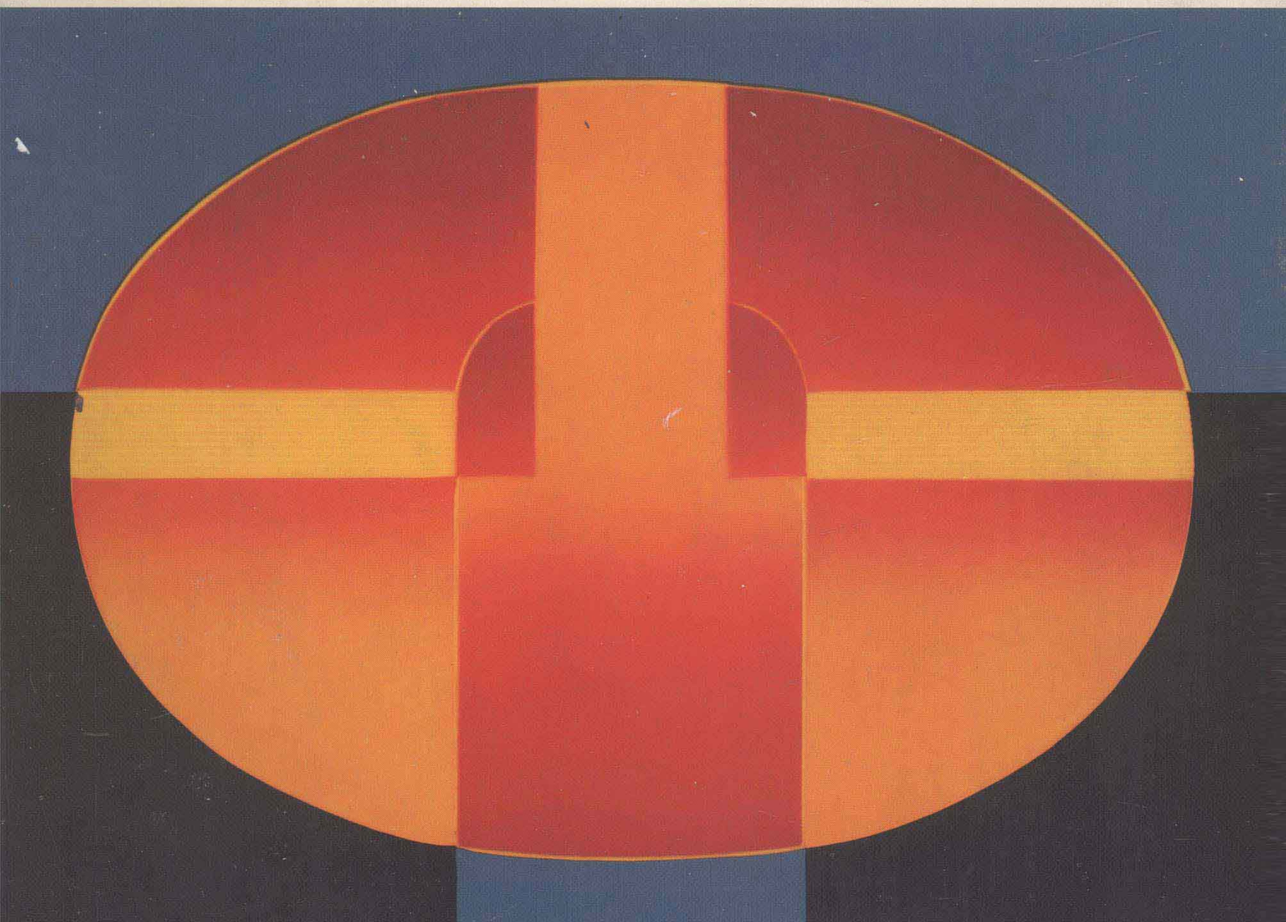
CALCULUS

With Analytic Geometry

Second Edition



Earl W. Swokowski

CALCULUS
With Analytic Geometry

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Preface to Second Edition

This edition has benefited greatly from comments of users of the first edition, and from the constructive criticism of those who reviewed the manuscript. Following their suggestions, many exercise sets were expanded by adding drill-type problems, together with some challenging ones for highly motivated students. There are now approximately 5500 exercises, almost 1500 more than appeared previously. The number of solved examples has also increased significantly. For more specific information regarding the differences between this edition and the previous edition the following will be useful.

