ANALYTIC TRIGONOMETRY with applications

Second Edition

RAYMOND A. BARNETT

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SECOND EDITION

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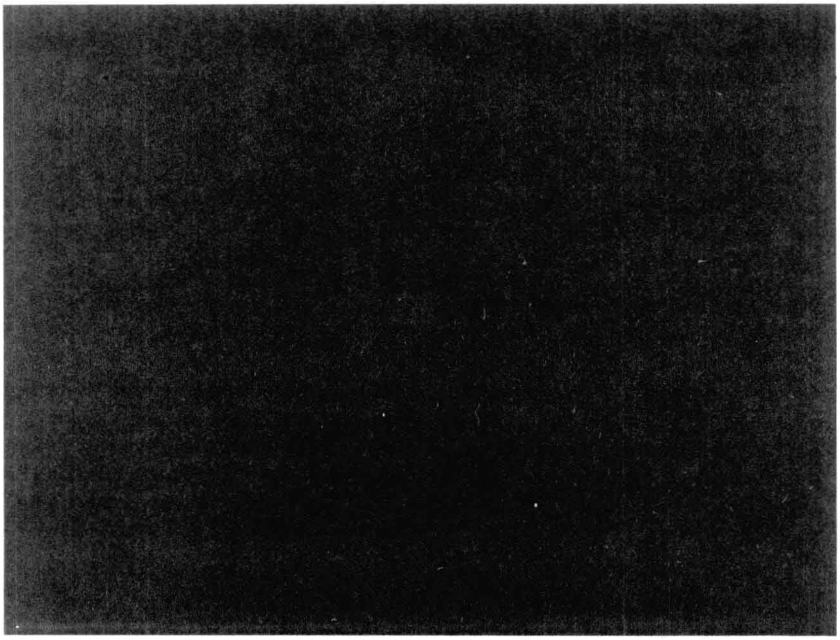
Merritt College



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Numbers in parentheses are section numbers where material is explained.

TRIGONOMETRIC IDENTITIES (4.1, 4.2, 4.3)

I-1 $\csc x = \dfrac{1}{\sin x}$

I-2 $\sec x = \dfrac{1}{\cos x}$

I-3 $\cot x = \dfrac{1}{\tan x}$

I-4 $\tan x = \dfrac{\sin x}{\cos x}$

I-5 $\cot x = \dfrac{\cos x}{\sin x}$

I-6 $\sin^2 x + \cos^2 x = 1$

I-7 $\tan^2 x + 1 = \sec^2 x$

I-8 $1 + \cot^2 x = \csc^2 x$

I-9 $\sin(-x) = -\sin x$

I-10 $\cos(-x) = \cos x$

I-11 $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

I-12 $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

I-13 $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

I-14 $\sin\left(\dfrac{\pi}{2} - y\right) = \cos y$

I-15 $\cos\left(\dfrac{\pi}{2} - y\right) = \sin y$

I-16 $2 \sin x \cos y = \sin(x + y) + \sin(x - y)$

I-17 $2 \cos x \cos y = \cos(x + y) + \cos(x - y)$

I-18 $2 \sin x \sin y = \cos(x - y) - \cos(x + y)$

I-19 $\sin 2x = 2 \sin x \cos x$

I-20 $\cos 2x = \cos^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

I-21 $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

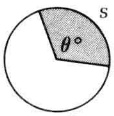
I-22 $\sin \dfrac{x}{2} = \pm\sqrt{\dfrac{1 - \cos x}{2}}$

I-23 $\cos \dfrac{x}{2} = \pm\sqrt{\dfrac{1 + \cos x}{2}}$

(Sign in I-22 and I-23 is determined by quadrant in which $x/2$ lies.)

