

A close-up photograph of a steam turbine's internal components. On the left, a vertical scale with markings like 1.000, 1.010, 1.020, and 1.030 is visible. To its right, several curved, polished metal blades are arranged in a radial pattern. The background is dark and out of focus.

# BLADE DESIGN & ANALYSIS

FOR STEAM TURBINES

MURARI SINGH, PH.D.  
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# Blade Design and Analysis for Steam Turbines

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**Mc  
Graw  
Hill**

New York Chicago San Francisco  
Lisbon London Madrid Mexico City  
Milan New Delhi San Juan  
Seoul Singapore Sydney Toronto

Cataloging-in-Publication Data is on file with the Library of Congress

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1 2 3 4 5 6 7 8 9 0 DOC/DOC 1 7 6 5 4 3 2 1

ISBN 978-0-07-163574-5

MHID 0-07-163574-2

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# Preface

**T**urbine engineers and designers have made remarkable improvements in the efficiency and reliability of industrial steam turbines over the last 30 years. Remarkable improvements have been achieved for products that already had over 100 years of technical development behind them. For most of those first 100 years, the analysis of turbine blades had concentrated on the behavior of individual blades. A key change, and one of the most significant advances in turbine reliability, was the development and application of analytical techniques that make it possible to characterize and explain the behavior not simply of individual turbine blades, but of entire bladed disk assemblies.

Advancements in modal analysis and testing, fatigue analysis, creep analysis, fracture mechanics, aerodynamic theories, and the development of many new materials and manufacturing processes cleared the path for the design of more powerful, more efficient, and more reliable turbines. It became evident that design of blades is a multidiscipline activity. For a proper reliability assessment of a design, one needs to understand many fields of science and these must be applied as need be. These advancements helped designers to extend the capabilities of designs beyond past experience. This also helped to explain past successes and failures of components.

The simultaneous development of powerful and inexpensive computers has made it practical to quickly and efficiently carry out the calculations necessary to apply these advanced analytical techniques to the routine design of new and replacement blades and rotors for industrial steam turbines. Nowhere have these advances had a greater influence than on the design of critical service process compressor drives for the refining and petrochemical industries. Large drivers for ethylene and LNG processes exceeding 75 MW in power are in successful service. Older designs using double-flow exhausts with short, but very strong, blades have been supplanted in newer designs by single-flow exhausts with taller, but more reliable and aerodynamically sophisticated, stages. Inlet pressure and temperatures of 2000 psig/1000°F (140 barg/540°C) have become almost common in new process drive applications.

The purpose of this book is to introduce these advances in a concise volume and provide an easy-to-understand reference for practicing engineers who are involved in the design, specification, and evaluation of industrial steam turbines in general, and critical process compressor drivers in particular. This text has also attempted to present a unified view of concepts and techniques needed in the understanding of blade design. It includes some advanced concepts such as life estimation. One chapter is dedicated in introducing the reader and designers to the effect of uncertainty of input variables on the reliability of the design. Probabilistic-type analysis is introduced for reliability estimation, as it is said that every design decision has some risk associated with it and risk may be managed if it is known.

We would like to thank each person and the many industries whose works have been referenced in the book. We also take this opportunity to apologize to those whose work might not have been referenced by mistake. Thanks to the many associates during our employment and consulting work whose thoughts guided the selection of many materials. We hope these will help readers in their work or at least make them think. Last but not the least, many thanks to Seema Singh for reading the manuscript word by word and making numerous suggestions for changes that made the work better and more readable.

*Murari P. Singh, Ph.D.*  
*George M. Lucas, PE*

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# Acknowledgments

Special thanks are extended to the following:

Advanced Turbomachine LLC	Steve Rashid
Arthaven	Barbara Lucas
Consultant	John Waggott
Dresser-Rand	Jay Scherbik, Randy Moll, Bob Voorhees, Jim Dello, Dan Flurschutz, David Nye, George Lentek, and Neeraj Bali
Elliott Co.	Robert Sloboda, Art Titus, and Brook Tolman
GE Oil & Gas	James Cencula and Leonardo Tognarelli
Mar-Test	Matt Webb
Metal Improvement Co.	Dave Massey

# **Blade Design and Analysis for Steam Turbines**



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# CHAPTER 1

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## Introduction

### 1.1 Importance of Blades in Steam Turbines

Structural integrity of all rotating components is the key for successful operation of any turbomachinery. This integrity depends on the successful resistance of the machine parts to the steady and alternating stresses imposed on them. The challenge with rotating equipment, such as turbomachinery, is often more severe due to the significance of the alternating loads that must be carried to satisfy their purpose.

One of the major classes of rotating machinery is the mechanical drive steam turbines, i.e., steam turbines that drive pumps and/or compressors. These steam turbines are differentiated from those that drive generators in that they operate at variable speeds. Steam turbines may operate from 1 to 5 hp up to several hundred thousand horsepower; they may operate with steam that ranges from vacuum to thousands of pounds per square inch; and blade tip speeds can exceed the force of the most severe hurricane (a large, last blade row with an 8-ft tip to tip diameter operating at 3600 rpm will experience tip speeds in excess of 1000 mph).

