

Ralph Martin  
Helmut Bez  
Malcolm Sabin (Eds.)

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# Mathematics of Surfaces XI

11th IMA International Conference  
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Proceedings



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# Mathematics of Surfaces XI

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Proceedings



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# Preface

This volume collects the papers accepted for presentation at the 11th IMA Conference on the Mathematics of Surfaces, held at Loughborough University, 5th–7th September 2005. As with all earlier conferences in the series, contributors to this volume come from many countries. The papers presented here reflect the interest in a subject of relevance to mathematics, engineering, and computer science, especially in domains such as computer-aided design, computer vision, and computer graphics.

The papers in the present volume include eight invited papers, as well as a larger number of submitted papers. They cover a range of ideas from underlying theoretical tools to industrial and medical uses of surfaces. The latter category includes such diverse topics as surfaces in car design, and modelling of teeth, while the former includes papers on Voronoi diagrams, linear systems, estimation of curvatures on meshes, operators on meshes, intersection of subdivision surfaces, approximate parameterization, condition numbers, Pythagorean hodographs, artifacts in B-spline surfaces, Bézier surfaces of minimal energy, line subdivision, subdivision surfaces, level sets and symmetry, the topology of algebraic surfaces, curve analysis, interpolation with positivity, and conversion of cyclides to NURBS. Other papers concentrate on particular algorithms arising from applications, such as embedding graphs in manifolds, recovery of 3D shape from shading, finding optimal feedrates for machining, detection of creases in range data, and filling holes in range data.

We would like to thank all those who attended the conference and helped to make it a success. We are particularly grateful to Lucy Nye at the Institute of Mathematics and Its Applications for her hard work in organizing many aspects of the conference, and to Alfred Hofmann and Frank Holzwarth of Springer for their help in publishing this volume. Following this Preface is a list of distinguished researchers who formed the International Programme Committee, and who freely gave their time in helping to assess papers for these proceedings. Due to their work, many of the papers have been considerably improved. Our thanks go to all of them, and to other people who they called upon to help with refereeing.

June 2005

Ralph Martin,  
Helmut Bez,  
Malcolm Sabin

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The Mathematics of Surfaces XI conference was organized by the Institute of Mathematics and Its Applications (Catherine Richards House, 16 Nelson St., Southend-on-Sea, Essex, SS1 1EF, UK), and Loughborough University.

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# Free-Form Surface Construction in a Commercial CAD/CAM System

Florian Albat and Rainer Müller

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**Abstract.** In automobile industry, free-form surfaces often have to be constructed and even more often have to be modified. Frequently, a surface model is given as a triangular mesh, which is converted to (polynomial) spline surfaces (reverse engineering). We show some arising problems and how they are solved in our software. Furthermore, we present some open theoretical problems.

## 1 Surface Construction in CAD

There are two main types of surfaces in CAD: free-form surfaces and standard surfaces. The latter describe spatial objects, that can be defined uniquely by quite simple rules, often in terms of two-dimensional drawings. Typically, such objects consist of planes, cylinders and similarly simple surfaces. The sharp edges between the single surfaces are rounded by simulating a rolling ball (fillet surfaces). These models dominate in machine-building. The hood or roof of a car, however, is shaped in such a way, that it cannot be defined by easy rules like “cylinder of height 10cm with radius 3cm”. Such surfaces are called free-form surfaces.

The automobile industry is not the only, but a very important field of industrial application for free-form surfaces. Many of our customers belong to this industry, both the car companies themselves and many suppliers of different kind. It is the main application field for surfaces of the highest quality requirements (‘Class A’, this implies e.g. curvature continuity). They are needed for the visible outer skin, that determines the potential buyer’s impression of a car. But these high-quality surfaces represent only a minority of all constructed surfaces in a car, most of which lie invisible in the inside like the one shown in Fig. 1 and do not need to be of such a high quality. However, even such surfaces need to be  $G^1$  continuous (within a tolerance of about  $0.5^\circ$ ), because they will finally be produced as sheet metals, and sheet metals cannot be shaped into sharp edges.