Chapter 2 was partially rewritten, keeping in mind the goals of presenting limits and continuity of functions in a mathematically sound manner, while providing students with a strong intuitive feeling for these important concepts. A noteworthy change is that the sections on limits at infinity and functions which become infinite have been moved to Chapter 4, where they are used in conjunction with graphs of rational functions. This move makes it possible to introduce the derivative earlier than before, and thus maintain student interest by using the limit notion in an important way very early in the course.

The proof of the Chain Rule in Chapter 3 has been simplified by avoiding the case where it is necessary to introduce an auxiliary function of some type. However, those who are interested in a complete proof will find one in Appendix II.

The discussion of extrema in Chapter 4 has been improved through the addition of examples and figures. Additional emphasis has been placed on end-point extrema. The section on applications to economics was rewritten.

In Chapter 5 the proof of the Fundamental Theorem of Calculus has been amplified, and error estimates for numerical integration are stressed more than in the earlier edition. The definite integral as a limit of a sum is strongly emphasized in Chapter 6; however, solutions to examples are constructed so that after this important idea is thoroughly understood, the student may bypass the subscript part of the procedure and merely "set up" the required integrals.

Chapter 8 on exponential and logarithmic functions was completely rewritten. The present version should make the development of these important functions much easier to follow than before.

The principal changes in Chapters 9 and 10 consist of the introduction of many new examples and exercises, a better discussion of hyperbolic functions, and a new section on the use of tables of integrals.

In Chapter 11, two physical examples are introduced to help motivate improper integrals. The section on Taylor's Formula has been clarified by graphically illustrating what happens if the number of terms of the approximating polynomial is increased.

Some of the material on infinite series in Chapter 12 has been rewritten and rearranged. Several new theorems, including the Root Test, were added, together with many new exercises. The discussion of power series representations of functions has also been enlarged.

The concept of the *direction* of a two-dimensional vector has been replaced by the simpler (and more easily generalizable) notions of the *same* or *opposite* direction of vectors. This, together with a reorganization of topics, leads to a smoother development in Chapter 14. A stronger emphasis has been placed on geometric problems. This is especially true with applications of the vector product.

The discussion of derivatives and integrals of vector-valued functions in Chapter 15 has been modified to help unify this material. A new section on Kepler's Laws was added to illustrate the power of vector techniques.

The changes in Chapters 16 and 17 consist primarily of additional examples, figures, exercises, and minor rewriting. In Chapter 18, two new sections have been added on transformations of coordinates, Jacobians, and change of variables in multiple integrals.

The order of topics is flexible. For instance, some users of the first edition introduced trigonometric functions very early in the course. A natural place to do so is after the discussion of the Chain Rule in Chapter 3. As a matter of fact, a remark at the end of Section 3.5 refers to the derivative of the sine function. This could be extended by stating formulas for derivatives of the remaining trigonometric functions, and then selecting appropriate exercises from Chapter 9 as students progress through subsequent sections. In like manner, integrals of the trigonometric functions may be introduced in Chapter 5.

Chapter 7, on analytic geometry, can be covered immediately after Chapter 1. Of course, in this event, exercises involving derivatives and integrals cannot be assigned. Chapter 19, on differential equations, may be discussed upon the completion of techniques of integration in Chapter 10, provided that the sections on exact equations and series solutions are omitted. It is also possible to discuss the material on vectors, in Chapter 14, prior to Chapter 13. If desired, Chapter 12, on infinite series, may be postponed until later in the text.

There is a review section at the end of each chapter consisting of a list of important topics together with pertinent exercises. The review exercises are similar in scope to those which appear throughout the text and may be used by students to prepare for examinations. Answers to odd-numbered exercises are given at the end of the text. Instructors may obtain an answer booklet for the even-numbered exercises from the publisher.

I wish to thank the following individuals, who reviewed all, or parts of, the manuscript for the second edition, and offered many helpful suggestions: Phillip W. Bean, Mercer University; Daniel D. Benice, Montgomery College; Delmar L. Boyer, University of Texas–El Paso; Ronald E. Bruck, University of Southern California; David C. Buchthal, The University of Akron; John E. Derwent, University of Notre Dame; William R. Fuller, Purdue University; Gary Haggard, University of Maine–Orono; Douglas Hall, Michigan State University; George Johnson, University of South Carolina; Andy Karantinos, University of South Dakota; G. Otis Kenny, Boise State University; Eleanor Killam, University of Massachusetts–Amherst;