ANGLES AND ARCS (1.2, 2.2)

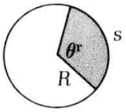
$\dfrac{\theta°}{360°} = \dfrac{s}{C}$



C (circumference)

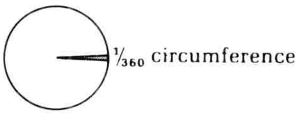
$\theta^r = \dfrac{s}{R}$

$s = R\theta^r$

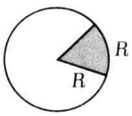


DEGREES AND RADIANS (1.2, 2.2)

$\dfrac{\theta°}{180°} = \dfrac{\theta^r}{\pi \text{ rad}}$



1°



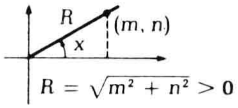
1 radian

TRIGONOMETRIC FUNCTIONS (2.3, 2.4, 2.5)

$\sin x = \dfrac{n}{R}$ $\csc x = \dfrac{R}{n}$

$\cos x = \dfrac{m}{R}$ $\sec x = \dfrac{R}{m}$

$\tan x = \dfrac{n}{m}$ $\cot x = \dfrac{m}{n}$

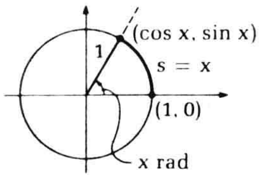


(x in degrees or radians)

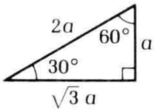
For x any real number and T any trigonometric function,

$T(x) = T(x \text{ rad})$

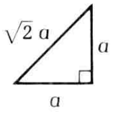
For a unit circle:



SPECIAL TRIANGLES (2.5)

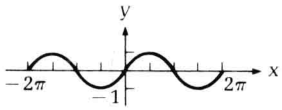


30°–60° triangle

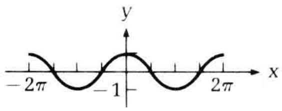


45° triangle

GRAPHING TRIGONOMETRIC FUNCTIONS



$y = \sin x$



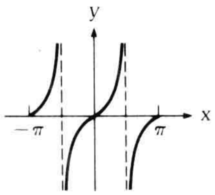
$y = \cos x$

$y = A\sin(Bx + C)$ $\quad y = A\cos(Bx + C)$

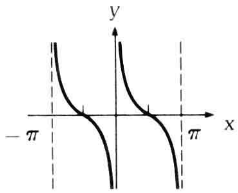
Amplitude $= |A|$ $\quad$ Period $= \dfrac{2\pi}{B}$

Frequency $= \dfrac{B}{2\pi}$

Phase shift $= \left|\dfrac{C}{B}\right| \begin{cases} \text{left if } C/B > 0 \\ \text{right if } C/B < 0 \end{cases}$



$y = \tan x$



$y = \cot x$

$y = A\tan(Bx + C)$ $\quad y = A\cot(Bx + C)$

Period $= \dfrac{\pi}{B}$

Phase shift $= \left|\dfrac{C}{B}\right| \begin{cases} \text{left if } C/B > 0 \\ \text{right if } C/B < 0 \end{cases}$

INVERSE TRIGONOMETRIC FUNCTIONS (5.1, 5.2)

$y = \mathrm{Sin}^{-1}\, x$ means $x = \sin y$ and $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$

$y = \mathrm{Cos}^{-1}\, x$ means $x = \cos y$ and $0 \le y \le \pi$

$y = \mathrm{Tan}^{-1}\, x$ means $x = \tan y$ and $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

(Continued inside back cover)

METRIC UNITS

STANDARD UNITS OF METRIC MEASURE

Meter (m):	Length (approx. 3.28 ft)
Liter (l):	Volume (approx. 1.06 qt)
Gram (g):	Weight (approx. 0.035 oz)

IMPORTANT PREFIXES

kilo	(× 1,000)	deci	(× 1/10)
hecto	(× 100)	centi	(× 1/100)
deka	(×10)	milli	(× 1/1,000)

ABBREVIATIONS

Length		*Volume*		*Weight*	
m	Meter	l	Liter	g	Gram
km	Kilometer	kl	Kiloliter	kg	Kilogram
hm	Hectometer	hl	Hectoliter	hg	Hectogram
dkm	Dekameter	dkl	Dekaliter	dkg	Dekagram
dm	Decimeter	dl	Deciliter	dg	Decigram
cm	Centimeter	cl	Centiliter	cg	Centigram
mm	Millimeter	ml	Milliliter	mg	Milligram

CALCULATOR CONVERSIONS

English to Metric

Length

Inches ⊠ 2.540 ⊟ Centimeters
Feet ⊠ 30.48 ⊟ Centimeters
Yards ⊠ 0.9144 ⊟ Meters
Miles ⊠ 1.609 ⊟ Kilometers

Volume (U.S.)

