# TRIGONOMETRY WITH APPLICATIONS

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**For the Student**

*To help you study and understand the course material, a* Solutions and Study Guide, *by Joseph Mutter, is available from your college bookstore. This book provides complete, step-by-step solutions to more than half of the odd-numbered exercises in the text, detailed chapter summaries, and practice chapter tests with complete solutions.*

*To Barbara, Barbara, Becky, Cindy, and Pam*

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# Preface

*Trigonometry with Applications* is designed to provide comprehensive coverage of the usual topics in trigonometry needed by students for later courses in mathematics, engineering, statistics, the natural sciences, or other fields. Students with two years of high school algebra or its equivalent should have the necessary prerequisite skills. The text is organized for maximum instructional flexibility. More than enough material is included for a one-term course.

Chapter 1 introduces trigonometric functions immediately, using the right triangle approach. The development of trigonometric functions of real numbers based on the unit circle follows in Chapter 2. Chapter 3 provides thorough coverage of the graphs of trigonometric functions; applications of right triangle trigonometry and vectors are presented in Chapter 4. Identities, which are first introduced in Chapter 2, are extensively treated in Chapter 5 along with trigonometric equations and inverse trigonometric functions. Polar coordinates and complex numbers are covered in Chapter 6. For added flexibility, the text concludes by examining exponential and logarithmic functions and topics in analytic geometry. A review of needed algebraic principles is provided in the appendix for students who require it; geometric concepts are reviewed within the body of the text when needed. Applications provide practical motivation throughout the book.

## FEATURES

The text is written informally; explanations are carefully worded to ensure student comprehension. Second color is used pedagogically to highlight important steps and emphasize methods and terminology. The many figures and graphs are labeled for easy reference and employ color to clarify the concepts presented. Cautions warn students of common mistakes and special problems, while Notes provide additional explanations or other pertinent information.

### Examples

The text contains over 350 carefully selected examples with detailed step-by-step solutions and helpful side annotations.

### Exercises

There are over 2400 exercises in the text. The exercise sets are carefully graded and begin with paired routine problems that are followed by a variety of challenging extension problems and numerous applications. A set of For Review exercises is included at the end of most exercise sets to help students review previously covered material or prepare for the next section. A collection of review exercises concludes each chapter. Answers to odd-numbered section exercises and to all For Review and Chapter Review exercises are included at the back of the book.

**Applications** To demonstrate the usefulness and practicality of mathematics, applications have been given special attention in this text. Over 300 relevant applied problems from such diverse areas as business, engineering, geology, physics, chemistry, medicine, and agriculture are included in the chapter introductions, examples, and exercises.

**Calculators** Calculators are discussed at appropriate places throughout the text, and illustrations are included for both Algebraic Logic and Reverse Polish Notation. Calculator exercises are not specifically marked, however, since students should learn when to use and when not to use calculators. Appendices on the use of trigonometric function and logarithmic tables are provided for instructors who prefer that their students learn these techniques.

## SUPPLEMENTS

### For the Instructor

The **Instructor's Guide** contains a Placement Test, four different but equivalent tests for each chapter, two final examinations, an extensive test bank of additional problems, and answers to all test items and even-numbered text exercises. As an alternative to the tests in the Instructor's Guide, the **Computer-Assisted Testing System (CATS)** can be used with Apple and IBM computers to construct and print tests. More than 50 **overhead transparencies** featuring key figures from the text are also provided for classroom lectures and presentations.

### For the Student

The **Solutions and Study Guide** contains complete, step-by-step solutions to more than half of the odd-numbered text exercises, detailed chapter summaries, and practice chapter tests with complete solutions.

## ACKNOWLEDGMENTS

We extend our sincere gratitude to the following mathematicians who helped develop this book by reviewing all or part of the manuscript: Richard D. Armstrong, St. Louis Community College; Edgar M. Chandler, Phoenix College; Mary Coughlin, University of Toledo; William Radulovich, Florida Community College; James V. Stewart, Oregon Institute of Technology; Eric Tellenbach, Citrus Community College. We have implemented many of your suggestions to the great benefit of the text.

We extend special appreciation to Joseph Mutter for reviewing the entire manuscript and offering numerous suggestions for improvements, for writing the supplements, and for checking the problems. Thanks go to Diana Denlinger Vanlandingham and Gail Dickerson for typing the manuscript and supplements.

