

HANDBOOKS IN ECONOMICS BOOK 2
EEEEEEEEEEEEEEEEEEEE

HANDBOOK OF ECONOMETRICS

VOLUME 1

Editors :
Zvi Griliches
Michael D. Intriligator

EEEEEEEEEEEEEEEEEEEE
NORTH-HOLLAND

HANDBOOK OF ECONOMETRICS

Edited by

Harvard University

MICHAEL D. INTRILIGATOR



NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM · NEW YORK · OXFORD

©NORTH-HOLLAND PUBLISHING COMPANY 1983

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the copyright owner.

ISBN North-Holland for this volume 0444 86185 8

ISBN North-Holland for this set 0444 86188 2

Publishers

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM • NEW YORK • OXFORD

Sole distributors for the U.S.A. and Canada

ELSEVIER SCIENCE PUBLISHING COMPANY, INC.
52 VANDERBILT AVENUE
NEW YORK, N.Y. 10017

Library of Congress Cataloging in Publication Data

Main entry under title:

Handbook of econometrics.

(Handbooks in economics; bk. 2)

Includes bibliographies.

1. Econometrics--Addresses, essays, lectures.

I. Griliches, Zvi, 1930- . II. Intriligator,
Michael D. III. Series.

HB139.H36 1983 330'.028 83-2396

ISBN 0-444-86185-8

PRINTED IN THE NETHERLANDS

INTRODUCTION TO THE SERIES

The aim of the *Handbooks in Economics* series is to produce Handbooks for various branches of economics, each of which is a definitive source, reference, and teaching supplement for use by professional researchers and advanced graduate students. Each Handbook provides self-contained surveys of the current state of a branch of economics in the form of chapters prepared by leading specialists on various aspects of this branch of economics. These surveys summarize not only received results but also newer developments, from recent journal articles and discussion papers. Some original material is also included, but the main goal is to provide comprehensive and accessible surveys. The Handbooks are intended to provide not only useful reference volumes for professional collections but also possible supplementary readings for advanced courses for graduate students in economics.

CONTENTS OF THE HANDBOOK

VOLUME I

Part 1 – MATHEMATICAL AND STATISTICAL METHODS IN ECONOMETRICS

Chapter 1

Linear Algebra and Matrix Methods in Econometrics
HENRI THEIL

Chapter 2

Statistical Theory and Econometrics
ARNOLD ZELLNER

Part 2 – ECONOMETRIC MODELS

Chapter 3

Economic and Econometric Models
MICHAEL D. INTRILIGATOR

Chapter 4

Identification
CHENG HSIAO

Chapter 5

Model Choice and Specification Analysis
EDWARD E. LEAMER

Part 3 – ESTIMATION AND COMPUTATION

Chapter 6

Nonlinear Regression Models
TAKESHI AMEMIYA

Chapter 7

Specification and Estimation of Simultaneous Equation Models
JERRY A. HAUSMAN

*Chapter 8***Exact Small Sample Theory in the Simultaneous Equations Model**

PETER C. B. PHILLIPS

*Chapter 9***Bayesian Analysis of Simultaneous Equation Systems**

JACQUES H. DREZE and JEAN-FRANÇOIS RICHARD

*Chapter 10***Biased Estimation**

G. G. JUDGE and M. E. BOCK

*Chapter 11***Estimation for Dirty Data and Flawed Models**

WILLIAM S. KRASKER, EDWIN KUH, and ROY E. WELSCH

*Chapter 12***Computational Problems and Methods**

RICHARD E. QUANDT

VOLUME II**Part 4 – TESTING***Chapter 13***Wald, Likelihood Ratio, and Lagrange Multiplier Tests in Econometrics**

ROBERT F. ENGLE

*Chapter 14***Multiple Hypothesis Testing**

N. E. SAVIN

*Chapter 15***Approximating the Distributions of Econometric Estimators and Test Statistics**

