



VOLUME FOURTY TWO

# EXPERIMENTAL METHODS IN THE PHYSICAL SCIENCES

## RADIOMETRIC TEMPERATURE MEASUREMENTS

### I. Fundamentals

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Academic Press is an imprint of Elsevier



Academic Press is an imprint of Elsevier  
Linacre House, Jordan Hill, Oxford OX2 8DP, UK  
84 Theobald's Road, London WC1X 8RR, UK  
Radarweg 29, PO Box 211, 1000 AE Amsterdam, The Netherlands  
30 Corporate Drive, Suite 400, Burlington, MA 01803, USA  
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#### Library of Congress Cataloging-in-Publication Data

Radiometric temperature measurements / edited by Zhang, Tsai, and Machin.  
p. cm.

Includes bibliographical references and index.

ISBN 978-0-12-374021-2

1. Temperature measurements. 2. Radiation pyrometers. 3. Radiation-Measurement. I. Zhang, Zhuomin M. II. Tsai, Benjamin K. III. Machin, Graham.

QC271.R336 2009

536'.5-dc22

2009026233

#### British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

ISBN: 978-0-12-374021-2

ISSN: 1079-4042

For information on all Academic Press publications  
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Printed and bound in United States of America

10 11 12 13 14 10 9 8 7 6 5 4 3 2 1


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MEASUREMENTS**

**I. Fundamentals**

# EXPERIMENTAL METHODS IN THE PHYSICAL SCIENCES

Thomas Lucatorto and Albert C. Parr, *Editors in Chief*

*Founding Editors*

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In memory of David P. DeWitt — a pioneer, teacher, and friend

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## PREFACE

Temperature measurement and control has played and continues to play a vital role in many scientific and technological advances. Radiometric temperature measurement, that is the measurement of temperature based on thermal radiative emission, has a long history from fundamental studies of Planckian emission, to many industrial applications including iron and steel production and materials and chemical processing, to playing a fundamental role in the realization and dissemination of successive international temperature scales. Radiation thermometry is attractive in many challenging temperature measurement situations because it is a noncontact, nonintrusive, and fast technique.

Thermal radiation is governed by the fundamental physical laws established over one hundred years ago by Kirchhoff, Stefan, Boltzmann, Wien, and, in particular, Planck. These laws directly link emitted blackbody radiation, total or spectrally resolved, to the thermodynamic temperature of the emitting source. Actual practical measurements by radiation thermometry, however, are prone to a number of uncertainties associated with, for example, surface emissivity and environmental effects such as absorption by dust or smoke and reflected ambient radiation. While a number of books have been published on thermometry, in general, no comprehensive book devoted to radiometric temperature measurement has been published since the publication in 1988 of *Theory and Practice of Radiation Thermometry*, edited by D.P. DeWitt and G.D. Nutter.

In recent years, there have been tremendous developments in instrumentation. For instance, infrared focal plane arrays can now produce images with a spatial resolution of order  $10\text{ }\mu\text{m}$  with a temperature resolution of  $0.01\text{ K}$ . While the expert in the field can keep abreast of these rapidly advancing techniques through the information presented at periodic international temperature symposia and through the technical literature, it is very difficult for a newcomer to find a definitive up-to-date summary of the practice of radiation thermometry. This book aims at filling that gap by covering basic theory, measurement fundamentals, standards and calibration, and summaries of current practice of radiation thermometry in different technical fields at a level accessible to the newcomer but also comprehensive enough to provide the information needed to understand and bring to bear the latest technique to a particular radiometric temperature measurement problem.

This two-volume set on *Radiometric Temperature Measurement* (I. Fundamentals and II. Applications) is written for those who will apply radiation

thermometers in industrial practice, who will use thermometers in scientific research, who design and develop thermometers for instrument manufacturers, and who will design the thermometers to address particular measurement challenges. These volumes are more than a practice guide. We hope that by presenting the fundamental principles and pointing out the pitfalls in applying radiation thermometry in various settings, our readers will gain knowledge in: (1) the proper selection of the type of thermometer; (2) the best practice in using radiation thermometers; (3) awareness of the uncertainty sources and subsequent appropriate procedure to reduce the overall measurement uncertainty; and (4) understanding of the calibration chain and its current limitations. We have also added a large number of references at the end of each chapter as a source for those seeking a deeper or more detailed understanding.

The author(s) of each chapter were chosen from a group of international scientists who are experts in the field and specialist(s) on the subject matter covered in the chapter. It is intended that together the two volumes will form a comprehensive summary of the current practice of radiation thermometry. The first volume concentrates on the fundamental aspects, while the second volume mainly focuses on the industrial and practical applications. In the fundamental volume, Chapter 1 provides a historical overview of radiation thermometry, explains the basic fundamentals and commonly used terms, and lists the various types of radiation thermometers. The concepts of temperature, its scale realization, calibration, traceability, measurement, uncertainty analysis, and future approaches, are extensively elaborated in Chapter 2. The basic theory on blackbody radiation, radiative properties, and the electromagnetic wave theory are discussed in Chapter 3. Chapter 4 focuses on the design and characterization of radiation thermometers. Chapter 5 addresses the theoretical and computational characterization of isothermal and nonisothermal blackbody cavities by analytical and Monte Carlo methods. In Chapter 6, radiance sources used for calibration such as fixed-point blackbodies, variable temperature blackbodies, cryogenic blackbodies, high stability and other tungsten-based lamps are described. Chapter 7 is an overview of some complementary surface temperature measurement techniques, such as thermal reflectance, interferometry, ellipsometry, and photothermal radiometry with application examples.