Besides sheet metals a car contains metal parts like the crank shaft, exhaust pipe and clutch shell plus plastic parts like the dashboard, which are molded. For all these parts CAD models must be constructed.

New objects account for only about 10% of construction work, roughly 90% of all constructions are modifications of existing models. So we will focus on such modifications. One reason for them is, that first versions of the fabrication

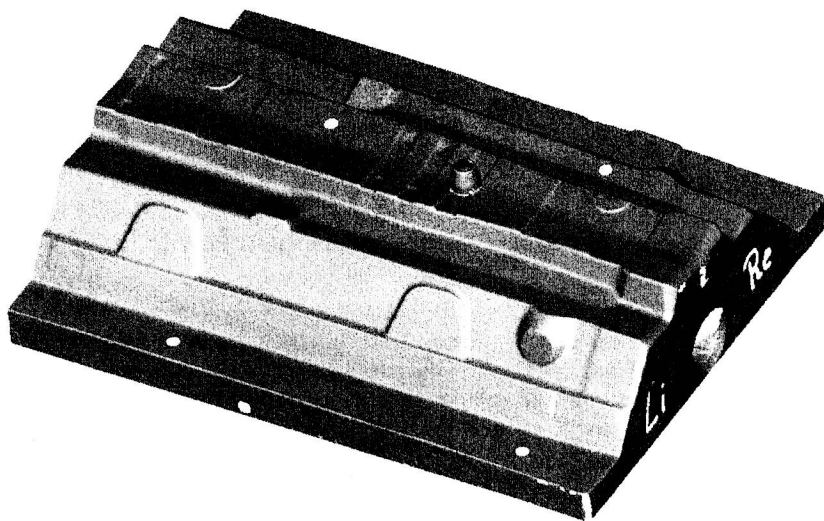


Fig. 1. Technical sheet metal from the inside of a car (scanned data)

tools are milled before the final design with all details is done. Another frequent reason is the compensation of a metal's springback.

Modifications can be either *real* or *virtual*. As real modifications we denote such, that are based on a really existing object. For example, the outer skin of a car is usually milled as a prototype, and the shape of this real model is modified by sanding off and spreading of material to satisfy certain aesthetic visions. This modified model is scanned, the scan points are triangulated and from this triangular mesh a new CAD model consisting of spline surfaces is constructed (reverse engineering). Virtual modifications we call such, that are performed directly on the CAD model without any real object being modified and scanned. A typical requirement is to raise a certain position of the roof for 1 cm. To do this, mostly neighbouring surfaces need to be adapted.

We will treat virtual modifications in the third section. Beforehand, we will elaborately treat real modifications, i.e. free-form surface construction by reverse engineering based on a triangular mesh. The question arises whether it is at all necessary to convert the mesh of scanned data into polynomial surfaces, or whether suitable new algorithms make it possible to work with meshes just as effectively. This question is interesting from a theoretical standpoint and not easily answered. However, it is (for the time being) irrelevant from a practical point of view, since industrial process chains are designed for work with 'smooth' surfaces. Changing to a continuous technique using meshes will not be possible until not only solutions for single steps exist, but really all design and production processes can be converted without suffering a loss in quality. Even then, the change will be done only when the savings gained outweigh the high cost of conversion. As a result, at least for the present and in the foreseeable future, it

will be unavoidable in industrial applications to convert mesh data into polynomial surfaces. Hence, there is a need for easily useable software to do reverse engineering.

Among others, this paper shall also show, that our point of view as a provider of commercial software differs in some points from that of more theoretically interested scientists. We have to sell the used algorithms in such a way, that our customers, of whom hardly any has studied mathematics or computer science, can use them as easily as possible. Some of the discussed points may be trivial for academically educated readers, but these are questions, which must be considered if one wants to have success in the market.

This paper does not want to give an overview of the vast literature about possible algorithms or similar, but is aimed to inform about practical experiences of different software users, which is not written in common scientific papers. Hence, we restrict our list of references to [1], which is a good short introduction to reverse engineering. More references can be found therein.

