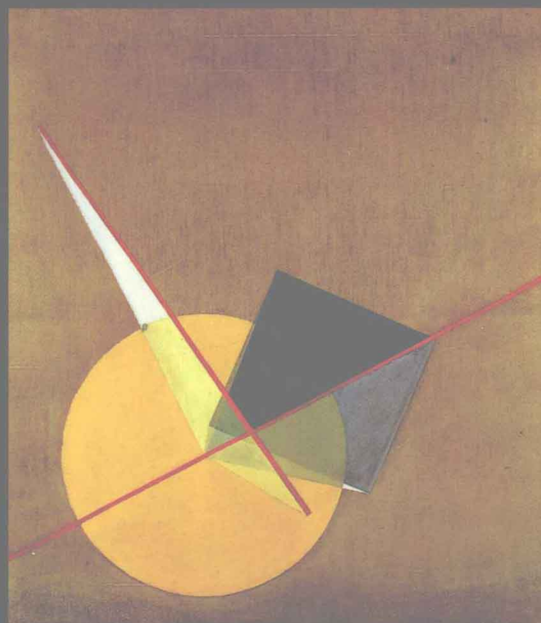
BRIEF CALCULUS *WITH* APPLICATIONS



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BRIEF CALCULUS WITH APPLICATIONS

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BRIEF CALCULUS WITH APPLICATIONS

To Cathy and "The Boy"—C.V.
To Jan and "The 3 K's"—C.S.

PREFACE

This book is designed to present the fundamentals of calculus to students in the management, life, social, and physical sciences. It is suitable for either a one-semester or two-quarter course. Its only prerequisite is two years of high-school algebra or a course in intermediate algebra.

APPROACH

Our objective has been a text that students will understand and instructors will find easy to use. Four years of class-testing versions of this text have helped us achieve that aim. They have shown us how to develop topics in a progression that students and teachers will find natural. Our approach is geometric and intuitive; although skills play an important role in the text, and applications to a wide variety of fields appear frequently, there is a significant emphasis on concepts.

We have been precise throughout, and important theorems are always proved or justified. Yet students should find the writing style friendly and informal. And we use figures extensively to illustrate, justify, and test the concepts.

Our approach can mean leaving out some necessary condition of a theorem. In Theorem 5.1 on page 194, for example, the hypothesis should really state that the function is differentiable on an open interval containing a, an interior point. But all of the functions in the text to which we apply the theorem satisfy that condition. So we have been brief and simple to decrease the odds that students will miss the forest for the trees. Our goal was to ensure that students have more than a reasonable chance of understanding and applying the mathematics.

ORGANIZATION OF THE TEXT

Our experience indicates that the organization of a text can make it easier to use for both students and instructors. We have given considerable attention to achieving a clear, natural topic sequence.

Chapter 1 contains the topics from college algebra that are needed for the successful study of calculus. It places heavy emphasis on graphs, which are central to the entire text. This chapter requires students to write equations involving specified variables and to write one quantity as a function of another. These skills are the foundation of related rates and maximum–minimum problems, which appear later. Exponential and logarithmic functions are introduced in Chapter 1, rather than in an isolated chapter later as in other texts, so that they can be included wherever appropriate. They are preceded by coverage of exponential and logarithmic equations, because these equations may be used to find the x-intercepts of the functions.

Chapter 2 focuses on the important concepts of limit, continuity, and the tangent line. The chapter relies heavily on graphs so that students understand these basic concepts, which other texts at this level often slight.

Chapter 3 begins by discussing the derivative, relying on the discussion of tangent lines in Chapter 2. The emphasis is on its interpretation as an instantaneous rate of change. The rest of the chapter is devoted to formulas for differentiation. We present the chain rule before the product and quotient rules; this approach increases the power of the latter and provides a convenient proof of the quotient rule.

Chapter 4 is arranged for maximum flexibility. Only the first section, on higher-order derivatives, is required later in the text. The rest of the chapter, which covers implicit differentiation and related rates, may be omitted or covered at any time.

Chapter 5 centers on two important applications of the derivative: curve sketching and maximum–minimum problems. The chapter emphasizes concepts as well as analytic skills and makes considerable use of graphs. Extra attention is given to determining whether a relative extreme point is absolute, an important detail omitted in many other texts.

Chapter 6 begins the discussion of integration. It defines the definite integral as the net change in an antiderivative, not as the limit of a Riemann sum. We believe that this makes more sense to students—who, of course, later see the connection between the two. All theorems concerning integrals are then tied to the corresponding theorems for derivatives in Chapter 3.