To everyone at Scott, Foresman and Company we are greatly indebted. Special thanks go to Jack Pritchard, Steve Quigley, Terry McGinnis, Sarah Joseph, and Ellen Pettengell.

Finally, we are deeply indebted to our families and in particular to our wives, Barbara and Barbara, who have given us unceasing support, time, and encouragement over the years.

L. Murphy Johnson

Arnold R. Steffensen

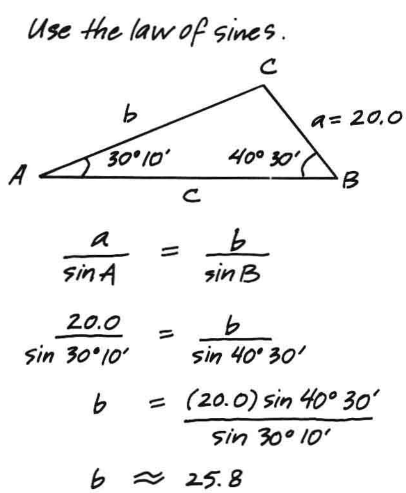
# To the Student

During the past several years we have taught trigonometry to more than 500 students having a variety of career choices. Some were taking mathematics to satisfy graduation requirements, while others were preparing for more advanced courses in mathematics, science, or engineering. Regardless of your educational goals, this text has been written with you, the student, in mind. The material is introduced gradually, building from basic to more advanced skills. We have tried to demonstrate the relevance and usefulness of mathematics throughout the text by including practical everyday applications. As you begin this course, keep in mind these guidelines that are both necessary and helpful.

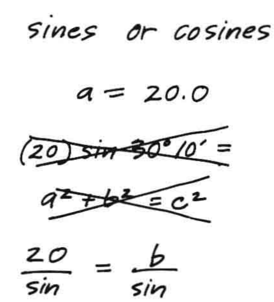
## GENERAL GUIDELINES

1. Mastering trigonometry requires motivation and dedication. Just as an athlete does not improve without commitment to his or her goal, a trigonometry student must be prepared to work hard and spend time studying.
2. Trigonometry is not learned simply by watching, listening, or reading; *it is learned by doing*. Use your pencil and practice. When your thoughts are organized and written in a neat and orderly way, you have taken a giant step toward success. Be complete and write out all details. The following are samples of two students' work on an applied problem. Can you tell which one was more successful in the course?

**Student A**




**Student F**



3. A calculator is useful in any course in trigonometry. Become familiar with your calculator by consulting your owner's manual. Use the calculator as a time-saving device for work with decimals or complicated functions, but do not become so dependent that you use it for simple calculations that can be done mentally. Learn when to use and when not to use your calculator. See "A Word About Calculators" for more information about how calculators can be used with this text.

## SPECIFIC GUIDELINES

1. As you begin to study each section, look through the material for a preview of what is coming.
2. Return to the beginning of the section and study the text and examples carefully. The side comments in color will help you if something is not clear.
3. Periodically you will encounter a CAUTION or a NOTE. The CAUTIONs warn you of common mistakes and special problems to avoid. The NOTEs provide pertinent information or additional explanations.
4. After you have completed the material in the section, check your mastery of the skills and apply what you have learned by working the exercises assigned by your instructor. Answers to the odd-numbered problems are at the back of the text. Complete worked-out solutions to selected odd-numbered problems are also available in the *Solutions and Study Guide*.
5. Exercises marked For Review, located at the end of most exercise sets, help keep previously covered materials fresh in your mind and often prepare you for the next section.
6. After you have completed a chapter, review each section and work the CHAPTER REVIEW EXERCISES. Answers to all these exercises are at the back of the text. To help you prepare for tests, additional chapter review material is also included in the *Solutions and Study Guide*.

If you follow these suggestions and work closely with your instructor, you will greatly improve your chances for success in the course.