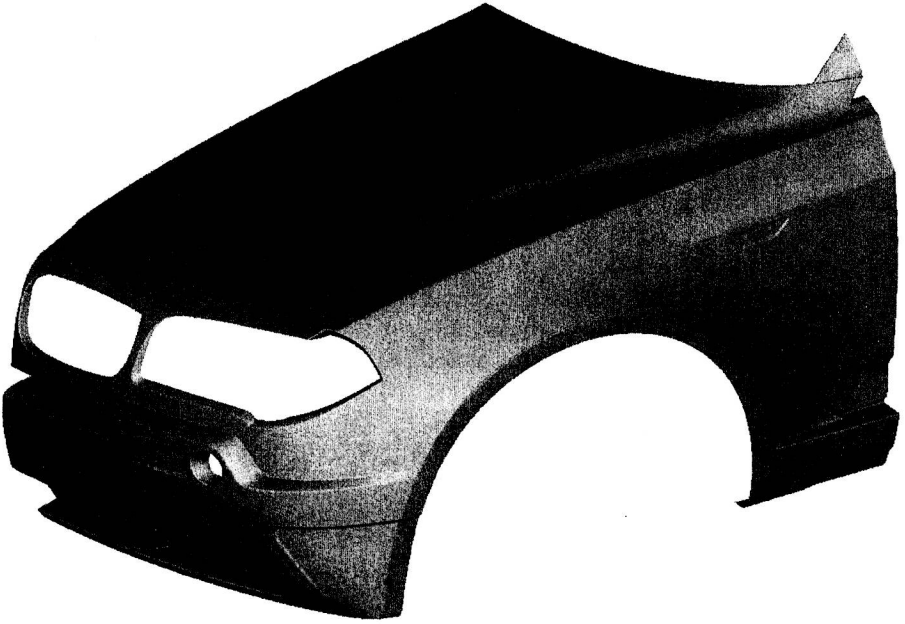
## 2 Surface Construction with Reverse Engineering

### 2.1 Structuring the Object

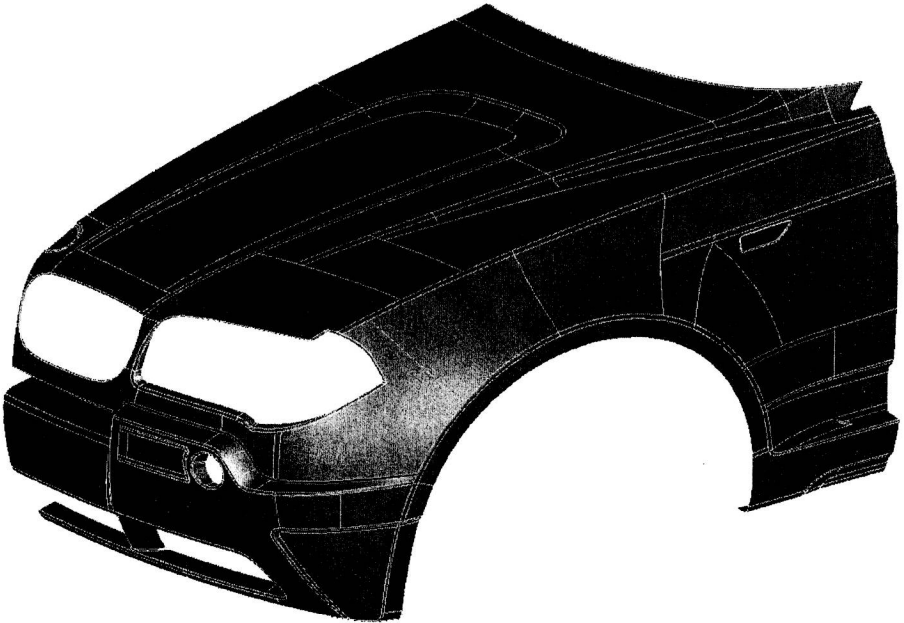
Fig. 2 shows a triangular mesh of the front part of a car. Because of the symmetry, it suffices to construct the left half of the car. A surface construction of this model is shown in Fig. 3. This model looks good, matches the mesh very well and is  $G^1$  continuous, which are very important properties of a surface model from reverse engineering. This model was constructed with our software package RSC (*Rapid Surface Creation*).