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In addition many instructors were kind enough to respond to a survey conducted by my publisher. Heron S. Collins, Louisiana State University–Baton Rouge; Karl Peterson, University of North Carolina; M. Evans Munroe, University of New Hampshire; John Berglund, Virginia Commonwealth University; George Johnson, University of South Carolina; Frank Quinn, University of Virginia; Lawrence Runyan, Shoreline College; Robert W. Owens, Lewis and Clark College; Karl Gentry, University of North Carolina–Greensboro; George Haborak, College of Charleston; James M. Sobota, University of Wisconsin–La Crosse; Gene A. de Both, St. Norbert College; David Greenstein, Northeastern Illinois University; Jerry Wagenblast, Valparaiso University; Gene Vanden Boss, Adrian College; Duane E. Deal, Ball State University; Carol Smith, Birmingham Southern University; Gary Eichelsdorfer, University of Scranton; Robert E. Spencer, Virginia Polytechnic Institute and State University; Albert L. Rabenstein, Washington and Jefferson College; Charles A. Grobe, Jr., Bowdoin College; Teisuke Ito, Northern Michigan University; Stanley R. Samsky, University of Delaware, were among those who sent in valuable information and ideas.

I also wish to acknowledge the advice of my colleagues at Marquette University, who offered numerous suggestions for improvements. Additional recognition is due Drs. Thomas Bronikowski and Michael Ziegler, for their careful work with solutions to the exercises.

I am grateful for the valuable assistance of the staff of Prindle, Weber & Schmidt. In particular, Elizabeth Thomson, Nancy Blodget, and Mary Le Quesne have been very helpful in the production of the text. As usual, Executive Editor John Martindale was a constant source of advice and encouragement.

Special thanks are due to my wife Shirley and the members of our family: Mary, Mark, John, Steven, Paul, Thomas, Robert, Nancy, and Judy. All have had an influence on the book—either directly, by working exercises, proofreading, or typing—or indirectly, through continued interest and moral support.

In addition to all of the persons named here, I express my sincere appreciation to the many unnamed students and teachers who have helped shape my views on how calculus should be presented in the classroom.

Earl W. Swokowski

Preface to First Edition

Most students study calculus in order to use it as a tool in areas other than mathematics. They desire information about *why* calculus is important, and *where* and *how* it can be applied. As I wrote this text, I tried to keep these facts in mind. In particular, before an important concept is defined, problems which require the concept are presented. After sufficient theory has been developed, there are many interesting physical and mathematical examples to draw upon. However, the difficulty is to arouse student interest at the *beginning* of a new subject.

To illustrate my approach to calculus, in this text the limit concept is motivated by referring to three practical problems, one from physics, another from chemistry, and the third from mathematics. The notion of limit is then discussed in an intuitive manner, using numerical examples. A precise definition is introduced a section later, but only after references are made to previous examples. The definition is then reinforced through the use of two different graphical techniques. I believe that students should not spend an entire semester or more repeating the words "closer and closer," nor should they be literally buried under epsilons and deltas! Limit theorems are stated and used in examples, but difficult proofs are placed in an appendix, where they may be left as reading assignments, discussed immediately, or postponed until a later time. A similar philosophy is followed when the derivative, the definite integral, and other important concepts are introduced.

In addition to achieving a good balance between rigor and intuition, my primary objective was to write a book which could be read and understood by college freshmen. Throughout each section I have striven for accuracy and clarity of exposition, together with a presentation which makes the transition from precalculus mathematics to calculus as smooth as possible.

This text contains sufficient material for any of the standard calculus sequences. The Table of Contents shows the order in which the material is presented. In general, Chapters 1 through 6 could constitute the equivalent of a one-semester course for students who only need a basic background consisting of limits, derivatives, and definite integrals of algebraic functions. Chapters 7 through 12 would ordinarily make up the second semester of work; however, Chapter 12 on infinite series could be postponed until the third semester. In this event, Chapter 13 on curves and polar coordinates, or parts of Chapter 14 on vectors could be substituted. The remainder of the text is intended for what is usually referred to as the third semester. Chapter 18 on vector calculus is somewhat unusual for a first course. Some instructors may wish to include this material and others not. For this reason it is placed near the end of the book, where portions may be omitted without interrupting the continuity of the text. The same is true for Chapter 19 on differential equations.

I wish to thank the following individuals, who reviewed the manuscript and offered many helpful suggestions: James Cornette, Iowa State University; August Garver, University of Missouri–Rolla; Douglas Hall, Michigan State University; Alan Heckenbach, Iowa State University; Simon Hellerstein, University of Wisconsin; David Mader, Ohio State University; William Meyers, California State University, San Jose; David Minda, University of Cincinnati; Chester Miracle, University of Minnesota; Ada Peluso, Hunter College of the City University of New York; Leonard Shapiro, University of Minnesota; Donald Sherbert, University of Illinois; Charles Van Gorden, Millersville State College; Dale Walston, University of Texas.