Chapter 7 covers topics related to integration. Included is an important application, the laws of growth and decay.

Chapter 8 briefly introduces the calculus of several variables. It draws heavily on the student's understanding of the calculus of one variable, noting parallels wherever possible.

PEDAGOGICAL AIDS

Numerous examples, exercises, and problems in each chapter reinforce student comprehension:

- Exercises—with answers—integrated into the narrative allow students to test their understanding. The exercises, which are always similar to preceding examples, may also be used as additional classroom illustrations.
- Sections then conclude with problems, carefully written to provide practice in skills, concepts, and applications. Most of the time, problems are matching pairs, and answers to the odd-numbered ones (excluding proofs) appear in a separate section at the end of the text.
- Further review problems span the chapter, beginning with true–false questions. Answers to the entire chapter review also appear at the end of the book.

Other pedagogical aids help bring out important concepts and skills:

- Figures play a major role in the text, as well as in the problem sets.
- Important results are always clearly set off.
- Students needing a quick review of intermediate algebra will benefit from the Topics for Review at the end of the text.

ANCILLARIES

The following ancillaries are available free to adopters:

- An *Instructor's Solutions Manual,* by the authors, provides instructors with answers to all even-numbered problems and step-by-step solutions to all proofs.
- *CalcTest,* an algorithm-based, computerized test generator, offers instructors a set of core questions for class testing. It allows selection of a large number of numerical variations on approximately 80 standard questions taken from the text.
- A *Test Bank,* based on CalcTest, shows hard copy of two versions of the questions for each chapter, along with student answer sheets and solutions.

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CONTENTS

CHAPTER 1

GRAPHS AND FUNCTIONS

CHAPTER OUTLINE

This chapter deals with some introductory ideas that are needed for a successful study of calculus. The first two sections focus on equations in two variables and their graphs. Section 1.3 follows, covering one of the most important topics in mathematics—the concept of functions. The remainder of the chapter introduces many graphs that are used throughout the text.

1.1 EQUATIONS IN TWO VARIABLES AND THEIR GRAPHS

In many applications of calculus, problems are stated in words, and one must produce a correct equation before proceeding to a solution. In the first part of this section we concentrate on writing the equation only.

In these problems we are told which algebraic symbols represent some of the unknown quantities. Upon finding a statement of equality, we may wish to introduce some additional variables. Then, using the symbols, we translate the statement of equality into an equation. If the statement is correctly translated, we should be able to identify what each term (or group of terms) represents in the problem. Writing the statement of equality in English before attempting to write the equation helps produce a correct equation. Finally, if necessary, we must substitute for any of the extra variables that we introduced.

It is essential that you *understand the problem*. Read the problem as many times as necessary to comprehend the information given. Draw a picture if helpful. If you introduce a variable not specified in the problem, define it. If the variable represents a measurable quantity, such as distance, length, or time, identify the units of measurement.

EXAMPLE 1

Ship A is steaming north at 10 miles per hour (mph), and ship B is steaming east at 15 mph. At 2:00 ship B is 5 miles due east of ship A. Let d represent the distance (in miles) between ships, and let t represent the elapsed time (in hours) since 2:00. Write an equation involving d and t only.

■ *SOLUTION*

We have the situation shown in Figure 1.1. We have let

$$x = \text{distance traveled by } B \text{ since 2:00 (in miles)},$$

$$y = \text{distance traveled by } A \text{ since 2:00 (in miles)}.$$

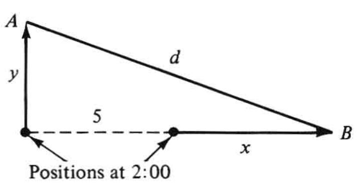


FIGURE 1.1

Note that a right triangle has been formed. The lengths of the two legs are y and $x + 5$, and the hypotenuse is d. By the Pythagorean theorem,

$$d^2 = y^2 + (x + 5)^2.$$

We have an equation that contains d but not t. To eliminate x and y we use the fact that

$$\text{distance} = \text{rate} \times \text{time}.$$

Since ship A is moving 10 mph we have

$$y = 10t.$$

Because ship B is traveling at 15 mph,

$$x = 15t.$$

Substituting for x and y thus gives the desired equation:

$$\begin{aligned} d^2 &= (10t)^2 + (15t + 5)^2 \\ &= 100t^2 + 225t^2 + 150t + 25 \\ &= 325t^2 + 150t + 25. \end{aligned}$$

EXERCISE

A certain baserunner runs at a top speed of 32 feet/second. The opposing catcher can throw the ball at a speed of 130 feet/second. The runner attempts to steal third base. The catcher (who is 90 feet from third base) throws the ball toward the third baseman when the runner is 30 feet from third base. Let d represent the distance (in feet) between the ball and the runner, and let t represent elapsed time (in seconds) after the catcher's throw. Write an equation involving d and t only.

