A Program for Self-instruction

McFadden

MODERN TRIGONOMETRY

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MYRA McFADDEN
Department of Mathematics
Bucknell University
Lewisburg, Pennsylvania

PROGRAM EDITORS

J. William Moore
Department of Education
Bucknell University
Lewisburg, Pennsylvania

Wendell I. Smith
Department of Psychology
Bucknell University
Lewisburg, Pennsylvania

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Preface

In a sense, the century in which we live has been, and is, a century of revolution—in particular, of revolutions in knowledge. Mathematics has been centrally involved in the revolutions in the sciences—in both the social and the natural sciences. Of equal importance to its participation in the scientific revolutions is the revolution within mathematics itself. Teachers of mathematics are rapidly becoming aware of the changing views of the appropriate content for courses in mathematics—courses bearing such familiar titles as algebra, geometry, trigonometry, and calculus. Through the impetus provided by the work of the School Mathematics Study Group, the Commission on Mathematics of the College Entrance Examination Board, the University of Illinois Committee on School Mathematics, and similar groups, text materials in modern mathematics for secondary-school and college curricula have been developed in the past half-dozen years.

What is "modern" mathematics? How does it differ from traditional mathematics? Why should we study it? What can be done with it? These are questions raised frequently by both teachers and students of the traditional mathematics. They deserve an explanation, although it is not an easy task to provide specific answers to such general questions. Much of modern mathematics is not new. Little of it was developed recently; in fact, many of the most important ideas in modern mathematics, the concepts of set, relationship, and function, for example, probably antedate man's recorded history. Further, the formal development and introduction of many of the concepts into mathematics during the nineteenth and twentieth centuries were preceded by some three hundred years of research. To mortal man, "modern" does not mean anything as old as a sixteenth-, seventeenth-, or eighteenth-century invention! The "new" mathematics is modern largely in the sense that its importance has become generally recognized by mathematicians only during this century. The great significance of the "new" mathematics has been widely recognized as a result of (1) the tremendous amount of research in mathematics and science that has been done in this century and (2) the technological revolution which goes on around us unabated.

The chief task of the creative mathematician is the development of theorems and the construction of proofs for them. In the research necessary for attaining this goal, the need for more languages which permit precise definition of terms has become apparent. Increasingly in the recent history of mathematics, the importance of formal logic has been recognized. The use of axioms, theorems, and proofs has been extended to all mathematics and not restricted simply to Euclid's geometry.

The extension of the logical system to the algebras has been particularly fruitful, since it has brought more attention to the *structure* of algebras and has helped to decrease the emphasis on an algebra as a means only of solving equations, with the accompanying reduction of overemphasis on the development of manipulative skills. This is not to imply that solving equations and manipulating a number system with skill are unimportant. On the contrary, the ability to solve and the ability to manipulate have great utility *providing they are applied with understanding.* Without understanding, one manipulates numbers and solves problems as though mathematics were a bag of tricks. The important developments in the mathematics underlying the technological revolution that has produced automation and the digital computer were not new "tricks." The increasing number of applications of mathematics in economics, biology, sociology, and psychology is not the result of transplanting old "tricks" to new subject matter.

The presentation of trigonometry in this book follows the recommendations of the Commission on Mathematics and includes the topics suggested by the School Mathematics Study Group.

Less time is given to the numerical solution of triangles and the usual work with logarithms, so that greater emphasis can be placed on the analytical and functional aspects of trigonometry, which have very important applications in science and industry.

The word *trigonometry* suggests triangle measurement (the numerical solution of triangles). Today, however, many of the most important applications of trigonometry do not involve angles or triangles specifically. The study of wave motion and periodic phenomena, such as sound waves, alternating electric currents, and business cycles, rests upon the principles developed in trigonometry, though not from the angles concept explicitly.

The geometric approach to trigonometry (through the study of angles) does make the introduction meaningful, and it is the one used in the first part of the course.

The only prerequisites which have been assumed for successful completion of this programmed text are (1) a desire to increase one's understanding of the "why" in mathematics and (2) two years of high-school algebra and one year of high-school geometry.

Graph paper, a ruler, and a compass are recommended for preparing all drawings.

HOW TO USE THIS TEXT

The material presented to you in this book is in the form of a program for self-instruction. The subject matter covered in this program has been broken into items or frames which permit you to learn efficiently by studying and answering each step or frame separately.