Special thanks are due to Dr. Thomas Bronikowski of Marquette University, who carefully read the entire manuscript, worked every exercise, and was responsible for many improvements in the text. In addition, he has written a student supplement which contains detailed solutions for approximately one-third of the exercises.

I am grateful to Carolyn Meitler for an excellent job of typing the manuscript, and to the staff of Prindle, Weber & Schmidt, for their painstaking work in the production of this book. In particular, John Martindale, a fine editor and friend, has been a constant source of encouragement during my association with the company. Above all, I owe a debt of thanks to my family, for their patience and understanding over long periods of writing.

Earl W. Swokowski

What is Calculus?

Calculus was invented in the seventeenth century to provide a tool for solving problems involving motion. The subject matter of geometry, algebra, and trigonometry is applicable to objects which move at constant speeds; however, methods introduced in calculus are required to study the orbits of planets, to calculate the flight of a rocket, to predict the path of a charged particle through an electromagnetic field and, for that matter, to deal with all aspects of motion.

In order to discuss objects in motion it is essential first to define what is meant by *velocity* and *acceleration.* Roughly speaking, the velocity of an object is a measure of the rate at which the distance traveled changes with respect to time. Acceleration is a measure of the rate at which velocity changes. Velocity may vary considerably, as is evident from the motion of a drag-strip racer or the descent of a space capsule as it reenters the Earth's atmosphere. In order to give precise meanings to the notions of velocity and acceleration it is necessary to use one of the fundamental concepts of calculus, the *derivative.*

Although calculus was introduced to help solve problems in physics, it has been applied to many different fields. One of the reasons for its versatility is the fact that the derivative is useful in the study of rates of change of many entities other than objects in motion. For example, a chemist may use derivatives to forecast the outcome of various chemical reactions. A biologist may employ it in the investigation of the rate of growth of bacteria in a culture. An electrical engineer uses the derivative to describe the change in current in an electric circuit. Economists have applied it to problems involving corporate profits and losses.

The derivative is also used to find tangent lines to curves. Although this has some independent geometric interest, the significance of tangent lines is of major importance in physical problems. For example, if a particle moves along a curve, then the tangent line indicates the direction of motion. If we restrict our attention to a sufficiently small portion of the curve, then in a certain sense the tangent line may be used to approximate the position of the particle.

Many problems involving maximum and minimum values may be attacked with the aid of the derivative. Some typical questions that can be answered are: At what angle of elevation should a projectile be fired in order to achieve its maximum range? If a tin can is to hold one gallon of a liquid, what dimensions require the least amount of tin? At what point between two light sources will the illumination be greatest? How can certain corporations maximize their revenue? How can a manufacturer minimize the cost of producing a given article?

Another fundamental concept of calculus is known as the *definite integral.* It, too, has many applications in the sciences. A physicist uses it to find the work required to stretch or compress a spring. An engineer may use it to find the center of mass or moment of inertia of a solid. The definite integral can be used by a biologist to calculate the flow of blood through an arteriole. An economist may employ it to estimate depreciation of equipment in a manufacturing plant. Mathematicians use definite integrals to investigate such concepts as areas of surfaces, volumes of geometric solids, and lengths of curves.

All the examples we have listed, and many more, will be discussed in detail as we progress through this book. There is literally no end to the applications of calculus. Indeed, in the future perhaps *you,* the reader, will discover new uses for this important branch of mathematics.

The derivative and the definite integral are defined in terms of certain limiting processes. The notion of limit is the initial idea which separates calculus from the more elementary branches of mathematics. Sir Isaac Newton (1642–1727) and Gottfried Wilhelm Leibniz (1646–1716) discovered the connection between derivatives and integrals. Because of this, and their other contributions to the subject, they are credited with the invention of calculus. Many other mathematicians have added a great deal to its development.

The preceding discussion has not answered the question "What is calculus?" Actually, there is no simple answer. Calculus could be called the study of limits, derivatives, and integrals; however, this statement is meaningless if definitions of the terms are unknown. Although we have given a few examples to illustrate what can be accomplished with derivatives and integrals, neither of these concepts has been given any meaning. Defining them will be one of the principal objectives of our early work in this text.

Table of Contents

Prerequisites for Calculus

This chapter contains topics necessary for the study of calculus. After a brief review of real numbers, coordinate systems, and graphs in two dimensions, we turn our attention to one of the most important concepts in mathematics—the notion of function.