THOMAS J. ROTHENBERG

*Chapter 16***Monte Carlo Experimentation in Econometrics**

DAVID F. HENDRY

Part 5 – TIME SERIES TOPICS*Chapter 17***Time Series and Spectral Methods in Econometrics**

C. W. J. GRANGER and MARK W. WATSON

Chapter 18

Dynamic Specification

DAVID F. HENDRY, ADRIAN R. PAGAN, and J. DENIS SARGAN

Chapter 19

Inference and Causality in Economic Time Series Models

JOHN GEWEKE

Chapter 20

Continuous Time Stochastic Models and Issues of Aggregation over Time

A. R. BERGSTROM

Chapter 21

Random and Changing Coefficient Models

GREGORY C. CHOW

Chapter 22

Panel Data

GARY CHAMBERLAIN

Part 6 – SPECIAL TOPICS IN ECONOMETRICS: 1

Chapter 23

Latent Variable Models in Econometrics

DENNIS J. AIGNER, CHENG HSIAO, ARIE KAPTEYN, and TOM WANSBEEK

Chapter 24

Econometric Analysis of Qualitative Response Models

DANIEL L. McFADDEN

VOLUME III

Part 7 – SPECIAL TOPICS IN ECONOMETRICS: 2

Chapter 25

Economic Data Issues

ZVI GRILICHES

Chapter 26

Functional Forms in Econometric Model Building

LAWRENCE LAU

Chapter 27

Limited Dependent Variables

PHOEBUS J. DHRYMES

*Chapter 28***Disequilibrium, Self Selection, and Switching Models**

G. S. MADDALA

*Chapter 29***Econometric Analysis of Longitudinal Data**

JAMES J. HECKMAN and BURTON SINGER

**Part 8—SELECTED APPLICATIONS AND USES
OF ECONOMETRICS***Chapter 30***Demand Analysis**

ANGUS DEATON

*Chapter 31***Econometric Methods for Modeling Producer Behavior**

DALE W. JORGENSON

*Chapter 32***Labor Econometrics**

JAMES J. HECKMAN and THOMAS E. MACURDY

*Chapter 33***Evaluating the Predictive Accuracy of Models**

RAY C. FAIR

*Chapter 34***Econometric Approaches to Stabilization Policy in Stochastic Models of
Macroeconomic Fluctuations**

JOHN B. TAYLOR

*Chapter 35***Economic Policy Formation: Theory and Implementation
(Applied Econometrics in the Public Sector)**

LAWRENCE R. KLEIN

PREFACE TO THE HANDBOOK

Purpose

The *Handbook of Econometrics* aims to serve as a source, reference, and teaching supplement for the field of econometrics, the branch of economics concerned with the empirical estimation of economic relationships. Econometrics is conceived broadly to include not only econometric models and estimation theory but also econometric data analysis, econometric applications in various substantive fields, and the uses of estimated econometric models. Our purpose has been to provide reasonably comprehensive and up-to-date surveys of recent developments and the state of various aspects of econometrics as of the early 1980s, written at a level intended for professional use by economists, econometricians, and statisticians and for use in advanced graduate econometrics courses.

Econometrics is the application of mathematics and statistical methods to the analysis of economic data. Mathematical models help us to structure our perceptions about the forces generating the data we want to analyze, while statistical methods help us to summarize the data, estimate the parameters of our models, and interpret the strength of the evidence for the various hypotheses that we wish to examine. The evidence provided by the data affects our ideas about the appropriateness of the original model and may result in significant revisions of such models. There is, thus, a continuous interplay in econometrics between mathematical-theoretical modeling of economic behavior, data collection, data summarizing, model fitting, and model evaluation. Theory suggests data to be sought and examined; data availability suggests new theoretical questions and stimulates the development of new statistical methods. The examination of theories in light of data leads to their revision. The examination of data in the light of theory leads often to new interpretations and sometimes to questions about its quality or relevance and to attempts to collect new and different data.