The material in the text will be arranged in this manner:

1 To assist the student to learn mathematics more efficiently, a self-instruction program provides units of new information in separate frames. The information is broken down into separate question-and-answer frames to make it easier for you to ________.

learn

2 Thus each program item, or ______, provides new information which you will read carefully and for which you provide an answer in writing. You will then be able to compare your answer with the correct one given below each question frame.

frame

Usually you will find that the most effective way to study a program for self-instruction is to read and study each frame carefully, covering with a sheet of paper or an index card all the material on the page below the frame which you are studying. It is best to study definitions and formulas thoroughly as you go along, so that you will be able to acquire new information, step by step, as you go through this self-instruction course.

After you have studied a question frame, write out your answer fully on a separate piece of paper. Then move the sheet of paper down the page until you uncover the correct answer below the question frame. Compare your answer with the answer given in the book. Later topics in the program build on the material covered in earlier sections; hence you will find it desirable to study again each question on which you make an error and to correct your first answer to that question.

"Optional frames" are provided throughout the book for the purpose of giving you additional experience with a concept or more difficult problems and applications. If you are in a hurry, you may skip them. If you enjoy a challenge, try them.

IMPORTANT THINGS TO REMEMBER

1. Always read the question frame first.

2. Write your answer out in complete form.

3. Compare your answer with the answer given in the answer frame of the text.

4. If you make an error, correct your answer before going on to the next question frame.

5. Learning is an active process. You must do something; that is, you must respond to each question by writing your answer before you read the answer provided.

REVIEW

To enable you to review the material in a programmed text, the author has made three provisions: (1) At intervals, *review frames* have been included. These will be recognized by the symbol $\bigtriangledown$ preceding the frame number. Review frames are also listed in the Contents. If you are unable to answer a review frame correctly, you should reread the teaching frames immediately preceding it to be certain that you have acquired the information before proceeding to the next set of teaching frames. (2) *Self-tests* have been provided for each section in the text. The answers to the test questions are given at the back of the book. It is advisable to work through each test before proceeding to the next section. If you find that you are not able to answer most of the questions correctly, you should reread the teaching frames which contain the information needed to answer correctly any question upon which you have not succeeded. (3) An *index* to the teaching frames in which key concepts are presented is provided at the back of the book.

Unlike nearly all textbooks, a self-instructional program is developed in a manner which maximizes its ability to teach. If the program has been properly constructed, the reader who carefully *follows the directions* for its use will be able to learn its contents with no other help.

SELF-TESTS

Six self-tests are provided in this text to assist you in measuring your progress. Each test *precedes* the unit of material of which it is the assessment measure. When the text is used for self-study, it is recommended that you take the test before reading the material with which it is concerned; if your score on the test is high, you may skip that section and proceed with the next. Self-test 1 is concerned with the material contained in Frames 1-398, Self-test 2 with Frames 399-774, Self-test 3 with Frames 775-1124, Self-test 4 with Frames 1125-1384, Self-test 5 with Frames 1385-1896, and Self-test 6 with the remainder of the book.

PANELS

At several points in the text, you will be asked to refer by number to one of a set of panels. Each panel contains information required for solving or understanding problems. These panels are placed at the end of the book for your convenience.

J. William Moore
Wendell I. Smith

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CONTENTS

SELF-TEST 1

1 (a) Which of the following are quadrantal angles? (b) Which are coterminal?

$180°$, $0°$, $-\frac{1}{2}$ rev, $\frac{3}{8}$ rev, $\frac{11}{8}$ rev, 1 rev

2 If a central angle θ cuts off an arc of $4\pi/3$ in. on a circle having a radius of 8 in., then θ = ____ radians.

3 (a) Change $\pi/3$ radians to degrees. (b) Change $150°$ to radians. (c) Change $225°$ to radians. (d) Change $11\pi/6$ radians to degrees. (e) Change $10{,}000°$ to radians. (f) Change -7π radians to degrees.

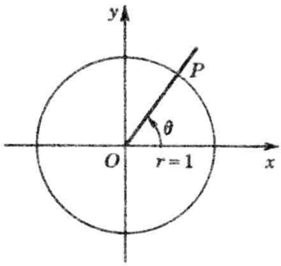
4 One radian is approximately ____°.

