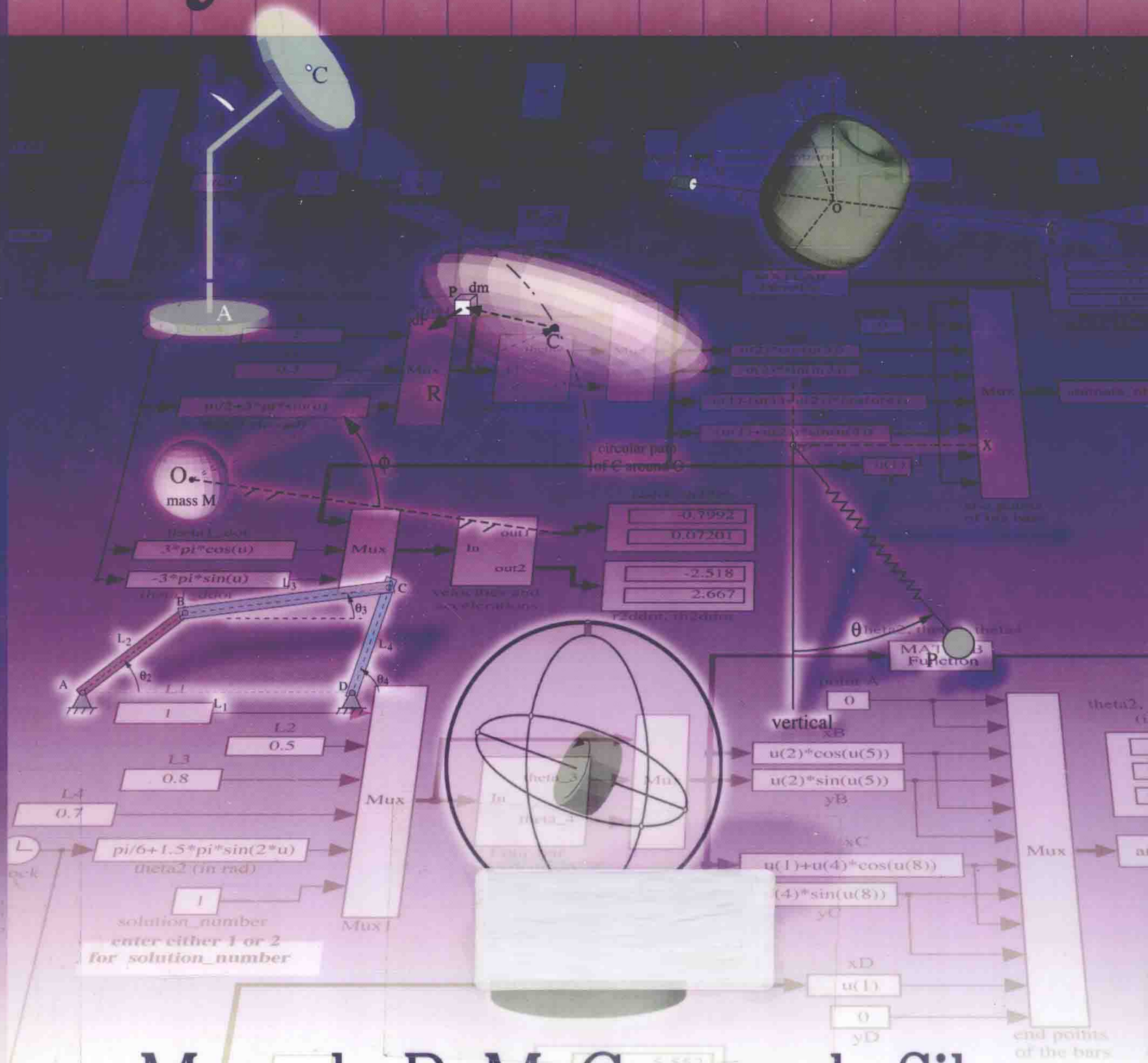
Intermediate Dynamics

Complemented with Simulations and Animations



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Intermediate Dynamics

Complemented with Simulations and Animations

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INTERMEDIATE DYNAMICS: COMPLEMENTED WITH SIMULATIONS AND ANIMATIONS

Published by McGraw-Hill, a business unit of The McGraw-Hill Companies, Inc., 1221 Avenue of the Americas, New York, NY 10020.

Some ancillaries, including electronic and print components, may not be available to customers outside the United States.

This book is printed on acid-free paper.