In this volume we review only a subset of what might be called “econometrics”. The mathematical-theoretical tools required for model building are discussed primarily in the *Handbook of Mathematical Economics*. Issues of sampling theory, survey design, data collection and editing, and computer programming, all important aspects of the daily life of a practicing econometrician, had, by and large, to be left out of the scope of this *Handbook*. We concentrate, instead, on statistical problems and economic interpretation issues associated with the modeling and estimation of economic behavioral relationships from already assembled and often badly collected data. If economists had access to good experimental data, or were able to design and to perform the relevant economic experiments,

the topics to be covered in such a *Handbook* would be quite different. The fact that the generation and collection of economic data is mostly outside the hands of the econometrician is the cause of many of the inferential problems which are discussed in this *Handbook*.

Organization

The organization of the *Handbook* follows in relatively systematic fashion the way an econometric study would proceed, starting from basic mathematical and statistical methods and econometric models, proceeding to estimation and computation, through testing, and ultimately to applications and uses. The *Handbook* also includes a fairly detailed development of time series topics and many other special topics. In particular:

Part 1 summarizes some basic tools used repeatedly in econometrics, including linear algebra, matrix methods, and statistical theory.

Part 2 deals with econometric models, their relationship to economic models, their identification, and the question of model choice and specification analysis.

Part 3 takes up more advanced topics in estimation and computation theory such as non-linear regression methods, biased estimation, and computational algorithms in econometrics. This part also includes a series of chapters on simultaneous equations models, their specification and estimation, distribution theory for such models, and their Bayesian analysis.

Part 4 considers testing of econometric estimators, including Wald, likelihood ratio, and Lagrange multiplier tests; multiple hypothesis testing; distribution theory for econometric estimators and associated test statistics; and Monte Carlo experimentation in econometrics.

Part 5 treats various topics in time series analysis, including time series and spectral methods in econometrics, dynamic specification, inference and causality in economic time series models, continuous time stochastic models, random and changing coefficient models, and the analysis of panel data.

Parts 6 and 7 present discussions of various special topics in econometrics, including latent variable, limited dependent variable, and discrete choice models; functional forms in econometric model building; economic data issues including longitudinal data issues; and disequilibrium, self selection, and switching models.

Finally, *Part 8* covers selected applications and uses of econometrics. Because of the extremely wide range of applications of econometrics, we could select only a few of the more prominent applications. (Other applications will be treated in later volumes in the "Handbooks in Economics" Series.) Applications discussed here include demand analysis, production and cost analysis, and labor economics. This part includes also chapters on evaluating the predictive accuracy of models, econometric approaches to stabilization policy, the formulation and estimation of

models with actors having rational expectations, and the use of econometric models for economic policy formation.

A brief history of econometrics

A brief review of the history of econometrics will put this *Handbook* in perspective. The historical evolution of econometrics was driven both by the increased availability of data and by the desire of generations of scientists to analyze such data in a rigorous and coherent fashion. There are many historical precursors to that which became “econometrics” in this century. Attempts to interpret economic data “scientifically” go back at least as far as Sir William Petty’s “political arithmetic” in the seventeenth century and Engel’s studies of household expenditures in the nineteenth. The results of the latter became known as Engel’s Law, stating that the proportion of total expenditures devoted to food falls as income rises. This “Law” has been tested extensively for many countries over various time periods, as discussed in Houthakker’s (1957) centenary article.

The development of statistical theory has played a critical role in the history of econometrics since econometric techniques are, to a large extent, based on multivariable statistics. Modern statistical theory starts with the work of Legendre and Gauss on least squares, motivated by the attempt to remove errors of observation in astronomy and geodesy. The next great impulse came from biology, in particular from evolutionary theory with, among others, Galton’s work on regression (a term he later invented). Later developments in mathematical statistics included Yule’s work on multiple regression, Karl Pearson’s formulation of the notions of probable error and of testing hypotheses, the more rigorous small-sample theory of Student and R. A. Fisher, R. A. Fisher’s work on the foundations of statistical inference, and the Neyman–Pearson theory of hypothesis testing. All of these developments in mathematical statistics had a significant influence on the development of econometrics.