5 What is the length of the radius of a circle if an arc of 6π in. is cut off by a central angle of $3\pi/2$ radians.

6 Find the area of a sector of a circle whose radius is 3 if the central angle is $\pi/4$.

7 On a rectangular coordinate system locate the following points: $P_1(3, 7)$, $P_2(-1, -3)$, $P_3(4, -2)$, and $P_4(-4, 0)$.

8 (a) In terms of ordinate and abscissa of point P in the sketch, give the definitions for sine and cosine of angle θ.
(b) In terms of x, y, and r, define sine and cosine of angle θ if $P(x, y)$ is a point on the terminal side of θ and r is the distance from P to O.

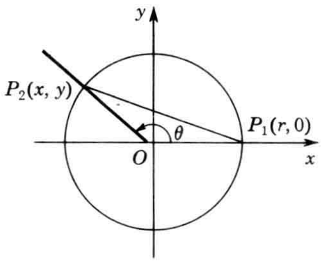


9 In which quadrants may an angle terminate if (a) the sine is negative and (b) the cosine is negative?

10 $P(6, -10)$ is a point on the terminal side of an angle θ in standard position. Sketch the angle; then find $\sin\theta$ and $\cos\theta$.

11 If $\cos\theta = -\frac{4}{5}$ and θ terminates in quadrant II, find (a) $\sin\theta$ and (b) $\tan\theta$. (Make a sketch.)

12 Two points on a circle of radius r are $P_2(x, y)$ and $P_1(r, 0)$ as shown in the sketch. Derive a formula for the length of the chord P_2P_1.



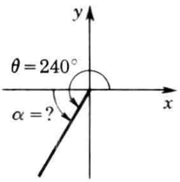
13 Fill in the following table without using Panel 1:

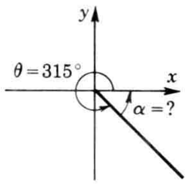
θ, radians	Degrees	sin θ	cos θ	tan θ
$\pi/6$				
$\pi/4$				
$\pi/2$				

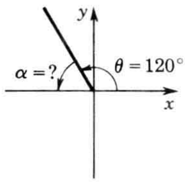
14 $P(-1, \sqrt{3})$ is a point on the terminal side of a positive angle in standard position. **(a)** Sketch the angle. **(b)** Report the radian measure of this positive angle. **(c)** Find sin θ, cos θ and tan θ.

15 Find the numerical value for $\sin 300^\circ + (\cos 30^\circ \cdot \sin 150^\circ)$.

16 Find the indicated angle in each sketch. Find the tangent of each angle θ.





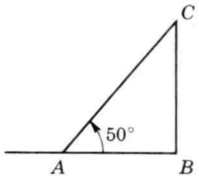


(1) (a) $\theta = 240^\circ$
$\alpha =$ ____
tan $\theta =$ ____.

(2) (b) $\theta = 315^\circ$
$\alpha =$ ____
tan $\theta =$ ____.

(3) (c) $\theta = 120^\circ$
$\alpha =$ ____
tan $\theta =$ ____.

17 Given the right triangle ABC with B the right angle and the length of BC = 12 ft and angle A = 50°, find **(a)** the measure of angle C and **(b)** the length of AC.



18 A tree casts a shadow 20 ft long at a time of day when the angle of elevation of the sun is 25°. How tall is the tree?

19 Simplify to a positive angle less than 360°: (a) $\sin 405^\circ = \sin$ ___, (b) $\cos 600^\circ = \cos$ ___, (c) $\tan(-90^\circ) = \tan$ ___, (d) $\sin 840^\circ = \sin$ ___.

20 $\theta = 14\pi/3$. Find α, $0 \leq \alpha < 2\pi$, coterminal with θ satisfying the statement $\theta = (\alpha + k \cdot 2\pi)$, k is an integer.

21 $\theta = 980^\circ$. (a) Make a sketch of this angle in standard position. (b) Show the reference angle you should use to determine $\sin 980^\circ$, $\cos 980^\circ$, and $\tan 980^\circ$. (c) Consult Panel 1 and give the numerical values.

22 If $\sin\theta = 0$ and $\cos\theta = -1$, (a) find the measure of θ in radians, $0 \leq \theta \leq 2\pi$, and (b) give θ in degrees.

23 As θ increases from 180 to 270°, what changes take place in $\sin\theta$? (Be sure to give the maximum and minimum values of $\sin\theta$ in your discussion.)