One of the causes of blade deterioration is static stress which is primarily the result of steam bending and centrifugal loads. Alternating stresses are imposed due to the vibration of the parts in question, e.g., blades and disks. If the combined loads become too large, vibration-induced fatigue of the rotor blades or disks is a major concern. In addition to the imposed loads, these forces are subject to resonant amplification caused by coincidence with natural frequencies. To put the scope of this problem into context, one must realize that there may be thousands of blades in a steam turbine. For example, there may be 10 to 20 rows of different blade designs with the possibility of each blade row having different dynamic characteristics.

Steam turbines have been in operation for more than 100 years and have always faced this problem. As may be imagined, the technology in engineering and physics to support these designs has grown dramatically over that time; tools have been enhanced and technological developments incorporated.

## 1.2 Brief Historical Perspective of Technological Development

The current state of design, as represented in the API standards for this class of machinery, sets a life of 30 years for all components. In many cases, this translates into a design requirement for infinite life and may exceed the needs of a specific installation. This requirement may be driven by the actual desire for infinite life, limitations in analysis techniques, tools that have existed over the years, and/or an incomplete understanding of the tools that have appeared in the recent past and are currently fully or partially available.

A common cause of vibration-related failure in steam turbine blading is resonant excitation of the blading occurring at an integral order, i.e., multiples of the rotational speed, nozzle passing frequency, and multiples thereof. The associated mode of failure is high cycle fatigue. A primary feature of resonant excitation is that dynamic stress amplitudes rise as the exciting frequency approaches the resonant speed and the response decreases after passing through the resonant speed. Hence, it is necessary to identify resonant frequencies of the system.

It is impossible to include all the work done by the numerous researchers and designers of steam turbine blades. Effort is made to include some of those that describe the progress and current methodology for steam turbine blade design. Many textbooks were published on steam turbines during the last century together with many technical publications dealing with all aspects of turbine design, specifically blade design. Early publications by Stodola (1905) and Kearton (1922) are worth mentioning because these two books are credited with setting the stage for detailed vibration and reliability analysis for blades. In many different ways designers followed the processes and methods outlined in these books. As the turbine design matured and manufacturers gained experience, methods were adjusted to include new technical methods and lessons learned from field experience and each manufacturer has evolved its own methods and criteria to achieve successful design. Hence, methods and criteria should not be expected to be consistent across manufacturers.

Blade design has evolved from the analysis of spring-mass systems to a single cantilever beam to a band of blades to a bladed disk. In addition, steam turbines have included bands of blades on a disk as a system. Throughout the years many effects of turbine speed to increase blade frequency were found, and it gave rise to the term *centrifugal stiffening*. Campbell (1925), while examining the failure (bursting) of disks, concluded that blades were broken due to axial vibration. This publication reported the results of an investigation conducted at General Electric to understand the wheel failures, mostly in wheels of large diameter, that could not be explained on the basis of high stress alone. About this time certain types of vibrations of standing



waves were investigated by means of sand pictures. This test was conducted by scattering sand over the wheel surface. Wheels were then excited by means of a magnet exciter, and the turbine wheel was placed in a horizontal position. An electromagnet was clamped with its poles close to the edge of the wheel, alternating current was passed through the coils of the magnet, and a series of pulls was exerted on the wheel. This resulted in deflecting the wheel in a transverse direction to the plane of the wheel. A variable-speed direct-current (dc) motor was used to drive the alternating-current (ac) generator and allowed the frequency of the pull of the magnet to be varied over a wide range. Frequency of excitation was varied until a sand pattern on the wheel appeared, and sand accumulated mostly in a radial line or pattern. When the frequency changed to some higher magnitude, a different sand pattern appeared on the wheel. These radial lines represented the location where the velocity of vibration was zero. The number of radial lines was always observed to be of an even number. These patterns, are known as nodal patterns, and two lines are taken as one diameter. It is now understood that the opposite radial lines might not be  $180^\circ$  apart. Frequencies at which these patterns are observed coincide with the natural frequency of the wheel in axial vibration associated with the mode shape represented by the sand pattern.

Figure 1.1 shows a picture of such a sand pattern. It is noticeable that sand has collected on certain portions of the wheel, and it forms a pattern showing four radial lines. This pattern is referred to as two nodal diameters mode. There are six radial lines in the pattern shown in Fig. 1.2. These modes are called three nodal diameters mode. Note that the radial lines pass through the balancing holes in the left picture while in the picture on right side these lines pass between the balancing holes. A detailed discussion of this phenomenon that forms the basis of bladed disk analysis is provided in Chap. 5.

Over time blades needed to be taller to accommodate the requirement of increasing power. This necessitated the blades to be joined together by a band of metal either at the tip or somewhere along the length of the blade. Kroon (1934) described a method to evaluate the effectiveness of such construction to reduce the dynamic response of the design under steam forces. Allen (1940) described design practices of blades in high-pressure and high-temperature stages. A detailed explanation for partial stage admission was included, as was one for full admission stage. Allen recommended limiting the number of blades per group to two for high-temperature service and argued that more blades in a group for high-temperature application tend to set up high stress. Two types of root attachment (axial entry vs. tangential entry) construction were explored, and the choice is dependent on the application, e.g., speed, power, and temperature. The effect of shrouding might be considerable for high-pressure blades. A reduction of 25 to 60 percent in bending stress may be achieved. The natural frequency