# A Word About Calculators

It is assumed that most students will have a hand calculator in this course. Although not absolutely essential, your work will be easier if you use a calculator. The major difference between the types of calculators is in the way they perform various operations. Perhaps more desirable at this level, since the order of operations is the same as in algebra, is the type that uses Algebraic Logic (ALG). The alternative system, Reverse Polish Notation (RPN), is preferred by many mathematicians and professionals, however. Each system, with its advantages and disadvantages, will perform the calculations necessary in this course. Throughout the text we illustrate both systems using ALG for Algebraic Logic and RPN for Reverse Polish Notation. As an example, we show the sequence of steps used in each system to compute

$$\frac{(2)(4.5) - (1.3)^2}{5\sqrt{3}}.$$

| | Display |
|---|---|
| ALG: 2 [×] 4.5 [−] 1.3 [$x^2$] [=] [÷] 5 [÷] 3 [$\sqrt{\ }$] [=] → | 0.8440861 |
| RPN: 2 [ENTER] 4.5 [×] 1.3 [$x^2$] [−] 5 [÷] 3 [$\sqrt{\ }$] [÷] → | 0.8440861 |

Notice that RPN calculators use an [ENTER] key instead of the [=] key found on ALG calculators. This is an essential difference between the two operating systems. Other variations in the types of keys are strictly notational. For example, to change the sign of a number (for entering negative numbers), some calculators have a [+/−] key, while others have a [CHS] key. Also, one calculator uses the [STO] key to place a number in memory, while another has an [M] key. We will try to point out some of the differences that arise as we consider various computations. However, since it is impossible to mention all of these differences, the best advice is to read your owner's manual.

With calculators, slight variations in accuracy due to rounding differences are bound to occur. Most of these will appear in the seventh or eighth decimal place and should not be of much concern. Throughout the text we have not rounded results until the final step, holding calculated values in memory. Even with this agreement, small variations due to individual calculator differences may arise. Don't panic if your calculator gives an answer that disagrees slightly with what we have shown.

Finally, keep in mind that a calculator is a tool for doing complicated computations; it does not think and only reacts to your input. Do not become so dependent on your calculator that you reach for it to make simple computations that can be made mentally. You must learn when a calculator should and should not be used and when your results are reasonable and appropriate.

# Contents

CHAPTER

# 1 INTRODUCTION TO TRIGONOMETRY

The word *trigonometry* is derived from Greek words which mean *three-angle measurement* or *triangle measurement*. Historically, trigonometry was developed as a tool for finding the measurements of parts (sides or angles) of a triangle when other parts were known. As a result, it became indispensable in areas such as navigation and surveying. More recently the study of trigonometry has taken a different direction. Although applications involving triangles remain important, viewing trigonometry in terms of functions of real numbers has expanded its usefulness. A broader spectrum of physical applications have been found, such as harmonic motion and orbits of atomic particles. Two types of problems that can be solved with trigonometry follow.

**FORESTRY ▶**

A small forest fire is sighted due south of a fire lookout tower on Woody Mountain. From a second tower, located 11.0 miles due east of the first, the bearing of the fire is S51°10′W. How far is the fire from the tower on Woody Mountain?


◀ NAVIGATION

An airplane leaves the East Coast of the United States and flies for two hours at 350 km/hr in a direction of 32°40′. Assuming that the East Coast is a straight north-south line, how far is the airplane from the coastline?

Both of these problems are solved with the properties of right triangles. The first is solved using bearing, and the second using navigational direction (see Examples 4 and 5 in Section 1.3).

The approach we have taken to the study of trigonometry includes both the traditional triangle approach in this chapter and the real-number functional approach in the next chapter. We begin with a discussion of angles and angular measure and then proceed to the trigonometric functions of acute angles and numerous applications.

## 1.1 Angles and Angular Measure

### Angles

An **angle** is generated by rotating a **half-line** or **ray** about its end point or **vertex.** See Figure 1. The initial position of the ray, denoted by $OA$, is called the **initial side** of the angle, and the terminal position of the ray, denoted by $OB$, is called the **terminal side.** The symbol $\angle$ represents the word *angle,* and the angle is denoted by $\angle AOB$.