In the first half of the twentieth century the increased availability of price and quantity data and the interest in price indexes aided by the development of family expenditure surveys generated interest both in theoretical modeling of demand structures and their empirical estimation. Particularly noteworthy were the demand studies of Moore (1914, 1917), Marschak (1931), and Schultz (1928, 1938) and studies of family expenditure by Allen and Bowley (1935). This period also witnessed the initial formulation of the identification problem in econometrics in E. Working (1927); studies of production functions by Cobb and Douglas (1928) [see also Douglas (1948)], and Marschak and Andrews (1944); studies of price determination in agricultural markets by H. Working (1922), Wright (1925), Hanau (1928), Bean (1928), and Waugh (1929), among others; and the statistical modeling of business cycles by Slutsky (1927) and Frisch (1933). Macroeconomet-

ric modeling also began in the 1930s by Tinbergen (1935, 1939) and was given additional impetus by the development of National Income Accounts in the United States and other countries and by Keynes' theoretical work.

The growth of data availability and the development of economic and statistical theory generated a demand for more extensive, more rigorous and higher quality data analysis efforts, stimulating significant research into the methodology of economic data analysis. Of great importance in this respect was the founding of the Econometric Society in 1930 and the publication, starting in 1933, of its journal *Econometrica*. Ragnar Frisch played a key role as the first editor of this Journal.

There was a great flourishing of econometric theory and applications in the period after World War II, particularly due to the work of the Cowles Commission at the University of Chicago. The development of the simultaneous equations model in Haavelmo (1943, 1944, 1947), Koopmans (1950), Hood and Koopmans (1953), Theil (1954) and Basmann (1957) provided econometricians with tools designed specifically for them, rather than for biologists and psychologists. The estimation of simultaneous equations and macroeconomic models in Klein (1950) and Klein and Goldberger (1955) started economic forecasting on a new path. This period also witnessed the important demand studies by Stone (1954a, 1954b) for the United Kingdom, and Wold and Jureen (1953) for Sweden, and the influential studies by Friedman (1957) of the consumption function, and by Theil (1958) of economic forecasts and policy. [For collections of historically important papers in econometrics see Zellner (1968), Hooper and Nerlove (1970), and Dowling and Glahe (1970).]

The more recent period of the 1960s and 1970s has witnessed many important developments in econometric theory and applications. Econometric theory has been refined and extended in many ways. Of particular note is the Bayesian approach to econometrics and the study of special features of econometric models, such as limited dependent variables, latent variables, and non-linear models. Great progress was also made in the statistical analysis of time series. In addition, the development of electronic computers, the great increase in computing power, and the development of sophisticated econometric software packages made it possible to pursue much more ambitious data analysis strategies. These developments expanded the range of applications of econometric methods greatly beyond the earlier applications to household expenditure, demand functions, production and cost functions, and macroeconomic models. Econometrics is now used in virtually every field of economics, including public finance, monetary economics, labor economics, international economics, economic history, health economics, studies of fertility, and studies of criminal behavior, just to mention a few. In all of these fields the greater use of econometric techniques, based in part on increased data availability and more powerful estimation techniques, has led to greater precision in the specification, estimation, and testing of economic data-based models.

Most of the important developments in econometric methods during the 1960s and 1970s are discussed in this *Handbook*. The significant topics under development in this period and the chapters treating them include:

(1) *Bayesian econometrics*, using Bayesian methods in the specification and estimation of econometric models. These topics are discussed in Chapter 2 by Zellner and Chapter 9 by Drèze and Richard.