Pints ⊠ 0.4732 ⊟ Liters
Quarts ⊠ 0.9464 ⊟ Liters
Gallons ⊠ 3.785 ⊟ Liters

Weight

Ounces ⊠ 28.35 ⊟ Grams
Pounds ⊠ 453.6 ⊟ Grams

Angle Measure

Degrees ⊠ 0.01745 ⊟ Radians

Temperature

Fahrenheit (°F) ⊟ 32 ⊟ ⊠ 5 ⊡ 9 ⊟ Celsius (°C)

Metric to English

Length

Centimeters ⊠ 0.3937 ⊟ Inches
Centimeters ⊠ 0.0328 ⊟ Feet
Meters ⊠ 1.0936 ⊟ Yards
Kilometers ⊠ 0.6215 ⊟ Miles

Volume

Liters ⊠ 2.1133 ⊟ Pints
Liters ⊠ 1.0567 ⊟ Quarts
Liters ⊠ 0.2642 ⊟ Gallons

Weight

Grams ⊠ 0.0353 ⊟ Ounces
Grams ⊠ 0.00221 ⊟ Pounds

Angle Measure

Radians ⊠ 57.30 ⊟ Degrees

Temperature

Celsius (°C) ⊠ 9 ⊡ 5 ⊞ 32 ⊟ Fahrenheit (°F)

$y = \text{Cot}^{-1} x$ means $x = \cot y$ and $0 < y < \pi$

$y = \text{Sec}^{-1} x$ means $x = \sec y$ and $0 \le y \le \pi$, $y \ne \frac{\pi}{2}$

$y = \text{Csc}^{-1} x$ means $x = \csc y$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, $y \ne 0$

(Principal values for $\text{Sec}^{-1} x$ and $\text{Csc}^{-1} x$ are not formally established.)

INVERSE TRIGONOMETRIC IDENTITIES (5.2)

For T any trigonometric function and T^{-1} its inverse function, $T[T^{-1}(x)] = x$ and $T^{-1}[T(x)] = x$.

$\text{Cot}^{-1} x = \text{Tan}^{-1}(1/x), \quad x > 0$

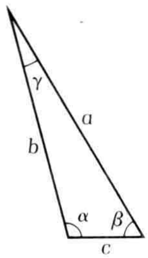
$\text{Cot}^{-1} x = \pi + \text{Tan}^{-1}(1/x), \quad x < 0$

$\text{Sec}^{-1} x = \text{Cos}^{-1}(1/x), \quad x \ge 1 \text{ or } x \le -1$

$\text{Csc}^{-1} x = \text{Sin}^{-1}(1/x), \quad x \ge 1 \text{ or } x \le -1$

LAW OF SINES (6.1)

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$



LAW OF COSINES (6.2)

$a^2 = b^2 + c^2 - 2bc \cos \alpha$

$b^2 = a^2 + c^2 - 2ac \cos \beta$

$c^2 = a^2 + b^2 - 2ab \cos \gamma$

If $\beta = 90°$, then

$b^2 = a^2 + c^2$ Pythagorean theorem

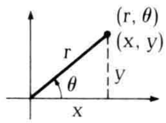
POLAR COORDINATES (7.1)