1 2 3 4 5 6 7 8 9 0 QPF/QPF 0 9 8 7 6 5 4 3

ISBN 0–07–285315–8

Publisher: *Elizabeth A. Jones*
Sponsoring editor: *Jonathan Plant*
Administrative assistant: *Rory Stein*
Marketing manager: *Sarah Martin*
Lead project manager: *Jill R. Peter*
Production supervisor: *Kara Kudronowicz*
Senior media project manager: *Stacy A. Patch*
Executive media technology producer: *Linda Meehan Avenarius*
Designer: *Rick D. Noel*
Cover designer: *John Rokusek Studios*
Cover illustrations by: *Marcelo R. M. Crespo da Silva*
Compositor: *Interactive Composition Corporation*
Typeface: *10.5/12 Times Roman*
Printer: *Quebecor World Fairfield, PA*

Library of Congress Cataloging-in-Publication Data

Crespo, da Silva, M. R. M.
Intermediate dynamics: complemented with simulations and animations/Marcelo R. M. Crespo da Silva. — 1st ed.
p. cm.
Includes bibliographical references and index.
ISBN 0–07–285315–8
1. Dynamics. I. Title.

QA845.D25 2004
531'.11—dc21 2003051273
CIP

www.mhhe.com

ABOUT THE AUTHOR

Marcelo R. M. Crespo da Silva Professor Crespo da Silva received his undergraduate engineering degree in 1963 from the *Universidade do Brasil* in Rio de Janeiro, Brazil (now, the *Universidade Federal do Rio de Janeiro*). After working in the electronics industry in Brazil during 1964 and 1965, he went to Stanford University, where he received his M.Sc. and Ph.D. degrees. His doctoral thesis dealt with satellite attitude dynamics. During 1969 and 1970 he was a National Research Council postdoctoral research associate at NASA Ames Research Center in California working on satellite dynamics. He joined the Rensselaer faculty in 1986 after rising through the ranks from assistant professor to professor at the University of Cincinnati, which he joined in 1971 as a faculty member in the aerospace engineering and engineering mechanics department. On a sabbatical leave during the first half of 1983 he continued research work on helicopter rotor blade dynamics at the Aeromechanics Laboratory at Ames Research Center. Professor Crespo da Silva's main interests are dynamics (including structural dynamics, satellite dynamics, and helicopter dynamics) and nonlinear oscillations. He has a number of publications in these areas and has received research grants from the National Aeronautics and Space Administration (NASA), the U.S. Air Force, the U.S. Army Research Office, and the National Science Foundation. He is a member of the American Academy of Mechanics, senior member of the American Institute of Aeronautics and Astronautics, and a fellow of the American Society of Mechanical Engineers.

A NOTE TO THE READER

Dynamics is a discipline that consists of the prediction and study of the motion of bodies under the influence of forces. It began to evolve as a rigorous scientific discipline based on solid mathematical principles during the seventeenth century when Sir Isaac Newton (1642–1727) published in July 1686 his famous book *Philosophiæ Naturalis Principia Mathematica*.[1] In that classical work he laid the foundations for dynamics by postulating his three famous laws of motion and his universal law of gravitation. The original work of Newton was directed to the motion of objects that were modeled as "point masses." A number of great scientists, including Leonhard Euler (1707–1783), Joseph Louis Lagrange (1736–1813), and William Rowan Hamilton (1805–1865), carried dynamics much further with their time-lasting contributions.

This book is aimed at an intermediate-level course in dynamics for college/university engineering students. It is also suitable for self-study or as a reference book. It is mostly a book on Newtonian dynamics, but, for completeness, a more advanced chapter (Chapter 7) covering the basics of *Lagrangian dynamics* is included for those readers who want to extend their capabilities in dynamics. As in the other chapters, many examples of application of the theory are also included in Chapter 7.

It is assumed that the reader has had courses in statics, calculus, and basic ordinary differential equations. A working knowledge of these subjects is a prerequisite for a solid understanding of dynamics. Hopefully, beginning-level engineering dynamics courses (which are generally taught in the second year of college) will evolve to be taught in a way to permit the usage, at that level, of this book or, at the very least, of the methodology presented here. The approach used is based on what an engineer actually needs to do to investigate dynamics problems.

This book stresses the use of *fundamentals* for setting up and solving each problem in dynamics, rather than the indiscriminate use of relatively elaborate "formulas" (which most humans will forget), the mere use of which does not contribute to the understanding of the subject matter.