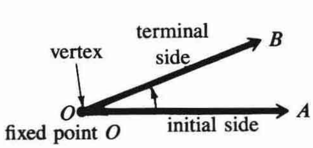



**Figure 1 Positive Angle**

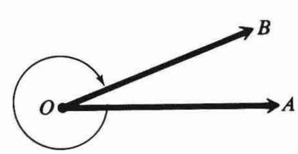



**Figure 2 Negative Angle**

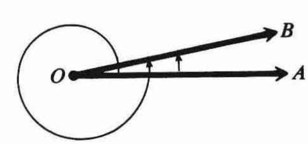



**Figure 3 Coterminal Angles**

If the rotation of the terminal side of an angle is counterclockwise (as in Figure 1), the angle is **positive.** If the rotation of the terminal side is clockwise (as in Figure 2), the angle is **negative.** In both cases an arrow indicates the direction of rotation. It is also possible for the terminal side to make one complete rotation and continue, as shown in color in Figure 3. Thus, the rays $OA$ and $OB$ may be the initial and terminal sides of many different angles, called **coterminal angles.**

## Degree Measure

If the initial side of an angle is placed along the positive $x$-axis in a Cartesian coordinate system, with its vertex at the origin (0, 0), the angle is said to be in **standard position.** Then, if the initial side of an angle in standard position is allowed to make one complete revolution so that the initial and terminal sides coincide, the resulting angle has measure 360 **degrees.** The symbol ° represents degrees. An angle of measure 180° is called a **straight angle,** and an angle of measure 90° is called a **right angle.** Figure 4 shows these angles in standard position along with other positive and negative angles.

