DEVELOPMENTAL MATHEMATICS

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PREFACE

Developmental Mathematics is designed to give students a review of the basic skills of mathematics and to provide an introduction to the study of algebra. Some of the material in this text has been drawn from *Fundamentals of Mathematics, Third Edition,* and from *Introductory Algebra, Fourth Edition,* with special emphasis given to the TASP™ skill requirements, including graph reading and geometry.

Informal yet carefully worded explanations, detailed examples with accompanying practice exercises, pedagogical second color, abundant exercises, and comprehensive chapter reviews are hallmarks of the book. The text has been written for maximum instructor flexibility. Both core and peripheral topics can be selected to fit individual course needs. An annotated instructor's edition, testing manual, and test generator are provided for the instructor. Interactive tutorial software and a set of instructional videotapes are also available.

FEATURES

STUDENT GUIDEPOSTS Designed to help students locate important concepts as they study or review, student guideposts specify the major topics, rules, and procedures in each section. The guideposts are listed at the beginning of the section, then each is repeated as the corresponding material is discussed in the section.

EXAMPLES More than 1200 carefully selected examples include detailed, step-by-step solutions and side annotations in color. Each example is headed by a brief descriptive title to help students focus on the concept being developed and to aid in review.

PRACTICE EXERCISES These parallel each example and keep students involved with the presentation by allowing them to check immediately their understanding of ideas. Answers immediately follow these exercises.

CAUTIONS This feature calls students' attention to common mistakes and special problems to avoid.

HELPFUL HINTS These have been supplied throughout the early portions of the text to emphasize useful memory devices or identify more important mechanical procedures.

FIGURES AND ILLUSTRATIONS Abundant use has been made of figures and illustrations in examples and exercises, particularly applications.

COLOR Pedagogical color highlights important information throughout the book. Key definitions, rules, and procedures are set off in colored boxes for increased emphasis. Figures and graphs utilize color to clarify the concepts presented. Examples present important steps and helpful side comments in color.

EXERCISES As a key feature of the text, nearly 10,000 exercises, including about 1800 applied problems, are provided. Two parallel exercise sets (A and B) and a collection of extension exercises (C) follow each section and offer a wealth of practice for students and abundant flexibility for instructors. Exercises ranging from the routine to the more challenging, including application and calculator problems, are provided.

Exercises A This set of exercises includes space for working the problems, with answers immediately following the exercises. Some of these problems, identified by a colored circle ④ , have their solutions worked out at the back of the book. Many of these solutions are to exercises that students frequently have difficulty solving.

Exercises B This set matches the exercises in set A problem for problem but is presented without work space or answers.

Exercises C This set is designed to give students an extra challenge. These problems extend the concepts of the section or demand more thought than exercises in sets A and B. Answers or hints are given for selected exercises in this set.

REVIEW EXERCISES To provide ample opportunities for review, the text features a variety of review exercises.

For Review exercises are located at the end of most A and B exercise sets. They not only encourage continuous review of previously covered material, but also often provide special review preparation for topics covered in the upcoming section.

Chapter Review Exercises and a practice **Chapter Test** conclude each chapter. The Chapter Review Exercises are divided into two parts: the problems in Part I are ordered and marked by section. Those in Part II are not referenced to the source section, and are thoroughly mixed to help students prepare for examinations. Answers to all review and test exercises are provided in the text.

Final Review Exercises, referenced to each chapter and with answers supplied, are located at the back of the book.

CHAPTER REVIEWS In addition to the Chapter Review Exercises and Chapter Tests, comprehensive chapter reviews also include **Key Words** and **Key Concepts**. Key Words, listed by section, have brief definitions. Key Concepts summarize the major points of each section.

CALCULATORS The use of a calculator is discouraged until Section 5.8, where a brief introduction is presented. In working exercises that involve dividing decimals or approximating irrational numbers, a calculator can be an invaluable tool, and it is important for students to learn to judge for themselves when a calculator should or should not be used. Some exercises, however, do specifically refer to calculator use and are marked with this calculator symbol: ▦

INSTRUCTIONAL FLEXIBILITY

Developmental Mathematics offers proven flexibility for a variety of teaching situations such as individualized instruction, lab instruction, lecture classes, or a combination of methods.