The volume on applications begins with a review of the state-of-the-art industrial applications of radiation thermometry, including a critique of multiwavelength thermometry (Chapter 1). Chapter 2 describes experimental characterization of blackbody cavities with an extensive survey on the measurement techniques. Chapter 3 focuses on the application of optical fiber thermometry for semiconductor processing, with an emphasis on rapid thermal processing and *in situ* calibration of lightpipe thermometers using thin-film thermocouples. Chapter 4 reviews the

state-of-the-art practice of radiation thermometry in the steel industry, highlighting specific manufacturing processes. Chapter 5 deals with thermal imaging in firefighting and other thermographic applications along with standards of measurement and application. Chapter 6 discusses remote sensing of earth and sea surface temperatures and reviews different instruments and their measuring capabilities. Finally, Chapter 7 covers four aspects of clinical radiation thermometry: ear thermometry, medical thermal imaging, medical pulsed photothermal radiometry, and microwave radiometry for clinical applications.

This two-volume set is a tribute to David DeWitt (1934–2005) who has been an inspiration for us and to many others in the radiation thermometry community. In his last eight years, he dedicated his research to temperature measurement and calibration for rapid thermal processing in microelectronics manufacturing industry. He will always be remembered as a leader in the fields of radiation thermometry and heat transfer engineering.

The editors sincerely thank all of the chapter authors for their outstanding contributions and hard work. We also express appreciation to Dr. Tom Lucatorto and Dr. Albert C. Parr, the series editors, for their constant encouragement during this process and their careful review of the chapter contents. Finally, we would like to thank our families for their full support and enduring patience throughout the writing and editing of this book.

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June 2009

# NOMENCLATURE

$A$	surface area ( $\text{m}^2$ )
$A_n$	projected area ( $\text{m}^2$ )
$A'$	directional absorptance of a semitransparent material
$a_\lambda$	absorption coefficient ( $\text{m}^{-1}$ )
<b>B</b>	magnetic induction or magnetic flux density (T or $\text{Wb}/\text{m}^2$ )
$c$	phase velocity of electromagnetic wave (m/s)
$c_0$	speed of light in vacuum (m/s)
$c_1$	first radiation constant ( $\text{W m}^2$ )
$c_{1L}$	first radiation constant for spectral radiance ( $\text{W m}^2/\text{sr}$ )
$c_2$	second radiation constant (mK)
$c_3$	third radiation constant (mK)
$c_p$	specific heat at constant pressure ( $\text{J}/(\text{kg K})$ )
$c_v$	specific heat at constant volume ( $\text{J}/(\text{kg K})$ )
<b>D</b>	electric displacement ( $\text{C}/\text{m}^2$ ); dynamic matrix
$D$	density of states
$D^*$	detectivity ( $\text{m Hz}^{1/2}/\text{W}$ )
$d$	diameter or film thickness (m)
<b>E</b>	electric field vector (N/C or V/m)
$E$	irradiance ( $\text{W}/\text{m}^2$ ); energy (J); magnitude of electric field (V/m)
$e$	elementary charge (C)
$f$	distribution function
$f_r$ or $f_t$	bidirectional reflectance (or transmittance) distribution function
<b>H</b>	magnetic field vector (A/m or C/(m s))
$h$	Planck's constant (J s)
$\hbar$	Planck's constant divided by $2\pi$ , $\hbar/2\pi$ , J s
$I$	Radiant intensity ( $\text{W}/\text{sr}$ )
$i$	$\sqrt{-1}$
<b>J</b>	electric current density ( $\text{A}/\text{m}^2$ )
<b>k</b>	wavevector ( $\text{m}^{-1}$ )
$k$	Boltzmann's constant (J/K); also magnitude of wavevector ( $\text{m}^{-1}$ )
$L$	radiance ( $\text{W}/(\text{m}^2 \text{sr})$ )
$L_\lambda$	spectral radiance ( $\text{W}/(\text{m}^2 \mu\text{m sr})$ )
<b>M</b>	magnetization vector (A/m or C/(m s))
$M$	exitance ( $\text{W}/\text{m}^2$ ); molecular weight (kg/kmol)
$M_{\text{em}}$	self-exitance or emitted exitance ( $\text{W}/\text{m}^2$ )
$m$	mass (kg)
$m_e$	electron mass (kg)
$N_A$	Avogadro constant ( $\text{mol}^{-1}$ )