(2) *Time series methods*, including specialized techniques and problems arising in the analysis of economic time series, such as spectral methods, dynamic specification, and causality. These techniques and problems are discussed in Part 5 on "Time Series Topics," including Chapter 17 by Granger and Watson; Chapter 18 by Hendry, Pagan and Sargan; Chapter 19 by Geweke; Chapter 20 by Bergstrom; and Chapter 21 by Chow. Related issues are discussed in Chapter 33 by Fair.

(3) *Discrete choice models*, in which there is a discrete choice of alternatives available, e.g. buy/don't buy decisions, yes/no responses, or alternative possibilities for urban transportation. Such models are discussed specifically in Chapter 27 by Dhrymes and Chapter 24 by McFadden and are also treated in Chapter 22 by Chamberlain, Chapter 28 by Maddala, and Chapter 29 by Heckman and Singer.

(4) *Latent variables models*, in which certain unmeasurable variables systematically influence measured phenomena, such as ability influencing earnings. This topic is treated in Chapter 23 by Aigner, Hsiao, Kapteyn, and Wansbeek and reappears in various guises in Chapter 22 by Chamberlain and Chapter 32 by Heckman and MaCurdy, among others.

(5) *Specification analysis*, involving problems of model choice and their specification and identification. These issues are treated in Chapter 3 by Intriligator, Chapter 4 by Hsiao, Chapter 5 by Leamer, Chapter 26 by Lau, and Chapter 28 by Maddala. This topic, of course, pervades many other chapters in this *Handbook* and overlaps with chapters which deal with testing and distribution theory.

(6) *Non-linear models and methods*, in which models that are intrinsically nonlinear are specified and estimated. Such models are discussed in Chapter 6 by Amemiya and Chapter 12 by Quandt and surface also in many of the other chapters of this *Handbook*.

(7) *Data analysis issues*, involving various problems with data and how they can be treated. These issues are treated in Chapter 10 by Judge and Bock, in Chapter 11 by Krasker, Kuh, and Welsch, Chapter 22 by Chamberlain, Chapter 25 by Griliches, and Chapter 29 by Heckman and Singer, among others.

(8) *Testing and small sample theory*, including various test procedures and Monte Carlo experimentation. These topics are treated in Part 4 on "Testing," including Chapter 13 by Engle, Chapter 14 by Savin, Chapter 15 by Rothenberg, and Chapter 16 by Hendry. Related issues are discussed in Chapter 8 by Phillips.

(9) *Rational expectations* models which treat economic agents as forming expectations in an optimal fashion, given the information available to them,

impose cross-equation constraints on parameters and lead to new problems of identification and estimation. This topic is discussed in Chapter 34 by Taylor.