$r^2 = x^2 + y^2$

$x = r \cos \theta$

$y = r \sin \theta$

$\tan \theta = y/x$



TRIGONOMETRIC FORM OF A COMPLEX NUMBER (7.3)

$$\begin{aligned} x + iy &= r[\cos(\theta + 2n\pi) + i \sin(\theta + 2n\pi)] \\ &= r \operatorname{cis}(\theta + 2n\pi), \quad n \in I \end{aligned}$$

DE MOIVRE'S THEOREM (7.4)

nth power of z:

$$z^n = (x + iy)^n = (r \operatorname{cis} \theta)^n = r^n \operatorname{cis} n\theta, \quad n \in N$$

nth roots of z:

$$r^{1/n} \operatorname{cis} \frac{\theta + 2k\pi}{n}, \quad k = 0, 1, \ldots, (n - 1)$$

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with applications

SECOND EDITION

PREFACE

The first edition of *Analytic Trigonometry with Applications* received many compliments from users for its clarity of exposition and wide selection of significant and interesting real-world applications. We hope that this new edition, reflecting users' experience and recommendations, will prove even more effective in the classroom.

The prerequisites for the course still remain $1\frac{1}{2}-2$ years of high school algebra and 1 year of high school geometry or their equivalents.

Key changes from the first edition include:

1. All review material has been placed in the appendixes (see the Contents). It can be referred to as needed or treated systematically, depending on the background of a class.
2. Chapters have been rearranged slightly for better continuity. For example, the chapter on miscellaneous applications (old Chapter 4) is now Chapter 8, the last chapter in the book.
3. All statements of theorems and definitions were carefully checked and modified where necessary to improve student understanding.
4. Material on trigonometric identities, inverse functions, and trigonometric equations was carefully reviewed, some parts rewritten, and many parts expanded. Identities are now presented earlier in the text.
5. The transition from the definition of trigonometric functions for a right triangle to trigonometric functions on a unit circle has been improved.
6. All exercise sets were thoroughly checked and many were expanded.
7. Calculator-related problems are more plentiful.
8. Computational accuracy and significant figures receive more attention. A new section on scientific notation and significant figures appears in Appendix A (Section A.1).
9. The use of metric units has been increased from about 50% to about 85%.
10. More applications have been distributed throughout the text. Chapter 8 (old Chapter 4) provides a sampler of applications from a variety of fields. This chapter may be used to supplement other chapters as interest and time permits.
11. A section on vectors has been added to Chapter 6.
12. A functional use of a second color has been introduced to increase clarity of exposition and figures.
13. Angles in decimal degrees now receive as much attention as angles in degrees and minutes.

The following important features from the first edition were retained:

1. The book is written for students. An open, easy-to-read, informal format is used. Most concepts are illustrated by examples, and almost every example is followed by a matching problem to encourage an active rather than passive involvement in the book. (Answers to matching problems are placed at the end of each section just before the exercise set.)
2. To gain student interest quickly, the text moves directly into trigonometric concepts. Review material from prerequisite courses is found in the appendixes and can be reviewed as needed or treated in a systematic way.
3. Problem sets, except in Chapter 8 and the appendixes, are divided into A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and theoretical) levels. Answers to most of the odd-numbered exercises and almost all the chapter review exercises are included at the end of the book.
4. Chapter review exercises are included at the end of each chapter except Chapter 8, which is a resource chapter on applications.
5. The content satisfies the requirements for many technical courses, including physics, analytic geometry, and the calculus sequence.
6. The trigonometric functions are defined first in terms of angle domains, using degree and radian measure side-by-side, and then in terms of real number domains. All of this is done early in the text and is reinforced throughout. By the end of the course students should be relatively comfortable with all three modes, and should be able to shift from one to the other without difficulty.
7. Periodic properties are emphasized, and many applications are included to illustrate the importance of these properties.
8. Computational procedures are set up to encourage the use of inexpensive hand calculators that can add, subtract, multiply, divide, and take square roots.
9. Historical remarks are included where appropriate to provide perspective.
10. A test booklet including quizzes, chapter tests, and final examinations (two forms of each) is available to instructors without charge, through the publisher. Included in the booklet are easy-to-grade solution keys. The format is $8\frac{1}{2}$ by 11 inches to facilitate reproduction.
11. An answer key for even-numbered text problems is included in the back of the test booklet.