In Fig. 3 the border lines of the single surfaces are drawn. We denote their entirety as *wire frame*. Many curves follow the object's *feature lines*, each of which separates different 'uniform' areas, e.g. the relatively flat part of the hood from the more curved fillet. As we will delve into later, it is very important for the quality of the approximation surfaces, that the feature lines can be found in the wire frame. In a classical construction without mesh, the feature lines are naturally incorporated in the process: the larger uniform surfaces, the so called main surfaces, are constructed first. Then they are intersected with each other, and the sharp edges are rounded by fillets. The border lines of fillets are feature lines.

When constructing the fillets, the main surfaces are trimmed by the fillet boundaries. Likewise, the wire frame in Fig. 3, which originated from reverse engineering, contains many trimmed surfaces, i.e. surfaces, whose border is not a quadrangle of isoparametric lines. This is the normal case, one can hardly find CAD constructions, which do not contain trimmed surfaces. Exceptions are constructions, that are made by some reverse engineering programs, who cannot handle trimmed surfaces, but only *natural surfaces*, i.e. quadrangular surfaces, whose boundaries are isoparametric lines. Of course, every wire frame can be subdivided into quadrangles, but such a face layout is not suitable for some subsequent process steps. As an example, fig. 4 shows on the left a part of the

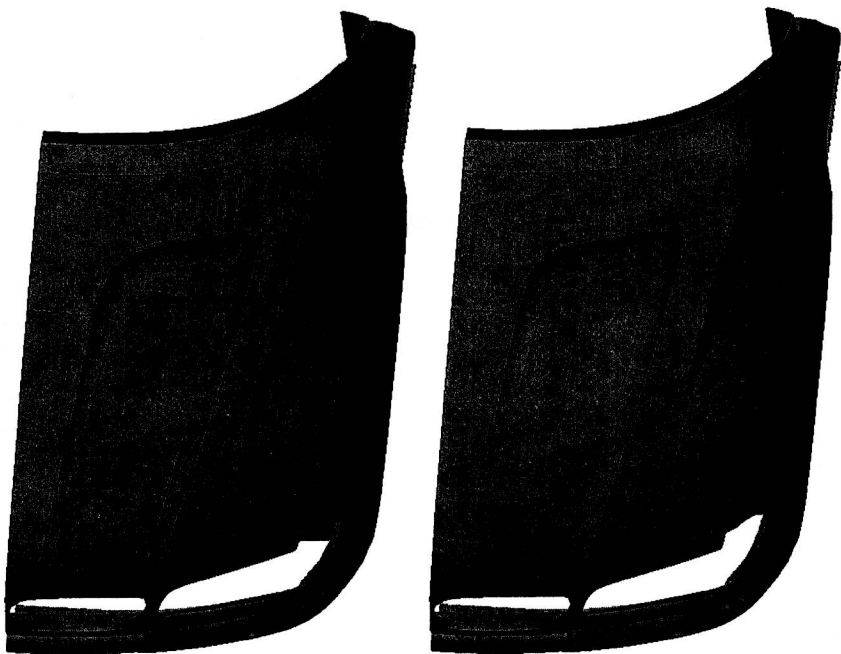


**Fig. 2.** Triangular mesh of a car



**Fig. 3.** Surface construction from the mesh in Fig. 2





**Fig. 4.** Part of the wire frame from Fig. 3(left) and example for quadrangulation (right)

wire frame from fig. 3 and on the right a subdivision of this wire frame, so that it does only contain quadrangles.

The most obvious disadvantage of quadrangulation is the 'unnaturalness': the human user is disturbed by the many additional border lines. Another disadvantage is, that it is difficult to enlarge a main surface, when it was divided into several quadrangles. Such an enlargement is e.g. necessary, when a neighbouring approximated fillet surface has to be replaced by an exact fillet surface with bigger or smaller radius (reduction of radius).

