
ADVANCED ENGINEERING ELECTROMAGNETICS

CONSTANTINE A. BALANIS

Arizona State University



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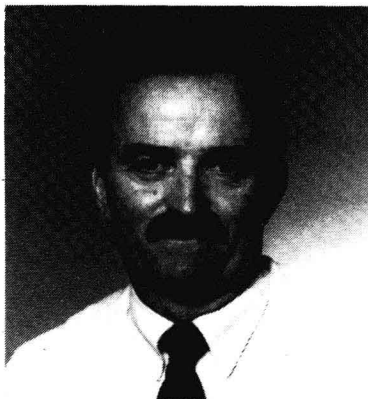
To Helen, Renie, and Stephanie

ABOUT THE AUTHOR

Constantine A. Balanis was born in Trikala, Greece. He received his B.S.E.E. degree from Virginia Polytechnic Institute, Blacksburg, in 1964, his M.E.E. degree from University of Virginia, Charlottesville, in 1966, and his Ph.D. degree in electrical engineering from Ohio State University, Columbus, in 1969.

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PREFACE

This book is designed for a two-semester sequence in time-harmonic electromagnetics that is beyond an introduction to electromagnetostatics. Although the first part of the book is intended primarily for undergraduates and beginning graduates in electrical engineering and physics, the last part is intended for advanced graduate students and practicing engineers and scientists. The majority of Chapters 1 to 10 can be covered in the first semester, and most of Chapters 11 to 14 can be covered in the second semester. To cover all of the material in the proposed time frame would be, in many instances, a very ambitious task. Sufficient topics have been included, however, to make the text complete and to allow the instructor the flexibility to emphasize, de-emphasize, or omit sections or chapters.

The discussion presumes that the student has a general knowledge of vector analysis, differential and integral calculus, and electromagnetostatics either from an introductory electrical engineering or physics course. Mathematical techniques required for understanding some advanced topics, mostly in the later chapters, are incorporated in the individual chapters or are included as appendixes.

This is a very detailed student-oriented book. The analytical detail, rigor, and thoroughness allows many of the topics to be traced to their origin. In addition to the coverage of traditional classical topics, the book includes state of the art advanced topics on Integral Equations (IE), Moment Method (MM), Geometrical Theory of Diffraction (GTD), and Green's functions. Electromagnetic theorems, as applied to the solution of boundary-value problems, are also included and discussed.

The material is presented in a sequential and unified manner, and each chapter is subdivided into sections or subsections for which the individual heading clearly identifies the topic discussed, examined, or illustrated. The book also includes numerous examples, illustrations, references, and end-of-chapter problems. The examples and end-of-chapter problems have been designed to illustrate basic principles and to challenge the knowledge of the student. An exhaustive list of references is included at the end of each chapter to allow the interested reader to research each topic. A number of appendixes of mathematical identities and special functions, some represented also in tabular and graphical forms, are included to aid the student in the solution of the examples and assigned problems. Moment Method (MM) and Geometrical Theory of Diffraction (GTD) FORTRAN computer programs are included, respectively, at the end of Chapters 12 and 13, and they can be used for the solution of simple and complex problems. Also, information is provided about other national moment method computer programs. A solutions manual for all end-of-chapter problems is available for the instructor.

In Chapter 1 the book covers the classical topics on Maxwell's equations, constitutive parameters and relations, circuit relations, boundary conditions, and power and energy relations. The electrical properties of matter, both direct-current and alternating-current, are covered in Chapter 2, and the wave equation and its solution in rectangular, cylindrical and spherical coordinates are discussed in Chapter 3. Electromagnetic wave propagation and polarization is introduced

in Chapter 4. Reflection and transmission at normal and oblique wave incidences are considered in Chapter 5. Chapter 6 covers the auxiliary vector potentials and their use toward the construction of solutions for radiation and scattering problems. The theorems of duality, uniqueness, image, reciprocity, reaction, volume and surface equivalences, induction, and physical and physical optics equivalents are introduced and applied in Chapter 7. Rectangular cross section waveguides and cavities, including dielectric slabs, striplines, and microstrips, are discussed in Chapter 8. Those of circular cross section, including the fiber optics cable, are examined in Chapter 9, and those of spherical geometry are introduced in Chapter 10. Scattering by strips, plates, circular cylinders, wedges, and spheres is analyzed in Chapter 11. Chapter 12 covers the basics and applications of Integral Equations (IE) and Moment Method (MM) and also includes a computer program for wire radiation and scattering. The techniques and applications for the Geometrical Theory of Diffraction (GTD) are introduced and discussed in Chapter 13, including computer programs for diffraction coefficients of conducting wedges. The classic topic of Green's functions is introduced and applied in Chapter 14.

Throughout the book an $e^{j\omega t}$ time convention is assumed, and it is suppressed in almost all of the chapters. The International System of Units, which is an expanded form of the rationalized MKS system, is used throughout the text discussion. In some instances, the units of length are given in meters (or centimeters) and feet (or inches). Numbers in parentheses () refer to equations, whereas those in brackets [] refer to references. For emphasis, the most important equations, once they are derived, are boxed.

I acknowledge the invaluable suggestions, corrections, and constructive criticisms of the reviewers of this book: Roger D. Radcliff of Ohio University, Christos G. Christodoulou of University of Central Florida, Prabhakar H. Pathak of Ohio State University, and Thomas E. Tice of Arizona State University. The writing of this book has been a very ambitious task, and its completion would not have been possible without the contributions of many of my graduate students. It is a pleasure to acknowledge those of Kefeng Liu for the development of the moment method computer programs and for the proofreading of parts of the manuscript, and those of Leslie A. Polka, Frank L. Whetten, Mark S. Frank, James P. Gilb, and Craig R. Birtcher for proofreading parts of the manuscript. A special tribute is owed to Thuy Griesser for the expert typing of all phases of the entire manuscript. I am also grateful to Christina Kamra, Wiley editor of electrical engineering and computer science, for her interest in the production and publication of this book. To the companies and authors that provided the copyright permissions, I am most appreciative. In a book of this size, there inevitably will be errors that have been overlooked. I would appreciate having any errors brought to my attention.

Constantine A. Balanis

Tempe, Arizona

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