Most of us really don't know what a book is like until it's been used in class, and we get feedback on how it works. That is why later editions of texts are better learning and teaching tools than earlier editions. The following reviewers have made an invaluable contribution to the reliability and teachability of this text:

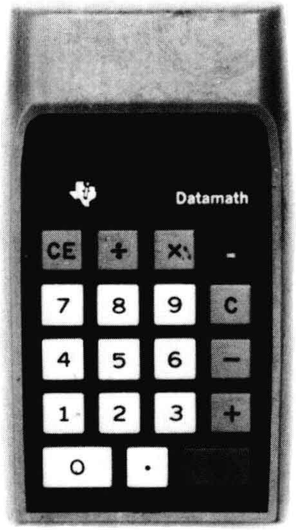
James C. Bolen, University of Texas; Robert A. Chaffer, Central Michigan University; Ted Cullen, California State Polytechnic University; Thaddeus Dillon, Youngstown State University; William V. Gamzon, Los Angeles Harbor College; Louis F. Hoelzle, Bucks County Community College; Gordon Hughes, California State University, Chico; Gordon Nipp, California State University, Los Angeles; Dave Sanders, Miami-Dade Community College; William Stannard, Eastern Montana College; Monty Strauss, Texas Tech University; E. E. Underwood, Utah State University; John Watson, Wingate College.

Special thanks go to Ray Westergard, Merritt College, for his preparation of an outstanding test booklet; to Gary Ling, Merritt College, for his careful checking of answers to all exercises, examples, and problems; and to Ikuko Workman for her expert typing of the final manuscript.

COMMENTS ON HAND CALCULATORS

Although a hand calculator is not required for the mastery of the material in this book, you will find certain sections more interesting if you have access to a calculator—even a very inexpensive (under $15) five-function ($+$, $-$, $\times$, $\div$, $\sqrt{\ }$) model will be helpful. Much of the computational drudgery will then be eliminated for these sections, and you will be able to focus more on important mathematical concepts and relationships and be able to work more realistic problems.

The kind of calculator you buy (should you decide to buy one) depends on your future plans. The inexpensive one described above is even useful



around the home, so it could be a good investment. If you plan to take many science courses and more advanced mathematics, then a scientific calculator with logarithmic, exponential, and trigonometric functions will prove worthwhile. There are also business calculators and combination business–scientific–statistics calculators that are very useful for certain types of applications. Look around a little before you buy. Talk to teachers and other students who own calculators.

Hand calculators are of two basic types relative to their internal logic (the way they compute): algebraic and Reverse Polish Notation (RPN). Let us compare how each would convert 23 minutes and 52 seconds to decimal hours. We need to compute $(23 \cdot 60 + 52)/3{,}600$.

	Press	Display
Algebraic:	[23] [×] [60] [+] [52] [=] [÷] [3,600] [=]	[0.39777778]
Polish:	[23] [ENTER] [60] [×] [52] [+] [3,600] [÷]	[0.39777778]

Some people prefer the algebraic logic and others prefer the Polish. Which is better is still being debated. The answer seems to rest with the type of problems one encounters and individual preferences. The author owns a calculator with Reverse Polish logic but knows many people who are quite happy with the other type.

In any case, when you buy a calculator, whether it is a simple inexpensive model or a somewhat more involved type, READ THE INSTRUCTION MANUAL that accompanies it. A large variety of calculators are on the market, and each is slightly different from the others. Therefore, it is imperative that you take the time to read the instruction manual. If you have a scientific or business calculator, don't try to read and understand everything the calculator can do. This will only tend to confuse you. Read only those sections pertaining to the operations you are or will be using; then return to the instruction manual as necessary when you encounter new operations.

It is important to remember that a calculator is not a substitute for thinking. It can save you a great deal of time in certain types of problems, but you still must know how and when to use it.

POSSIBLE COURSES

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