Especially for the visible surfaces of a car (roof etc.) a surface calculated automatically by approximation of digitized data is hardly ever good enough for the high quality standards. Here is always manual postprocessing necessary, e.g. by manipulation of Bézier points. For that, a subdivision of the surfaces into quadrangles is completely unsuitable, one needs a face layout like in Fig. 3.

The designer's great dream is of course an algorithm, that is given the mesh and creates a wire frame like that in Fig. 3. We are working on this and are optimistic, that there will be considerable progress soon, but until now, such an algorithm does not exist. As a surface model implies a wire frame, no automatic procedure exists, that constructs a surface model like the one shown directly from the mesh. Sure, there are commercial programs for this task, but the programs known to us can only handle quadrangles and therefore have the mentioned disadvantages.

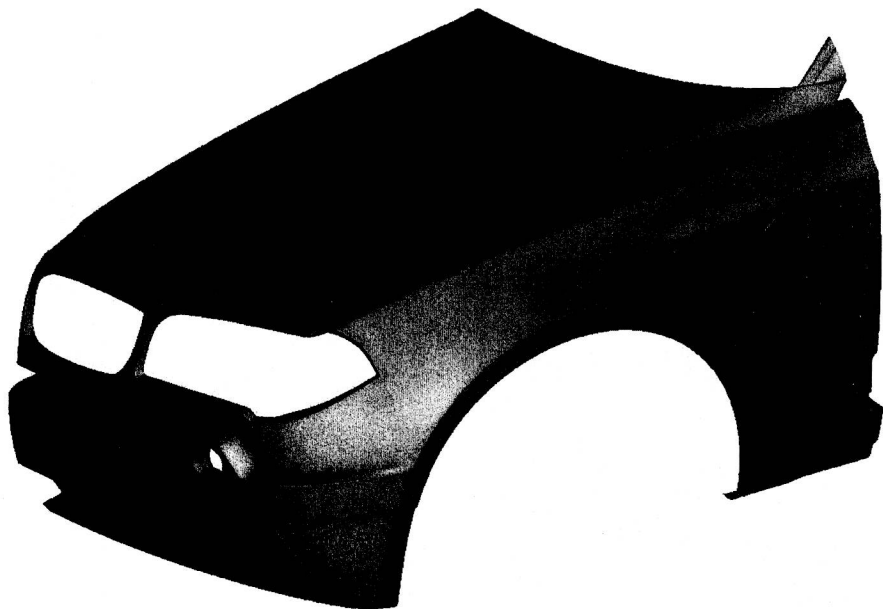


Fig. 5. Curvature representation of the mesh in Fig. 2

Until now, no program can create a wire frame nearly as well as the human designer, so the most reliable method, which is also implemented in our software, is to let the user construct the wire frame by himself. In general, it is difficult to construct spatial free-form curves, but in this application, it is much easier, as the curves are not absolutely free, but must lie on the mesh. By projecting the input automatically, our software enables the user to create curves directly on the mesh. This has proven to be both intuitive and reliable.

As support, we offer half-automatic functions for construction of feature lines, which e.g. recognize pair of fillet border curves or curves with constant (approximated) mesh curvature on positions selected by the user. For visualization of the object's structure it has proven useful to approximate the mesh's curvature and show it as a color spectrum, see Fig. 5.

## 2.2 Calculation of Surfaces

By construction of the wire frame the user decomposes the mesh into several facets, in which then surfaces can be calculated.

The surface, which is calculated for a facet, depends on the mesh and the facet's boundary curves, that define the approximated region. To achieve  $G^1$  continuity, it also depends on surfaces in adjacent facets, if such exist. Letting the user specify all these parameters explicitly, is not only cumbersome, but also prone to error. A user-friendly program should ease this as far as possible. To do so, we use in our software a data structure 'RSC manifold', which contains besides geometric data also topological data, namely neighbourhood