics of rigid bodies is also dealt with in this chapter. The basic concepts and terminology used in mechanical vibration analysis, classification of vibration and elements of vibrating systems are discussed. The free vibration analysis of single degree of freedom undamped translational and torsional systems are presented in Chapter 2. The concept of damping in mechanical systems, including viscous, structural, and Coulomb damping are introduced. In Chapter 3, the response to harmonic excitations is presented in frequency domain, through magnitude and phase angle frequency response plots. The approach to the response of systems to harmonic excitations is extended to periodic excitations through the use of Fourier transforms. Chapter 3 also discusses the application such as systems with rotating eccentric masses, systems with harmonically moving support and vibration isolation. Chapter 4 is concerned with the response of a single degree of freedom system under general forcing functions. Methods discussed include Fourier series, the convolution integral, Laplace transform, and numerical solution. In Chapter 5, the linear theory of free and forced vibration of two degree of freedom systems is presented. Matrix methods are introduced to make this chapter a foundation for the systems with multiple degrees of freedom to be studied in Chapter 6. Coordinate coupling and principle coordinates, orthogonality of modes, and beat phenomenon are also discussed. Matrix methods of analysis are described for the presentation of the theory. The modal analysis procedure is used for the solution of forced vibration problems. In Chapter 7, a brief introduction to Lagrangian dynamics is presented. Using the concepts of generalized coordinates, principle of virtual work, and generalized forces, Lagrange's equations of motion are then derived for single and multi degree of freedom systems in terms of scalar energy and work quantities. Chapter 8 is an introduction to the theory of vibration of continuous systems. The vibration analysis of continuous systems, including strings, bars, shafts, beams, and membranes are presented. Approximate methods for distributed parameter systems including the Rayleigh's principle, and the Rayleigh-Ritz method are presented in Chapter 8. There are several methods available for finding the natural frequencies and mode shapes of undamped vibrating systems. Dunkerley's equation, Rayleigh method, Rayleigh-Ritz method, Holzer method, Jacobi diagonalisation method, Cholesky decomposition method, and iteration methods, which include the matrix iteration method, inverse matrix method, simultaneous iteration method, and the subspace iteration method were presented in this chapter.

Chapter 10 presents several direct numerical integration techniques for finding the dynamic response of discrete and continuous systems. The central difference, Runge-Kutta, Houbolt, Wilson Theta, Newmark Beta methods are summarized and illustrated.

Chapter 11 presents MATLAB basics including commands. MATLAB is considered as the software of choice. MATLAB can be used interactively and has an inventory of routines, called as functions, which minimize the task of programming even more. Further information on MATLAB can be obtained from: The MathWorks, Inc., 3 Apple Hill Drive, Natick, MA 01760. Chapter 12 covers a large number of MATLAB tutorial problems on vibration analysis. In addition, a great number of MATLAB problems are included. The MATLAB material in this book is designed to enhance the study of vibration analysis. MATLAB use and application is highly recommended.

I sincerely hope that the final outcome of this book helps the students in developing an appreciation for the topic of mechanical vibrations.

A basic review of Vector Algebra, Matrix Algebra, Fourier Series, and Laplace Transforms are outlined respectively, in Appendices A, B, C, and D. This book contains SI units and conversion of SI and US/English units are presented in Appendix E, followed by Glossary of Terms, Glossary of Symbols, Bibliography, and Answers to Problems. All end-of chapter problems are fully solved in the Solution Manual available only to Instructors.

Notation and Units

Both the SI and the US/English system of units have been used throughout the book.

Acknowledgements

I am grateful to all those who have had a direct impact on this work. Many people working in the general areas of design, dynamics, and vibrations have influenced the format of this book. I am indebted to my colleague, Dr. J. Srinivas and to numerous authors who have made contributions to the literature in this field. I would also like to thank and recognize all the graduate students in the Master of Science in Management of Technology (MSMOT) program and all the undergraduate mechanical engineering students at Fairfield University over the years with whom I had the good fortune to teach and work and who contributed in some ways and feedback to the development of the material of this book. In addition, I greatly owe my indebtedness to all the authors of the articles listed in the bibliography of this book. I also like to thank the reviewers for their efforts and for the comments and suggestions, which have well served to compile the best possible book for the intent and targeted audience. I gratefully acknowledge the continuous support and encouragement given to me by all my colleagues at the School of Engineering, Fairfield University. Finally, I would very much like to acknowledge the encouragement, patience, and support provided by my family members: my wife, Sudha, my family members, Ravi, Madhavi, Anand, and Ashwin who have also shared in all the pain, frustration, and fun of producing a manuscript.

I would appreciate being informed of errors, or receiving other comments about the book. Please write to the Authors' Fairfield University address or send e-mail to Rdukkipati@mail.fairfield.edu.