A variety of solved problems is included in each chapter, and, for each problem, an organized approach to the formulation and analysis of the differential equations of motion of dynamical systems is presented. The methodology adopted here may be applied to a wide range of problems (from the very simple to the complex) encountered in engineering and in basic research. It is hoped that the repetitive approach used in this book will help eliminate the *mystery* that many students see in dynamics and the *fear* that many seem to have for the subject.

[1] *The Mathematical Principles of Natural Philosophy*. "Natural philosophy" is what is called "physical science" today.

Dynamics is a fascinating and powerful subject, and the author has been teaching it to university students, and working with it in his own research, for many years.

The differential equations that govern the motion of dynamical systems are generally nonlinear and cannot be solved analytically. In actual engineering practice, computers are used for integrating them numerically. The same is done in this book, especially for problems for which either the motion cannot be investigated analytically or advanced analytical methods that are beyond the scope of this book are needed for such investigations (such as "perturbation methods" for dealing with nonlinear oscillation problems, that are taught at the masters and doctoral levels). The implementation of the numerical integration of differential equations of motion is presented in the form of *block diagrams,* which are constructed simply by connecting appropriate blocks that perform some obvious operation to its input to generate an output. Examples are a time integration, which is done by an *integrator block;* the multiplication of the input by a desired constant, which is done by a *gain block,* etc. Inputs and outputs of blocks are represented by arrows into and out of the block. There is a software called Simulink that is actually based on fully operational block diagrams. It is the block diagram implementation of MATLAB,[2] and both software are widely used at many universities and companies worldwide. Simulink block diagram *models* are portable between different computers. Because of its simplicity, and because it does not involve the writing of computer programs, Simulink is the software used with this book for numerically integrating differential equations and for displaying solutions. Displays, such as plots, are simply obtained by including in a block diagram a block called a *scope* (which stands for "oscilloscope"). A tutorial is presented in Appendix A.

Although Simulink is widely used in this book because of its flexibility and ease of use, and mostly because preparing block diagrams is much less involved than writing computer programs and/or calling functions from a mathematical package for performing the same task, it must be emphasized again that this is a book on dynamics. As such, its main contents, which is dynamics and analysis of motion, should not be viewed as tied to any particular software. Readers and instructors using this book can, of course, use any other appropriate software of their choosing for performing the numerical tasks presented in this book, writing programs using their preferred language.

Several *animation MATLAB* files that can be simply added as another block to a number of Simulink block diagram models accompany this book. They were prepared by the author. Such blocks display, on the computer screen, drawings that resemble certain systems in order to impart to them a motion *governed by the differential equations that were formulated for the system* using the material presented in this book. Two special examples of such blocks are the `animate_nbars` and `animate3d_nbars` blocks. Both can be used in a variety of models (including point mass problems) that consist of an arbitrary number of interconnected straight bars. The `animate_nbars` block is for two-dimensional motion, and the `animate3d_nbars` block animates motion in

[2] Simulink and MATLAB are registered trademark of The MathWorks, Inc.

three-dimensional space. These blocks are described in Appendix A. Such a visualization capability is very helpful for better understanding the resulting motion governed by the differential equations of motion that are formulated for an object to model its dynamic behavior and to perform a number of numerical and visual experiments using such equations. By using this capability you will be able to change parameters and initial conditions in the differential equations of motion that you formulate and see the effect such changes have on the object's behavior. In engineering practice, such "numerical experimentations" are used in design involving dynamical systems and in research work.

A formidable combination for engineers, physicists, and others, for analyzing dynamical systems should have the following three ingredients, all three of which are part of this book:

1. Knowledge of dynamics for formulating the differential equations of motion of a system
2. Knowledge of approximate analytical methods for yielding essential information about the response of the system
3. Use of numerical software as a tool for solving the differential equations that govern the motion, and even for animating the motion

> The CD that comes with this book contains a number of Simulink models, animation blocks, and other files. They are listed and described in Appendix E. To use them, follow the instructions in Section A.2.

Chapter 1 presents essential material that is of fundamental importance for the study of subsequent chapters. It also introduces the notation used throughout the book. It is assumed that you have already learned most of that material from other courses and that you are familiar with it. However, you should read it carefully and study it thoroughly, even if some of the material may appear, at the beginning, to be too basic and simple to you at that stage of your reading. Not doing so may cause you difficulties in appreciating and understanding dynamics and the rest of the material in this book when you proceed to more advanced chapters. The same comment applies to the basic material presented at the beginning of the second chapter. Some of the material in Sections 1.12 to 1.15 of Chapter 1 may be new to you, though. If it is, you should study it more carefully since that material is essential for understanding the behavior of dynamical systems and for analyzing their motion.