ZVI GRILICHES
Harvard University

MICHAEL D. INTRILIGATOR
University of California, Los Angeles

References

- Allen, R. G. D. and A. L. Bowley (1935) *Family Expenditure*. London: P. S. King
- Basmann, R. L. (1957) "A Generalized Classical Method of Linear Estimation of Coefficients in a Structural Equation", *Econometrica*, 25, 77-83.
- Bean, L. H. (1928) "Some Interrelationships between the Supply, Price, and Consumption of Cotton", *USDA*, mimeographed.
- Cobb, C. W. and P. H. Douglas (1928) "A Theory of Production", *American Economic Review*, 18 (supplement), 139-165.
- Douglas, P. H. (1948) "Are There Laws of Production?", *American Economic Review*, 38, 1-41.
- Dowling, J. M. and F. R. Glahe (eds.) (1970) *Readings in Econometric Theory*. Boulder: Colorado Associated University Press.
- Friedman, M. (1957) *A Theory of the Consumption Function*, National Bureau of Economic Research. Princeton: Princeton University Press.
- Frisch, R. (1933) "Propagation Problems and Impulse Problems in Dynamic Economics", in: *Economic Essays in Honor of Gustav Cassel*. London: George Allen & Unwin, pp. 171-205.
- Haavelmo, T. (1943) "The Statistical Implications of a System of Simultaneous Equations", *Econometrica*, 11, 1-12.
- Haavelmo, T. (1944) "The Probability Approach in Econometrics", *Econometrica*, 12 (supplement), 1-115.
- Haavelmo, T. (1947) "Methods of Measuring the Marginal Propensity to Consume", *Journal of the American Statistical Association*, 42, 105-122 (reprinted in Hood and Koopmans (eds) (1953)).
- Hanau, A. (1928) "Die Prognose der Schweinepreise", *Vierteljahrshfte zur Konjunkturforschung*, Sonderheft 7, Berlin.
- Hood, W. C. and T. C. Koopmans (eds.) (1953) *Studies in Econometric Method*, Cowles Commission Monograph No. 14, New York: John Wiley & Sons.
- Hooper, J. W. and M. Nerlove (eds.) (1970) *Selected Readings in Econometrics from Econometrica*. Cambridge: MIT Press.
- Houthakker, H. S. (1957) "An International Comparison of Household Expenditure Patterns, Commemorating the Centenary of Engel's Law", *Econometrica*, 25, 532-551.
- Klein, L. R. (1950) *Economic Fluctuations in the United States, 1921-1941*, Cowles Commission Monograph No. 11. New York: John Wiley & Sons.
- Klein, L. R. and A. S. Goldberger (1955) *An Econometric Model of the United States, 1929-1952*. Amsterdam: North-Holland Publishing Co.
- Koopmans, T. C. (ed.) (1950) *Statistical Inference in Dynamic Economic Models*, Cowles Commission Monograph No. 10. New York: John Wiley & Sons.
- Marschak, J. (1931) *Elastizität der Nachfrage*. Tübingen: J. C. B. Mohr.
- Marschak, J. and W. H. Andrews (1944) "Random Simultaneous Equations and the Theory of Production", *Econometrica* 12, 143-205.
- Moore, H. L. (1914) *Economic Cycles: Their Law and Cause*. New York: The Macmillan Company.
- Moore, H. L. (1917) *Forecasting the Yield and Price of Cotton*. New York: The Macmillan Company.
- Schultz, H. (1928) *Statistical Laws of Demand and Supply*. Chicago: University of Chicago Press.
- Schultz, H. (1938) *The Theory and Measurement of Demand*, Chicago: University of Chicago Press.

- Slutsky, E. (1927) "The Summation of Random Causes as the Source of Cyclic Processes" (Russian with English summary), in: *Problems of Economic Conditions*, vol. 3. Moscow: Rev. English edn., 1937; *Econometrica*, 5, 105–146.
- Stone, R. (1954a) "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand", *Economic Journal*, 64, 511–527.
- Stone, R. (1954b) *The Measurement of Consumers' Expenditure and Behavior in the United Kingdom, 1920–1938*. New York: Cambridge University Press.
- Theil, H. (1954) "Estimation of Parameters of Econometric Models", *Bulletin of the International Statistics Institute*, 34, 122–128.
- Theil, H. (1958) *Economic Forecasts and Policy*. Amsterdam: North-Holland Publishing Co. (Second Edition, 1961).
- Tinbergen, J. (1935) "Quantitative Fragen der Konjunkturpolitik", *Weltwirtschaftliches Archiv*, 42, 316–399.
- Tinbergen, J. (1939) *Statistical Testing of Business Cycle Theories*. Vol. 1: *A Method and its Application to Investment Activity*; Vol. 2: *Business Cycles in the United States of America, 1919–1932*. Geneva: League of Nations.
- Waugh, F. V. (1929) *Quality as a Determinant of Vegetable Prices*. New York: Columbia University Press.
- Wold, H. and L. Jureen (1953) *Demand Analysis*. New York: John Wiley & Sons.
- Working, E. J. (1927) "What do Statistical 'Demand Curves' Show?", *Quarterly Journal of Economics*, 41, 212–235.
- Working, H. (1922) "Factors Determining the Price of Potatoes in St. Paul and Minneapolis", University of Minnesota Agricultural Experiment Station Technical Bulletin 10.
- Wright, S. (1925) *Corn and Hog Correlations*, Washington, USDA, Bul. 1300.
- Zellner, A., Ed. (1968) *Readings in Economic Statistics and Econometrics*. Boston: Little, Brown.