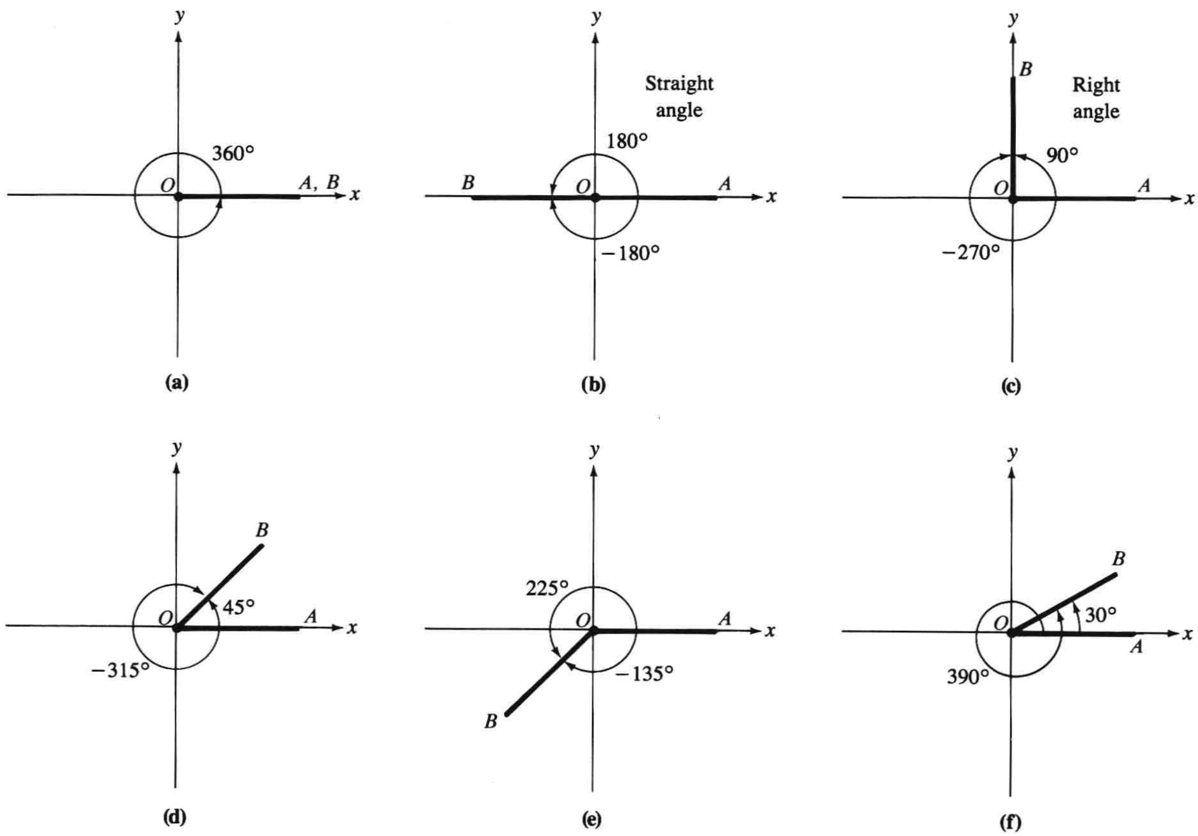



**Figure 4 Angles in Standard Position**

If an angle in standard position has measure between 0° and 90°, its terminal side is in quadrant I. If the angle is between 90° and 180°, its terminal side is in quadrant II, and so forth. If the terminal side of an angle lies on one of the coordinate axes, such as angles 0°, 90°, 180°, and −90°, the angle is called a **quadrantal angle.** An angle measuring between 0° and 90° is **acute,** and one measuring between 90° and 180° is **obtuse.** Two acute angles are called **complementary** if the sum of their measures is 90°, and two positive angles are **supplementary** if their measures add to 180°. For example, the angles 35° and 55° are complementary, and 35° and 145° are supplementary.

**EXAMPLE 1** Give two angles that are coterminal with a 30° angle, one positive and one negative.

Figure 4(f) shows that the positive angle 390° is coterminal with 30°. The angle −330° is also coterminal with 30°. In fact, to find any positive angle coterminal with 30°, we can add any positive multiple of 360° to 30°. Similarly, any negative multiple of 360° added to 30° will result in a negative coterminal angle. ■

When a greater degree of precision is needed, 1 degree can be divided into 60 equal parts, called **minutes** (denoted by ′), and one minute can be divided into 60 equal parts, called **seconds** (denoted by ″). Thus, the angle 45°15′35″ has measure 45 degrees, 15 minutes, and 35 seconds. Another way to obtain more precise measurement is to express parts of a degree as decimals. As we shall see, with the use of calculators, this method is often more convenient.

**EXAMPLE 2** **(a)** Express 45°15′35″ as degrees and the decimal part of a degree.

Since $15' = \frac{15}{60^\circ}$, dividing gives 0.25. Also, since $35'' = \frac{35'}{60} = \frac{35^\circ}{3600}$, dividing gives 0.0097222. Adding these two decimals to 45 yields the required value. The value can be obtained with a calculator, starting with 35″, by following these steps.

ALG: 35 [÷] 3600 [+] 15 [÷] 60 [+] 45 [=] → [45.259722]

RPN: 35 [ENTER] 3600 [÷] 15 [ENTER] 60 [÷] [+] 45 [+] → [45.259722]

Thus, 45°15′35″ = 45.26°, correct to two decimal places.

**(b)** Express 123.67° using degrees, minutes, and seconds.

To convert 0.67° into minutes, we multiply by 60.

$$(60)(0.67^\circ) = 40.2'$$

To convert 0.2′ into seconds, we multiply by 60.

$$(60)(0.2') = 12''$$

Thus, 123.67° = 123°40′12″. ■

## 1.1 EXERCISES

State whether the given angles are complementary, supplementary, or neither.

**1.** 26.5°, 153.5° **2.** 55.8°, 34.2° **3.** 14°20′, 75°20′ **4.** 48°10′, 131°50′

Are the angles given in Exercises 5–16 coterminal?

**5.** 45°, −45° **6.** 90°, −270° **7.** 60°, 420° **8.** 225°, −225°

**9.** 35°, −325° **10.** 180°, −180° **11.** 45°30′, 405°30′ **12.** 16.7°, −343.3°

**13.** 88.4°, 448.6° **14.** 242°20′, −118°20′ **15.** 720°50′, −359°10′ **16.** 66.8°, 786.8°

In Exercises 17–20, express each angle in degrees, correct to the nearest hundredth of a degree.

**17.** 38°41′13″ **18.** 135°50′25″ **19.** −52°35′21″ **20.** −247°27′52″

In Exercises 21–24, express each angle using degrees, minutes, and seconds.

**21.** 85.42° **22.** 135.65° **23.** −48.18° **24.** −390.76°

## For Review

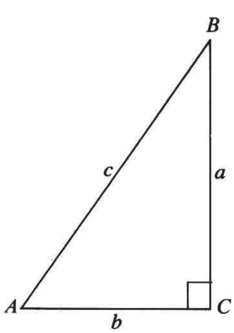
The next section begins our work with the trigonometric functions of an acute angle. To prepare, review the material on functions in Appendix I.