> As you feel the material presented becomes progressively more complex, you should read it several times as your need dictates. As you read it again, make notes as you go along, put it aside for a little while and think about it, and then come back and study it again until things start to make more sense to you. This is actually unspoken commonsense. As you progress through the subsequent chapters, your patience and perseverance should start paying dividends to you. Hopefully, some of you will also become dynamicists and will start contributing to the dynamics literature yourselves.

The material presented progresses in a natural manner from kinematics and dynamics of a point mass (Chapter 2) to a system of point masses (Chapter 4), and then to continuum rigid bodies, which are bodies with an infinite number of point masses that are constrained so that the distance between any two of their points is constant.

Unlike in other books, kinematics and dynamics are not separated into different chapters, except in Chapter 3, which deals only with kinematic analysis of mechanisms whose motion takes place either in a plane or in parallel planes. In that chapter, the motion of one of the parts of the mechanism is specified without any concern for determining the external forces that cause the motion. The material in Chapter 3 is a natural extension of the application of the analysis involving velocity and acceleration of a point, and also the use of rotating unit vectors, that first appears in Chapter 2.

The first two sections of Chapter 4 deal with important properties of the motion of a system of point masses. Such properties involve the motion of the center of mass of the system, and the relation involving the moment of the forces applied to the system and the time rate of change of angular momentum of the system. The concept of angular momentum for a system of particles is introduced in that chapter. Chapter 4 constitutes a transition to the study of the motion of rigid bodies. In addition, two important classical problems in celestial mechanics and spacecraft trajectory dynamics are treated in reasonable detail in Chapter 4. Thus, by the time you have studied the material up to that chapter, you will have analyzed important problems in mechanical engineering and in aerospace engineering and been introduced to applications of the fundamentals of vibration analysis that were first presented in Chapter 1.

Chapter 5 deals with dynamics of rigid bodies in "simpler" planar motion so that the analysis of rotational motion involves only one moment of inertia of the body. Such motions are defined on p. 266 in that chapter. Dynamics of rigid bodies that are able to move in any manner in three-dimensional space is presented in Chapter 6, all at an intermediate level of sophistication. Examples 6.8.3 and 6.8.4 in Section 6.8 deal with important problems in spacecraft dynamics and further introduce you to that field of study. They are slightly more advanced but are still appropriate for an intermediate dynamics course.

Chapter 7 is a more advanced chapter that covers the basics of Lagrangian dynamics and is included in this book for those readers who want to extend their capabilities in dynamics. At least some of that material is covered in intermediate dynamics courses at several universities and colleges, especially when it comes to using the Lagrangian formulation for obtaining the differential equations of motion of dynamical systems. In contrast with the Newtonian methodology, the Lagrangian methodology does not involve the formulation of acceleration. Its fundamental quantities are the kinetic energy of the motion and the *virtual work* (which is defined in that chapter) done by the forces applied to the system, and both are scalar quantities. The Lagrangian methodology is actually simpler *to apply* to problems than the Newtonian methodology, and *classical integrals of the motion* (or *constants of the motion*) are easily obtained (when they exist, of course!) using that methodology. Free-body diagrams are rarely needed for working with the Lagrangian methodology. As in other chapters, many examples of the application of the methodology are also included in Chapter 7.

The subject of vibrations/oscillations is introduced throughout the book, starting in Chapter 1, as a natural application of "investigation of motion." Equilibrium solutions, linearization for small motions about any equilibrium, and the concept of stability are also presented in that chapter. Several examples throughout the book deal with the motion of dynamical systems, both linear and nonlinear, frequently accompanied by numerical integration of the differential equations that govern the motion.

Chapter 8 is short and complements the rest of the book with additional material on vibrations/oscillations, including the analysis of the response of a linear system to a sinusoidal forcing function, and a very brief introduction to the homogeneous response of multiple-degree-of-freedom linear systems. Two classical problems are presented in Chapter 8, namely, the analysis of the *Foucault pendulum* and the analysis of a *vibration absorber*. The Foucault pendulum demonstrates the effect of the rotation of the earth on the motion of a simple pendulum. It also brings into evidence when one can reasonably neglect such an effect and when one must take it into consideration when motion relative to a rotating planet as viewed by an observer on that planet is observed for a longer time. Vibration absorbers are practical devices used in engineering, and the one considered in Chapter 8 is one particular type of such device.