■ *ANSWER*
$d^2 = (30 - 32t)^2 + (90 - 130t)^2 = 9000 - 25{,}320t + 17{,}924t^2$

We shall now consider equations in the variables x and y only. Examples include

$$2x - y = x + y - 3, \quad y = x^2, \quad \text{and} \quad x^2 + y^2 = 9.$$

The symbolism (a,b) is used to designate an **ordered pair**, a pair of real numbers a and b where the order is important. Thus (1,2), (5,−3), and (2,1) are examples of ordered pairs. And because the order is significant, (1,2) is different from (2,1).

When $a < b$, the notation (a,b) also denotes the open interval from a to b. (See Appendix A.1 for a discussion of intervals.) But within the context of a particular problem, it will be apparent whether the symbolism refers to the open interval or to the ordered pair.

A **solution** of an equation in x and y is an ordered pair (a,b) which makes the equation true when x is replaced by a and y is replaced by b. Hence (2,1) is a solution of $2x + y = 5$, but (1,2) is not.

Given an equation in x and y, we use its solutions to give us a "picture." To do so we must first consider two perpendicular number lines intersecting at their origins. Such a system, as in Figure 1.2, is called a **Cartesian coordinate system.** By convention the number lines are vertical and horizontal with the positive directions up and right. It is useful to have

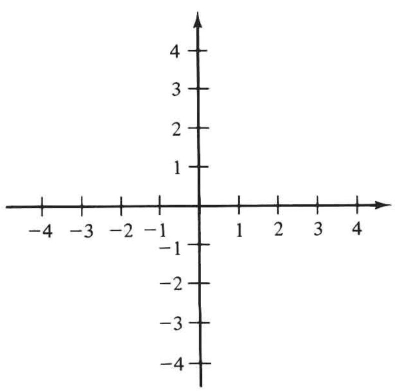


FIGURE 1.2

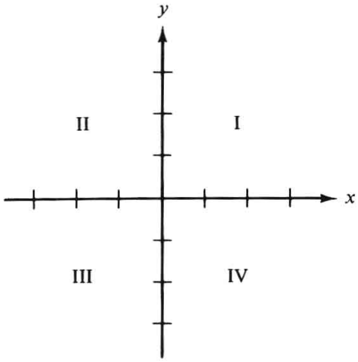


FIGURE 1.3

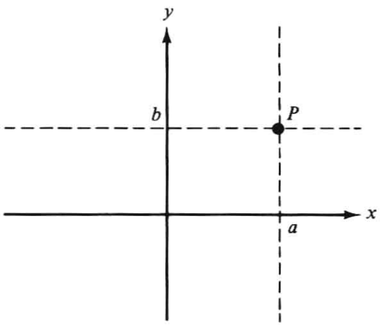


FIGURE 1.4

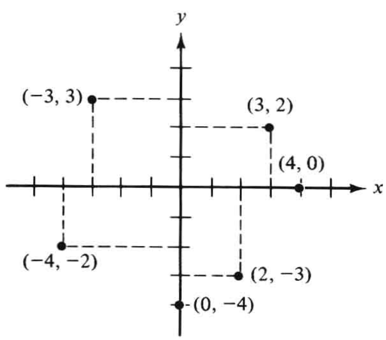


FIGURE 1.5

the unit lengths of each line the same, but this is not always practical or convenient.

In this system each number line is called an **axis.** The horizontal line is called the x-axis, and the vertical line is called the y-axis. The point of intersection is called the **origin.** The plane determined by such a system is called the **Cartesian** or **coordinate plane** or the ***xy*-plane.** The two axes divide the plane into four distinct regions called **quadrants,** numbered I through IV, as indicated in Figure 1.3. The points on the axes are not part of any quadrant.

With this system each point in the plane is associated with a unique ordered pair, and each ordered pair corresponds uniquely to a point in the plane. Given a point P, the ordered pair associated with it can be found as follows. Construct the line passing through P perpendicular to the x-axis. The x-axis coordinate of the point of intersection of the x-axis with the constructed line is the first entry of the ordered pair that corresponds to P. Construct the line passing through P perpendicular to the y-axis. The y-axis coordinate of the point of intersection of the y-axis with the constructed line is the second entry of the ordered pair corresponding to P. In Figure 1.4 we see that the point P is associated with (a,b).