$n$	real part of refractive index or refractive index; number density ( $\text{m}^{-3}$ )
$\tilde{n}$	complex refractive index
$\mathbf{P}$	dipole moment per unit volume ( $\text{C}/\text{m}^2$ ); propagation matrix
$\mathbf{p}$	momentum vector ( $m\mathbf{v}$ or $\hbar\mathbf{k}$ ) ( $\text{kg m/s}$ ); dipole moment ( $\text{C m}$ )
$p$	pressure ( $\text{Pa}$ or $\text{N}/\text{m}^2$ ); magnitude of momentum ( $\text{kg m/s}$ ); magnitude of dipole moment ( $\text{C m}$ )
$Q$	heat or radiant energy ( $\text{J}$ )
$q$	heat transfer rate ( $\text{W}$ )
$q''$	radiative heat flux ( $\text{W}/\text{m}^2$ )
$R$	universal gas constant ( $\text{J}/(\text{mol K})$ ); gas constant ( $\text{J}/(\text{kg K})$ ); electrical resistance ( $\Omega$ or $\text{V/A}$ ); detector responsivity ( $\text{V/W}$ or $\text{A/W}$ )
$R'$	directional-hemispherical reflectance
$R_e$	electrical resistivity ( $\Omega \text{ m}$ )
$r$	radius or radial coordinate ( $\text{m}$ ); also Fresnel reflection coefficient
$\mathbf{S}$	Poynting vector ( $\text{W}/\text{m}^2$ )
$S$	entropy ( $\text{J/K}$ ); detector output signal ( $\text{V}$ or $\text{A}$ )
$S_j$	strength of the $j$ th phonon oscillator
$T$	temperature ( $\text{K}$ )
$T'$	directional-hemispherical transmittance
$T_\lambda$	radiance temperature ( $\text{K}$ or $^\circ\text{C}$ )
$T_t$	total radiance temperature ( $\text{K}$ or $^\circ\text{C}$ )
$t$	temperature ( $^\circ\text{C}$ ); time ( $\text{s}$ ); Fresnel transmission coefficient
$u$	energy density ( $\text{J}/\text{m}^3$ )
$V$	volume ( $\text{m}^3$ ); voltage ( $\text{V}$ )
$\mathbf{v}$	velocity ( $\text{m/s}$ )
$\nu$	specific volume ( $\text{m}^3/\text{kg}$ ), also speed ( $\text{m/s}$ )
$x, y, z$	coordinates ( $\text{m}$ )

## GREEK SYMBOLS

$\alpha'$	directional absorptivity
$\beta$	phase shift ( $\text{rad}$ )
$\beta_\lambda$	attenuation coefficient ( $\text{m}^{-1}$ )
$\gamma$	scattering rate ( $\text{rad/s}$ )
$\delta_\lambda$	radiation penetration depth ( $\text{m}$ )
$\varepsilon$	dielectric function or relative permittivity; also emissivity
$\varepsilon_0$	electric permittivity of vacuum ( $\text{F/m}$ or $\text{A}^2 \text{ s}/(\text{N m}^2)$ )
$\tilde{\varepsilon}$	complex dielectric function
$\varepsilon'$	directional emissivity
$\eta$	efficiency
$\theta$	polar or zenith angle ( $\text{rad}$ )
$\theta_B$	Brewster's angle ( $\text{rad}$ )
$\theta_c$	critical angle ( $\text{rad}$ )



$\kappa$	extinction coefficient (i.e., imaginary part of the refractive index)
$\lambda$	wavelength (m)
$\mu$	relative magnetic permeability; also mobility ( $\text{m}^2/(\text{V s})$ )
$\mu_0$	magnetic permeability in vacuum ( $\text{N/A}^2$ )
$\nu$	frequency (Hz)
$\rho$	density ( $\text{kg/m}^3$ ); electric charge density ( $\text{C/m}^3$ )
$\rho'$	directional-hemispherical reflectivity
$\sigma$	Stefan–Boltzmann constant ( $\text{W}/(\text{m}^2 \text{K}^4)$ ); electric conductivity ( $\text{A/V}$ )
$\sigma_\lambda$	scattering coefficient ( $\text{m}^{-1}$ )
$\tau$	relaxation time (s); internal transmissivity; spectral transmittance
$\Phi$	scattering phase function
$\phi$	azimuthal angle (rad)
$\Omega$	solid angle (sr)
$\omega$	angular frequency (rad/s); solid angle (sr); scattering albedo
$\omega_p$	plasma frequency (rad/s)

## SUBSCRIPTS

0	vacuum
1,2,3	medium 1,2,3
B, b or bb	blackbody
e	electron
em	emitted
h	hole
i	incident
L	radiance
m	medium
r	reflected
t	total (integration over all wavelengths or frequencies)
$\lambda, \nu$	spectral property in terms of wavelength or frequency

## SUPERSCRIPTS

h	hemispherical
$p$	TM wave or $p$ (parallel) polarization
$s$	TE wave or $s$ (perpendicular) polarization
'	used to signify a directional radiative property
*	complex conjugate