The book has several appendices. The first one, Appendix A, is a Simulink tutorial and a brief introduction to MATLAB. The best time to read that material is when you encounter the use of Simulink in Chapter 1. The use of the animation blocks mentioned earlier is also described in Appendix A. A third animation block, for use with the two-body problem of space mechanics, is presented in that appendix and described in Chapter 4. Appendices B and C consist of important material, but that material is somewhat more advanced and is optional. A suggested sample of computer problems, using numerical integration with Simulink and that can be used in formal courses, are included in Appendix D. Appendix E consists of the listing of the files included on the CD that accompanies the book, with a brief explanation about them. Answers to some selected problems are given in Appendix F. Several references for more advanced studies are listed in Appendix G.

The author has been extensively using the material in this book in two intermediate dynamics one-semester courses at Rensselaer taken by mechanical engineering seniors and by aerospace engineering seniors. Graduate students have also frequently taken them. The material in Chapter 1, parts of Chapter 2, the first two sections of Chapter 4, and Appendices A and B is common to both courses. The material in Appendix A is given in two or three hands-on tutorial classes (for a total of 3 hours or so) with the students having access to a computer. The material in Chapter 3 and parts of Chapters 5, 6, and 8 is covered in the mechanical engineering course. The material in Chapter 4 and parts of Chapters 5, 6, and 8 is covered in the aerospace engineering course. The material in Chapter 7 has been optional for some more advanced students and has been included as an additional assignment to a number of graduate students. Home assignments involving numerical integration of differential equations of motion formulated by the students has been an integral part of the learning experience in such courses. Lagrange's equation is also used in other senior-level courses. This book allows instructors to design their own course contents using selected parts of it.

Marcelo R. M. Crespo da Silva

ACKNOWLEDGMENTS AND DEDICATION

I am grateful to several anonymous reviewers for their useful suggestions and to Jonathan Plant, senior sponsoring editor at McGraw-Hill, for his interest in this book. Several students at Rensselaer provided me with very useful comments. In particular, I express my sincere appreciation to Mark Jensenius for checking most of my solutions to the proposed problems at the end of the chapters. A special thank you goes to Mrs. Harriet Borton, at Rensselaer, for her expert advice every time I could not solve a LaTeX problem on my own while typesetting the original manuscript, and to Ms. Elizabeth Doocey, from my department at Rensselaer, for her cheerful and expert answers to several of my editorial questions. I am also grateful to Ms. Courtney Esposito at The Mathworks, Inc., for her assistance and pivotal role in securing permission for me to include on the enclosed CD the *Auto-Scale Graph* scope block mentioned in Appendix A and two of the files mentioned in Section E.2. The entire book team did an excellent job in the production of this book and I sincerely appreciate their efforts.

I made many personal sacrifices while writing this book, and I am glad my cheerful wife Linda d'Escoffier Crespo da Silva has been at my side and that I have her support. I dedicate this book to her.

Marcelo R. M. Crespo da Silva

SYSTEM OF UNITS USED IN THE BOOK

This book uses the MKS (meter-kilogram-second) International System of units. As per standard convention, these units are abbreviated as m, kg, and s, respectively. The unit of force in this system, which is the newton, is abbreviated, in accordance with international convention, as N, where $1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2$.

In some of the problems, lengths are also quoted either in centimeters (cm) or in millimeters (mm), and mass in grams (g).

In some examples, especially those involving space applications in Chapter 4, the equivalent distance in miles is also given after the distance is quoted in kilometers (km). In such cases, velocity may be quoted in both kilometers per hour (km/h) and in miles per hour (mi/h).

WEBSITE FOR THIS BOOK

A website for the book is available at *www.mhhe.com/crespodasilva*. This site contains a complete instructor's guide with solutions for all chapter problems, and further information on using the book and software for typical intermediate-level dynamics courses. Instructors using the text need to receive password information from their McGraw-Hill sales representative to access this protected information.

HOW TO USE THE SOFTWARE ON THE ENCLOSED CD

Follow the instructions in Section A.2 on p. 507. A list of all the files, and their descriptions, is given in Appendix E.