Material in each section of the book is presented in a well-paced, easy-to-follow sequence. Students in a tutorial or lab instruction setting, aided by the student guideposts, can work through a section completely by reading the explanation, following the detailed steps in the examples, working the practice exercises, and then doing the exercises in set A.

The book can also serve as the basis for, or as a supplement to, classroom lectures. The straightforward presentation of material, numerous examples, practice exercises, and three sets of exercises offer the traditional lecture class an alternative approach within the convenient workbook format.

CONTENT HIGHLIGHTS

- Diagnostic Pretests for Chapters 1 and 2 help the instructor determine the extent of coverage necessary for a particular class.
- The proportion approach to percent is used with the percent equation in a side-by-side presentation in solving percent problems.
- Consistent emphasis is placed on real-world applications.
- A separate chapter on statistics (Chapter 7) provides extensive coverage of this important area.
- The development of operations with signed numbers has been given special consideration to provide a thorough discussion of the topic.
- An introduction to inductive and deductive reasoning has been placed at the beginning of the geometry chapter (Chapter 9) and referenced throughout.
- Special right triangles and the Pythagorean theorem are considered in detail in the chapter on radicals (Chapter 17).

SUPPLEMENTS

An extensive supplemental package is available for use with *Developmental Mathematics*.

The **ANNOTATED INSTRUCTOR'S EDITION** provides instructors with immediate access to the answers to every exercise in the text; each answer is printed in color next to the corresponding text exercise.

The **INSTRUCTOR'S TESTING MANUAL** contains a series of **ready-to-duplicate tests,** including a Placement Test, six different but equivalent tests for each chapter (four open-response and two multiple-choice), and two final exams, all with answers supplied. Section-by-section **teaching tips** provide suggestions for content implementation that an instructor, tutor, or teaching assistant might find helpful.

HARPERCOLLINS TEST GENERATOR FOR MATHEMATICS   Available in Apple, IBM, and Macintosh versions, the test generator enables instructors to select questions by objective, section, or chapter, or to use a ready-made test for each chapter. Instructors may generate tests in multiple-choice or open-response formats, scramble the order of questions while printing, and produce multiple versions of each test (up to 9 with Apple, up to 25 with IBM and Macintosh). The system features printed graphics and accurate mathematics symbols. It also features a preview option that allows instructors to view questions before printing, to regenerate variables, and to replace or skip questions if desired. The IBM version includes an editor that allows instructors to add their own problems to existing data disks.

VIDEOTAPES   A new videotape series, ALGEBRA CONNECTION: *The Developmental Mathematics Course,* has been developed to accompany *Developmental Mathematics*. Produced by an Emmy Award–winning team in consultation with a task force of academicians from both two-year and four-year colleges, the tapes cover all objectives, topics, and problem-solving techniques within the text. In addition, each lesson is preceded by motivational ''launchers'' that connect classroom activity to real-world applications.

INTERACTIVE TUTORIAL SOFTWARE   This innovative package is also available in Apple, IBM, and Macintosh versions. It offers interactive modular units, specifically linked to the text, for reinforcement of selected topics. The tutorial is self-paced and provides unlimited opportunities to review lessons and to practice problem solving. When students give a wrong answer, they can request to see the problem worked out. The program is menu-driven for ease of use, and on-screen help can be obtained at any time with a single keystroke. Students' scores are automatically recorded and can be printed for a permanent record.

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TO THE INSTRUCTOR

Students in this course traditionally have a variety of backgrounds. For some, the pace of Chapters 1 and 2 will be appropriate and provide a thorough review in preparation for the more complex topics that follow. On the other hand, some students will be able to review operations on whole numbers at a faster rate or omit it altogether, leaving more time for the remaining sections. As a result, we have provided the following two Diagnostic Pretests for Chapters 1 and 2 to help you determine the extent of coverage necessary for your particular class. Since the questions in each chapter are grouped by section, you may discover that some topics will need to be discussed, while others can be omitted. Answers to the Diagnostic Pretests are given in the Instructor's Guide that accompanies this text and in the Annotated Instructor's Edition. Also, multiple-choice versions of the Pretests have been included in the Instructor's Guide for those who wish to use this format of testing.
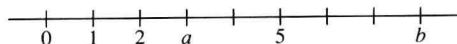
CHAPTER 1 DIAGNOSTIC PRETEST

1.1   **1.** Give the first five whole numbers.