Note that every point on the x-axis is associated with an ordered pair of the form $(a,0)$ for some a, and every point on the y-axis is associated with an ordered pair of the form $(0,b)$ for some b. The ordered pair corresponding to the origin is $(0,0)$. Some points and associated ordered pairs are indicated in Figure 1.5.

Whenever the axis coordinates do not appear, as in Figure 1.5, we assume that the distance between consecutive hash marks is one unit. If the axes are not labeled, we assume that the horizontal axis is the x-axis and that the vertical axis is the y-axis.

Because of the one-to-one relationship that exists between points in the plane and ordered pairs, we frequently interchange the terms. And when we refer to the point $P(a,b)$, we mean that the point is labeled P and its corresponding ordered pair is (a,b). Furthermore, we call the first entry of an ordered pair the ***x*-coordinate** and the second entry the ***y*-coordinate.**

The **graph of an equation** in x and y is the set of points in the xy-plane which are solutions of the equation. Most equations have infinitely many solutions; hence the graph cannot be constructed by plotting them all. We plot a few solutions and connect them with a curve that follows the general trend of the plotted points.

EXAMPLE 2

Graph $y = x + 1$.

■ *SOLUTION*

We begin by constructing a table that lists some of the infinitely many solutions of this equation. We choose some values for x and find the cor-

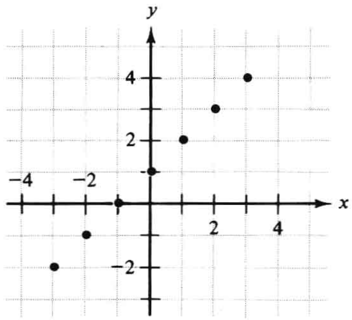
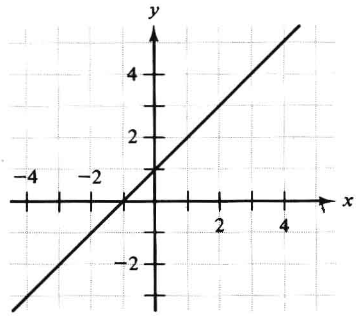


FIGURE 1.6



$y = x + 1$

FIGURE 1.7

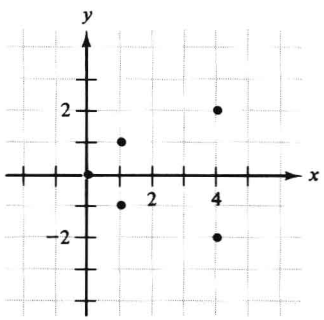
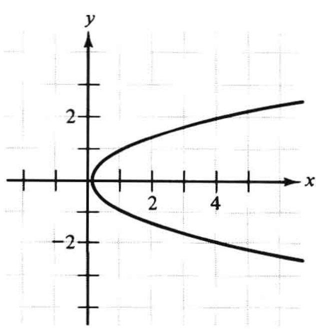


FIGURE 1.8



$x = y^2$

FIGURE 1.9

responding values for y. For example, when $x = -3$, $y = -3 + 1 = -2$. When $x = -2$, $y = -2 + 1 = -1$, and so on. We get the following table:

x	-3	-2	-1	0	1	2	3
y	-2	-1	0	1	2	3	4

This table is simply another way of representing the ordered pairs. Each column refers to an ordered pair, with the entry in the top row being the x-coordinate and the entry in the bottom row being the y-coordinate. The graph of these ordered pairs is shown in Figure 1.6.

Since the solution set is infinite, and these solutions appear to lie along a straight line, we suspect that the graph of $y = x + 1$ is the line determined by these points. See Figure 1.7.

When graphing equations by finding arbitrary solutions, how can we be certain that we have plotted representative points? And how can we be sure that we have connected them properly to give us an accurate sketch? The fact is that we cannot unless we have a general idea of what the graph looks like before we begin. In Sections 1.2, 1.4, and 1.6, we obtain a general idea of the graph by examining the equation. Later, in Section 5.5, we use calculus techniques to sketch the graph. Meanwhile, suggested values for plotting points are included with the problems.

EXAMPLE 3

Graph $x = y^2$.

■ *SOLUTION*

We construct a table that lists some of the solutions by selecting some values for y and finding the corresponding values for x. We get the following:

x	4	1	0	1	4
y	-2	-1	0	1	2

The graph of these ordered pairs is shown in Figure 1.8. Although these points do not lie along a straight line, we can still make a guess about the graph. See Figure 1.9.