# 1.2 Trigonometric Functions of an Acute Angle

## Right Triangles

In Section 1.1 we discussed the measurement of angles. Now we are ready to define six trigonometric functions of an acute angle in a right triangle. A **right triangle** is a triangle containing a right angle (90°); the side opposite the right angle is called the **hypotenuse** of the triangle. Since the sum of the measures of the angles of a triangle is 180°, the remaining two angles are complementary acute angles. The sides opposite these angles are called **legs** of the triangle.

We often label the angles of a triangle with capital letters. The sides of the triangle are indicated by lower-case letters corresponding to the capital letter of the angle opposite the side. For example, the right triangle with angles $A$, $B$, and $C$ and sides $a$, $b$, and $c$ is shown in Figure 5. We shall agree (unless specified otherwise) to identify the right angle in a triangle with the letter $C$ (⊿ denotes a right angle). The side **opposite** $A$ is $a$, and the side **adjacent** to $A$ is $b$.




$A + B = 90°$
$C = 90°$
$c$ is the hypotenuse
$a$ is opposite $A$
$b$ is adjacent to $A$
$b$ is opposite $B$
$a$ is adjacent to $B$

**Figure 5 Right Triangle**

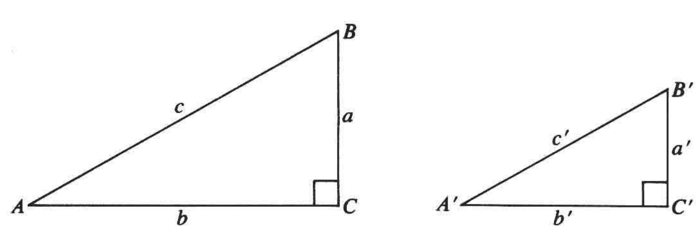



**Figure 6 Similar Right Triangles**

## Similar Right Triangles

Two triangles are **similar** if the angles of one triangle are equal to the corresponding angles of the other. As a result, two right triangles are similar if an acute angle of one is equal to an acute angle of the other. A theorem in plane geometry states that the corresponding sides of similar triangles are proportional. Suppose that two right triangles $ABC$ and $A'B'C'$ are similar as shown in Figure 6. Then $a/c = a'/c'$, $b/c = b'/c'$, $a/b = a'/b'$, $b/a = b'/a'$, $c/b = c'/b'$, and $c/a = c'/a'$.

## The Six Trigonometric Functions

The six possible ratios of the lengths of the sides of a right triangle are given particular names relative to the acute angle $A$. Since the six ratios are independent of the size of the right triangle containing $A$, these ratios can be used to define six **trigonometric functions** of an acute angle, the **sine, cosine, tangent, cotangent, secant,** and **cosecant** functions.

### The Six Trigonometric Functions

Let $A$ be an acute angle in right triangle $ABC$. (See Figure 7.)

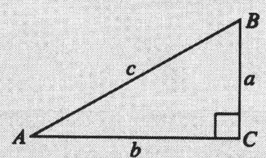



**Figure 7**

| **Function name** | **Abbreviation** | **Value of function at $A$** |
|---|---|---|
| **sine** | **sin** | $\sin A = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{a}{c}$ |
| **cosine** | **cos** | $\cos A = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{b}{c}$ |
| **tangent** | **tan** | $\tan A = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{a}{b}$ |
| **cotangent** | **cot** | $\cot A = \dfrac{\text{adjacent side}}{\text{opposite side}} = \dfrac{b}{a}$ |
| **secant** | **sec** | $\sec A = \dfrac{\text{hypotenuse}}{\text{adjacent side}} = \dfrac{c}{b}$ |
| **cosecant** | **csc** | $\csc A = \dfrac{\text{hypotenuse}}{\text{opposite side}} = \dfrac{c}{a}$ |

Since $$\sin A = \frac{a}{c} \quad \text{and} \quad \csc A = \frac{c}{a},$$

$\sin A$ and $\csc A$ are reciprocals. That is,

$$\sin A = \frac{a}{c} = \frac{1}{\frac{c}{a}} = \frac{1}{\csc A}$$

and $$\csc A = \frac{c}{a} = \frac{1}{\frac{a}{c}} = \frac{1}{\sin A}.$$

Similarly, $\cos A = 1/\sec A$ and $\sec A = 1/\cos A$, and also $\tan A = 1/\cot A$ and $\cot A = 1/\tan A$. Thus,