CONTENTS

CHAPTER 1

Essential Material for Dynamics

The fundamental quantities that appear in Newtonian dynamics for analyzing the motion of a point mass (or mass particle[1]) are the *absolute acceleration* of the particle and the *resultant force acting on the particle,* and both are vector quantities. The absolute acceleration is, by definition, a vector that is equal to the time derivative of the *absolute velocity* of the particle (which is, in turn, the time derivative of the *absolute position vector* for the particle). The resultant force is determined directly from the free-body diagram for the particle, which is a diagram showing all the individual forces acting on the particle. These fundamental quantities, and the meaning of the adjective *absolute,* are introduced in Chapter 2.

This chapter presents the basic material that is essential in the study of dynamics. It includes a basic review of vectors and differential equations. Reviews of free-body diagrams, Newton's laws of motion, and Newton's law of universal gravitation are also included in this chapter. Details of the calculation of the gravitational force acting on a body are presented in Section 1.8, together with a discussion of the meaning of *weightlessness*.

1.1 Vector and Scalar Quantities: A Brief Review

A *scalar* quantity is one that can be represented by a number, a variable, or a function of one or more variables. Examples of scalar quantities are the mass, temperature, energy of a body and the speed of a car. Such quantities may be functions of a scalar variable (or variables), such as time. In such a case, one has a *scalar function.*

A *vector* is a quantity that has a magnitude (i.e., a "length") as well as a direction. In addition, to qualify as vectors, such quantities must obey the *parallelogram rule of addition* and be *commutative* (i.e., the result of the sum

[1] A *mass particle,* which is simply referred to as a *particle,* is an idealized point having a constant mass.

of two vectors must be independent of the order of summation). For example, to locate a point B relative to a point A in space one needs to specify not only the distance from A to B but also the direction of the line segment from A to B. The vector (which specifies distance and direction) that indicates the location of a point B relative to a point A is called the *position vector* of B relative to A. Additional examples of vectors are the velocity and acceleration of a point and the force acting on a body. Notice that, contrary to everyday speech, there is a distinction in dynamics between the velocity and speed of a point. *Speed* is a scalar quantity, always positive, that is equal to the magnitude of the velocity vector. The *velocity* of a point is a vector and, as such, contains information about the direction of the motion of the point.

In books and technical journals, vectors are commonly represented by boldface letters (e.g., **A**, **v**), but in this book they are represented by a letter (or by two letters written side by side) with an arrow on top (e.g., $\vec{r}$, $\vec{v}$, $\overrightarrow{AB}$). The representation adopted here is the one everyone uses either on their own or in classrooms because of the obvious inconvenience of using boldface symbols in those situations.

A *unit vector* is a vector of unit magnitude and no dimensions (i.e., no units). A unit vector is represented in this book by a symbol with a caret on top (e.g., $\hat{r}$, $\hat{e}_1$). Clearly, for any vector $\vec{r}$, whose magnitude is $|\vec{r}|$, the vector $\vec{r}/|\vec{r}|$ is the same as the unit vector $\hat{r}$ in the direction of $\vec{r}$.

Any vector $\vec{r}$ in three-dimensional space may be expressed in terms of any three independent basis vectors. However, it is most convenient to choose the basis vectors to be three orthogonal unit vectors, each one of which is chosen simply to indicate a reference direction in space. Such sets of unit vectors are called *unit vector triads,* or simply *triads*. For example, the vector $\vec{r} = \overrightarrow{AB}$ shown in Fig. 1.1 may be represented in the $\{\hat{x}, \hat{y}, \hat{z}\}$ triad (also shown in Fig. 1.1) as $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$, where x, y, and z are distances measured from point A, along the three orthogonal axes that are parallel to the $\hat{x}$, $\hat{y}$, and $\hat{z}$ unit vectors shown in the figure.

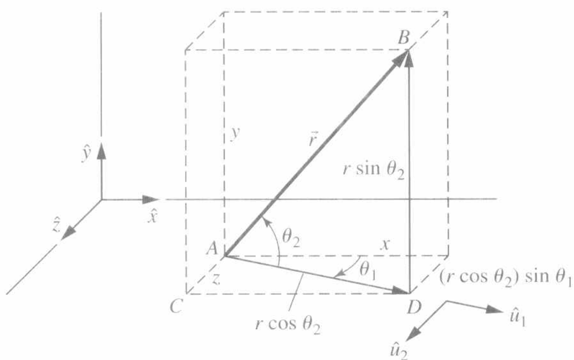

Figure 1.1 An arbitrary vector $\vec{r}$.