Rao V. Dukkipati

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1

Introduction

1.1 Introduction

Vibration is the motion of a particle or a body or system of connected bodies displaced from a position of equilibrium. Most vibrations are undesirable in machines and structures because they prôduce increased stresses, energy losses, cause added wear, increase bearing loads, induce fatigue, create passenger discomfort in vehicles, and absorb energy from the system. Rotating machine parts need careful balancing in order to prevent damage from vibrations.

Vibration occurs when a system is displaced from position of stable equilibrium. The system tends to return to this equilibrium position under the action of restoring forces (such as the elastic forces, as for a mass attached to a spring, or gravitational forces, as for a simple pendulum). The system keeps moving back and forth across its position of equilibrium. A *system* is a combination of elements intended to act together to accomplish an objective. For example, an automobile is a system whose elements are the wheels, suspension, car body, and so forth. A *static* element is one whose output at any given time depends only on the input at that time while a *dynamic* element is one whose present output depends on past *inputs*. In the same way we also speak of *static* and *dynamic systems*. A *static system* contains all elements while a *dynamic system* contains at least one dynamic element.

A physical system undergoing a time-varying interchange or dissipation of energy among or within its elementary storage or dissipative devices is said to be in a *dynamic state*. All of the elements in general are called *passive*, i.e., they are incapable of generating net energy. A dynamic system composed of a finite number of storage elements is said to be *lumped* or *discrete*, while a system containing elements, which are dense in physical space, is called *continuous*. The analytical description of the dynamics of the discrete case is a set of ordinary differential equations, while for the continuous case it is a set of partial differential equations. The analytical formation of a dynamic system depends upon the kinematic or geometric constraints and the physical laws governing the behaviour of the system.

1.2 Classification of Dynamic System Models

In order to deal in an efficient and systematic way with problems involving time dependent behaviour, we must have a description of the objects or processes involved and such a description is called a *model*. The model used most frequently is the *mathematical model*, which is a description in terms of mathematical relations, and represents an idealization of the actual physical system. For describing a dynamic system, these relations will consist of differential or difference equations. Predicting he performance from a model is called *analysis*. The model's purpose partly determines its form so that the purpose influences the type of analytical techniques used to predict the dynamic system's behaviour. There are many types of analytical techniques available and their applicability depends on the purpose of the analysis. The physical properties, or characteristics, of a dynamic system are known as *parameters*. In general, real systems are continuous and their parameters distributed. However, in most cases, it is possible to replace the distributed characteristics of a system by discrete ones. In other words, many variables in a physical system are functions of location as well as time. If we ignore the spatial dependence by choosing a single representative value, then the process is called *lumping*, and the model of a lumped element or system is called a lumped-parameter model. In a dynamic system the independent variable in the model then would be time only. The model will be an ordinary differential equation, which includes time derivatives but not spatial derivatives. If spatial dependence is included then the resulting model is known as a distributed-parameter model in which the independent variables are the spatial coordinates as well as time. It consists of one or a set of partial differential equations containing partial derivatives with respect to the independent variables. Discrete systems are simpler to analyse than distributed ones.

Vibrating systems are classified according to their behaviour as either *linear* or *non-linear*. If the *dependent variables* in the system differential equation(s) appear to the first power only and there are no cross products thereof, then the system is called *linear*. If there are fractional or higher powers, then the system is *non-linear*. On the other hand, if the systems contain terms in which the independent variables appear to powers higher than one or two fractional powers, then they are known as *systems with variable coefficients*. Thus, the presence of a time varying coefficient does not make a model non-linear. Models with constant coefficients are known as *time-invariant* or *stationary models*, while those with variable coefficients are *time-variant* or *non-stationary*. Fig.1.1 shows the relationships between the various model types. If there is uncertainty in the value of the model's coefficients or inputs then often, *stochastic* models are used. In a stochastic model, the inputs and coefficients are described in terms of probability distributions involving their means and variances, etc.

1.3 Constraints, Generalized Coordinates, and Degrees of Freedom

The position of a system of particles is called its *configuration*. Usually, because of constraints on the system, actual coordinates need not be assigned to each particle. In a dynamic system, constraints may be at its boundary or at points internal to the system. Constraints may be either static or kinematic in nature. The static constraints result from relationships among forces, whereas the kinematic constraints are due to the relationships among displacements. In selecting the coordinates to describe a dynamic system, the static and kinematic constraints must be considered. Relationships among coordinates, which exist because of constraints on the system, are termed *constraint equations*. Based on this discussion, it can be said that systems of *unconstrained* or *independent* coordinates exist. In general, this is true in dynamic systems, and such a system can be described by a system of constrained coordinates. As an example, we can consider a dynamic system that is defined in terms of M coordinates. If there are R constrained displacements, then R coordinates can be expressed in terms of the remaining M-R coordinates, which are independent. Thus, if

$$N = M - R$$

N is the number of independent coordinates, and the forces and displacements are fully defined by these N coordinates. The independent coordinates required to specify completely the configuration of a dynamic system are called *generalized coordinates*. IT is assumed that the generalized coordinates may be varied arbitrarily and independently without violating the constraints. Such a dynamic system is called a *holonomic system*. The number of generalized coordinates is called the *number of degree of freedom* of a dynamic system.