1. _____

2. Consider the number line.

2. (a) _____

    (a) What number corresponds to b?

    (b) Place the appropriate symbol ($<$ or $>$) between b and 4.

$$b \underline{\quad ? \quad} 4$$

(b) _____

(c) _____

    (c) Place the appropriate symbol ($<$ or $>$) between a and b.

$$a \underline{\quad ? \quad} b$$

3. Write $300{,}000 + 5000 + 200 + 6$ in standard notation.

3. _____

4. In the number $62{,}451{,}903$, what digit represents the following places?

    (a) millions   **(b)** ten thousands   **(c)** tens

4. (a) _____

(b) _____

(c) _____

5. Write $23{,}401$ in expanded notation.

5. _____

1.2   **6.** What law of addition is illustrated by $6 + 10 = 10 + 6$?

6. _____

7. What is the missing number? $2 + \underline{\quad ? \quad} = 2$

7. _____

1.3   *Find the following sums.*

8.	**9.**	**10.**	**11.**
21	648	2049	4235
+ 49	+ 37	+ 1167	217
			+ 6258

8. _____

9. _____

10. _____

11. _____

12. What law of addition is illustrated by

$$(2 + 3) + 7 = 2 + (3 + 7)?$$

12. _____

1.4   **13.** Round $43{,}584$ to the nearest

    (a) ten   **(b)** hundred   **(c)** thousand.

13. (a) _____

(b) _____

(c) _____

14. Estimate   2459 by rounding to the nearest
                  + 375

    (a) ten   **(b)** hundred.

14. (a) _____

     (b) _____

15. During a two-game championship, 8176 people attended the first game, and 9857 attended the second. First estimate the total attendance, and then find the exact attendance.

15. _____

1.5 **16.** What is the missing number? $7 - \underline{\ ?\ } = 2$

16. _____

17. What is the missing number? $\underline{\ ?\ } - 0 = 24$

17. _____

1.6 *Find the following differences.*

18.   42   **19.** 643   **20.** 805   **21.** 9003
    − 35   − 97   − 486   − 157

18. _____

19. _____

20. _____

21. _____

1.7 **22.** Professor Perko has three classes with 32, 45, and 13 students, respectively. How many students does Professor Perko have?

22. _____

23. On the first day of football practice, 137 players reported. After one week, 48 players had been cut or quit the team. How many players were still on the team at the end of the week?

23. _____

24. Cari received two checks for her graduation, one for $250 and the other for $325. She used the money to buy a sweater for $68, a skirt for $45, and a book for $7. She kept $50 in cash and deposited the rest of the money in her savings account. How much did she deposit?

24. _____

CHAPTER 2 DIAGNOSTIC PRETEST

2.1   **1.** Give the first three multiples of 5.

1. _____

    2. What law of multiplication is illustrated by

$$5 \times 9 = 9 \times 5?$$

2. _____

    3. What is the missing number? $3 \times \underline{} = 3$

3. _____

    4. What is the missing number? $3 \times \underline{} = 0$

4. _____

2.2   *Find the following products.*

    5.   296   **6.**   43   **7.**   651   **8.**   4351
        $\times \quad 5$ $\times 15$ $\times 202$ $\times \quad 600$

5. _____
6. _____
7. _____
8. _____

2.3   **9.** What law of multiplication is illustrated by

$$(2 \times 3) \times 8 = 2 \times (3 \times 8)?$$

9. _____

    10. Estimate the product 897×603 by rounding to the nearest hundred.

10. _____

    11. Without actually finding the product, use estimation to determine whether the work shown appears to be correct.

                         409
                    $\times\,817$
                    2863
                    409
                  3902
             397,153

11. _____

2.4   **12.** What is the missing number? $15 \div 3 = \underline{}$

12. _____

    13. What is the missing number? $12 \div \underline{} = 6$

13. _____

    14. What is the missing number? $0 \div 7 = \underline{}$

14. _____

    15. What number cannot be used as a divisor?

15. _____

2.5   *Find the following quotients and remainders.*

    16. $7\overline{)168}$ **17.** $21\overline{)745}$ **18.** $12\overline{)3100}$

16. _____

17. _____

18. _____

2.6 **19.** Dr. Besnette has 3 children and he wishes to divide equally 19. _____
an inheritance of $7695 among them. How much will
each child receive?