1.1 REAL NUMBERS

Real numbers are used considerably in precalculus mathematics and it will be assumed that the reader is familiar with the fundamental properties of addition, subtraction, multiplication, division, exponents, radicals, and so on. Throughout this chapter, unless otherwise specified, lower-case letters $a, b, c, \ldots$ will denote real numbers.

The **positive integers** $1, 2, 3, 4, \ldots$ may be obtained by adding the real number 1 successively to itself. The **integers** consist of all positive and negative integers together with the real number 0. A **rational number** is a real number that can be expressed as a quotient a/b, where a and b are integers and $b \neq 0$. Real numbers that are not rational are called **irrational**. The ratio of the circumference of a circle to its diameter is irrational. This real number is denoted by π and the notation $\pi \approx 3.1416$ is used to indicate that π is *approximately equal* to 3.1416. Another example of an irrational number is $\sqrt{2}$.

Real numbers may be represented by nonterminating decimals. For example, the decimal representation for the rational number 7434/2310 is found by long division to be 3.2181818..., where the digits 1 and 8 repeat indefinitely. Rational numbers may always be represented by repeating decimals. Decimal representations for irrational numbers may also be obtained; however, they are nonterminating and nonrepeating.

It is possible to associate real numbers with points on a line l in such a way that to each real number a there corresponds one and only one point, and conversely, to each point P there corresponds precisely one real number. Such an association between two sets is referred to as a **one-to-one correspondence**. We first choose an arbitrary point O, called the **origin**, and associate with it the real number 0. Points associated with the integers are then determined by laying off successive line segments of equal length on either side of O as illustrated in Figure 1.1. The points corresponding to rational numbers such as 23/5 and $-1/2$ are obtained by subdividing the equal line segments. Points associated with certain irrational numbers, such as $\sqrt{2}$, can be found by geometric construction. For other irrational numbers such as π, no construction is possible. However, the point corresponding to π can be approximated to any degree of accuracy by locating successively the points corresponding to 3, 3.1, 3.14, 3.141, 3.1415, 3.14159, and so on. It can be shown that to every irrational number there corresponds a unique point on l and, conversely, every point that is not associated with a rational number corresponds to an irrational number.

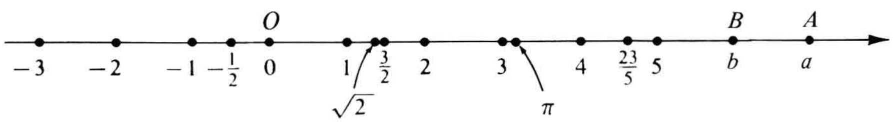

Figure 1.1

The number a that is associated with a point A on l is called the **coordinate** of A. An assignment of coordinates to points on l is called a **coordinate system** for l, and l is called a **coordinate line**, or a **real line**. A direction can be assigned to l by taking the **positive direction** to the right and the **negative direction** to the left. The positive direction is noted by placing an arrowhead on l as shown in Figure 1.1.

The real numbers which correspond to points to the right of O in Figure 1.1 are called **positive real numbers**, whereas those which correspond to points to the left of O are **negative real numbers**. The real number 0 is neither positive nor negative. The collection of positive real numbers is **closed** relative to addition and multiplication; that is, if a and b are positive, then so is the sum $a + b$ and the product ab.

If a and b are real numbers, and $a - b$ is positive, we say that ***a* is greater than *b*** and write $a > b$. An equivalent statement is ***b* is less than *a***, written $b < a$. The symbols $>$ or $<$ are called **inequality signs** and expressions such as $a > b$ or $b < a$ are called **inequalities**. From the manner in which we constructed the coordinate line l in Figure 1.1, we see that if A and B are points with coordinates a and b, respectively, then $a > b$ (or $b < a$) *if and only if A lies to the right of B*. Since $a - 0 = a$, it follows that $a > 0$ if and only if a is positive. Similarly, $a < 0$ means that a is negative. The following rules can be proved.

(1.1)

If $a > b$ and $b > c$, then $a > c$.

If $a > b$, then $a + c > b + c$.

If $a > b$ and $c > 0$, then $ac > bc$.

If $a > b$ and $c < 0$, then $ac < bc$.

Analogous rules for "less than" can also be established.

The symbol $a \geq b$, which is read ***a* is greater than or equal to *b***, means that either $a > b$ or $a = b$. The symbol $a < b < c$ means that $a < b$ and $b < c$, in which case we say that ***b* is *between* a and *c***. The notations $a \leq b$, $a < b \leq c$, $a \leq b < c$, $a \leq b \leq c$, and so on, have similar meanings.