To illustrate a dynamic system with constant, we consider a rigid body attached to a point that is constrained to translate in the y direction, as shown in Fig.1.2. In three-dimensional space, five coordinates, i.e., two translations, one each along the x and z-axes, and three rotations would describe the motion of the rigid body about the x, y, and z axes, respectively. In this case, the number of degrees of freedom for the system is five. Let us suppose that the rigid body is further constrained and that it undergoes motion in the x-y plane only, as shown in Fig.1.3. The rigid body in a planar motion configuration would require two degrees of freedom to describe its motion. These degrees of freedom would correspond to the translation along x-axis and the rotation about the z-axis.

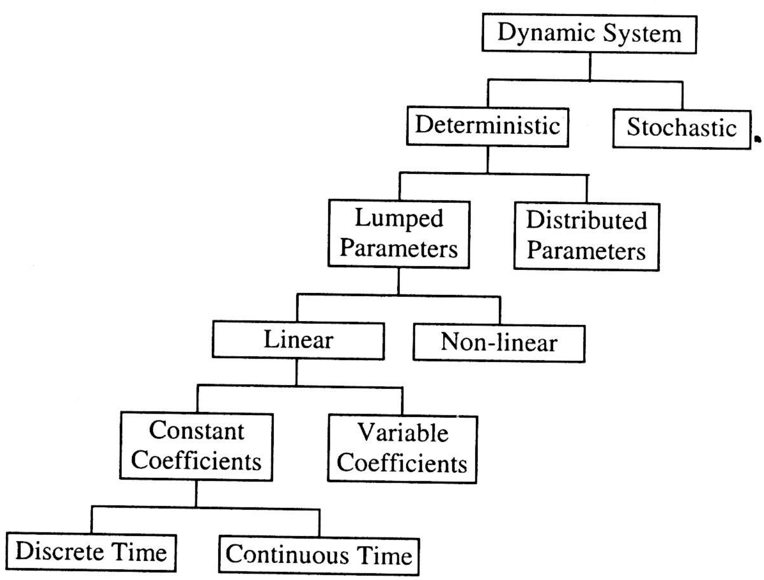

Fig. 1.1 Classification of dynamic system mathematical models

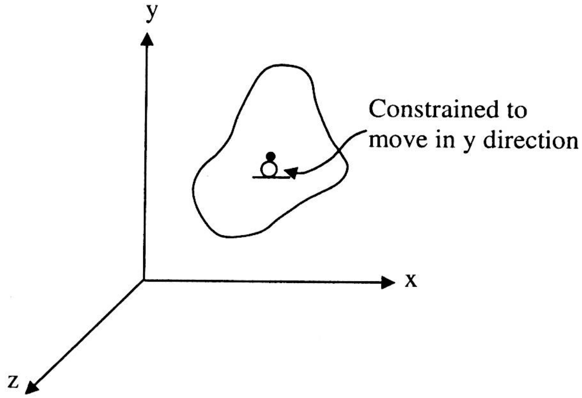

Fig. 1.2 Rigid body in general motion (five degree of freedom)

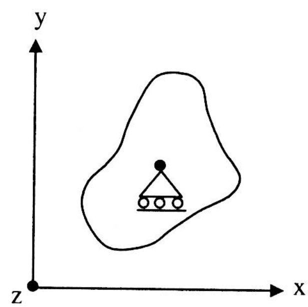

Fig. 1.3 Rigid body in planar motion (two degree of freedom)

The number of *degrees of freedom* of a vibratory system is the number of *independent spatial coordinates* necessary to define its *configuration*. A *configuration* is defined as the geometric location of all the masses of the system. A rigid body in space requires six coordinates for its complete identification: three coordinates to define the rectilinear positions and three for the angular positions. Generally speaking, the masses in a system are constrained to move only in a certain manner. Thus, the *constraints* limit the degrees of freedom of a system. The independent spatial coordinates describing the configuration of a dynamic system are also called generalized coordinates. Thus the number of degrees of a dynamic system is equal to the number of *generalized coordinates*. Alternatively, the number of degrees of freedom of a system can be defined as the number of spatial coordinates required, to completely specify its configuration minus the number of *equations of constraint*. Such a system is called a *holonomic* system.

Energy enters a dynamic system through the application of an excitation. The excitation varies in according wit h a prescribed function of time. The vibratory behaviour of dynamic system is characterized by the motions caused by these excitations and is referred to as the system *response*. The response motion is