20. Nancy purchases 7 books at $16 each and 3 posters at $4 20. _____
each. How much change will she receive if she pays with
seven 20-dollar bills?

2.7 **21.** Is 17 a prime number? Explain. 21. _____

22. What is the only even prime number? 22. _____

23. Find the prime factorization of 150. 23. _____

24. Is 1357 divisible by 3? 24. _____

25. What property do numbers that are divisible by 10 have? 25. _____

26. Numbers that are divisible by 2 or that are multiples of 2 have a 26. _____
special name. What is that name?

2.8 **27.** Is 16 a perfect square? 27. _____

28. What is the square of 25? 28. _____

29. What is the square root of 25? 29. _____

30. Evaluate 2^4. 30. _____

31. Write $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ using an exponent. 31. _____

Perform the indicated operations.

32. $3 - 2 + 8 \div 4$ 32. _____

33. $(2 + 3)^2 - 2^2 + 3^2$ 33. _____

34. $5 \cdot 2^3$ 34. _____

35. $\sqrt{9} + 3$ 35. _____

During the past few years we have taught developmental mathematics to over 2000 students and have heard the following comments more than just a few times. "I've always been afraid of math and have avoided it as much as possible." "I don't like math, but it's required to graduate." "I can't do percent problems!" If you have ever made a similar statement, now is the time to think positively, stop making negative comments, and start down the path toward success in mathematics. Don't worry about this course as a whole. The material in the text is presented in a way that lets you take one small step at a time. Here are some general and specific guidelines that are necessary and helpful.

GENERAL GUIDELINES

1. Mastering mathematics requires motivation and dedication. A successful athlete does not become a champion without commitment to his or her goal. The same is true for the successful student of mathematics. Be prepared to work hard and spend time studying.

2. Mathematics is not learned simply by watching, listening, or reading; *it is learned by doing.* Use your pencil and practice. When your thoughts are organized and written in a neat and orderly fashion, you have taken a giant step toward success. Be complete and write out all details. The following are samples of two students' work on a word problem. Can you tell which one was the most successful in the course?

STUDENT A   **STUDENT F**

3. The use of a calculator is introduced in Section 5.8. Become familiar with the features of your calculator by consulting the owner's manual. When working problems that involve computing with decimals, a calculator can be a time-saving device. On the other hand, you should not be so dependent on a calculator that you use it to perform simple calculations that might be done mentally. For example, it would be ridiculous to use a calculator to find $8 \div 2$, while it would be helpful in finding $7343.84 \div 1.12$. It is important that you learn when to use and when not to use your calculator.

SPECIFIC GUIDELINES

1. As you begin to study each section, glance through the material and obtain a preview of what is coming.

2. Return to the beginning of the section and start reading slowly. The STUDENT GUIDEPOSTS will help you locate important concepts as you progress or as you review.

3. Read through each EXAMPLE and make sure that you understand each step. The side comments in color will help you if something is not quite clear.

4. After reading each example, work the parallel PRACTICE EXERCISE. This will reinforce what you have just read and start the process of practice.

5. Periodically you will encounter a CAUTION. These warn you of common mistakes and special problems that should be avoided.

6. The HELPFUL HINTS in the earlier portions of the text are designed to give you a better understanding of material or a more direct way of solving a problem.

7. After you have completed the material in the body of a section, you must test your mastery of the skills and practice, practice, practice! Begin with the exercises in set A. Answers to all of the problems are placed at the end of the set for easy reference. Some of the problems, identified by colored exercise numbers, have complete solutions at the back of the book. After trying these problems, refer to the step-by-step solutions if you have difficulty. You may want to practice more by doing the exercises in set B or to challenge yourself by trying the exercises in set C.

8. After you have completed all of the sections in the chapter, read the CHAPTER REVIEW that contains key words and key concepts. The review exercises provide additional practice before you take the CHAPTER TEST. Answers to the tests are at the back of the book.

9. To aid you in studying for your final examination, we have concluded the book with a comprehensive set of FINAL REVIEW EXERCISES.

10. To review the definitions of important terms covered previously, refer to the GLOSSARY that is also located at the back of the text.

If you follow these steps and work closely with your instructor, you will greatly improve your chance of success in mathematics.

Best of luck, and remember that you can do it!

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