CONTENTS OF VOLUME I

| | |
|--|-----|
| Introduction to the Series | v |
| Contents of the Handbook | vii |
| Preface to the Handbook | xi |
| Part 1 – MATHEMATICAL AND STATISTICAL METHODS IN ECONOMETRICS | |
| <i>Chapter 1</i> | |
| Linear Algebra and Matrix Methods in Econometrics | |
| HENRI THEIL | 3 |
| 1. Introduction | 5 |
| 2. Why are matrix methods useful in econometrics? | 5 |
| 2.1. Linear systems and quadratic forms | 5 |
| 2.2. Vectors and matrices in statistical theory | 7 |
| 2.3. Least squares in the standard linear model | 8 |
| 2.4. Vectors and matrices in consumption theory | 10 |
| 3. Partitioned matrices | 12 |
| 3.1. The algebra of partitioned matrices | 12 |
| 3.2. Block-recursive systems | 14 |
| 3.3. Income and price derivatives revisited | 15 |
| 4. Kronecker products and the vectorization of matrices | 16 |
| 4.1. The algebra of Kronecker products | 16 |
| 4.2. Joint generalized least-squares estimation of several equations | 17 |
| 4.3. Vectorization of matrices | 19 |
| 5. Differential demand and supply systems | 20 |
| 5.1. A differential consumer demand system | 20 |
| 5.2. A comparison with simultaneous equation systems | 22 |
| 5.3. An extension to the inputs of a firm: A singularity problem | 23 |
| 5.4. A differential input demand system | 23 |
| 5.5. Allocation systems | 25 |
| 5.6. Extensions | 25 |
| 6. Definite and semidefinite square matrices | 27 |
| 6.1. Covariance matrices and Gauss–Markov further considered | 27 |
| 6.2. Maxima and minima | 29 |
| 6.3. Block-diagonal definite matrices | 30 |

| | |
|--|-----------|
| 7. Diagonalizations | 30 |
| 7.1. The standard diagonalization of a square matrix | 30 |
| 7.2. Special cases | 32 |
| 7.3. Aitken's theorem | 33 |
| 7.4. The Cholesky decomposition | 34 |
| 7.5. Vectors written as diagonal matrices | 34 |
| 7.6. A simultaneous diagonalization of two square matrices | 35 |
| 7.7. Latent roots of an asymmetric matrix | 36 |
| 8. Principal components and extensions | 37 |
| 8.1. Principal components | 37 |
| 8.2. Derivations | 38 |
| 8.3. Further discussion of principal components | 40 |
| 8.4. The independence transformation in microeconomic theory | 40 |
| 8.5. An example | 43 |
| 8.6. A principal component interpretation | 44 |
| 9. The modeling of a disturbance covariance matrix | 45 |
| 9.1. Rational random behavior | 46 |
| 9.2. The asymptotics of rational random behavior | 47 |
| 9.3. Applications to demand and supply | 49 |
| 10. The Moore–Penrose inverse | 51 |
| 10.1. Proof of the existence and uniqueness | 51 |
| 10.2. Special cases | 52 |
| 10.3. A generalization of Aitken's theorem | 53 |
| 10.4. Deleting an equation from an allocation model | 56 |
| Appendix A: Linear independence and related topics | 57 |
| Appendix B: The independence transformation | 58 |
| Appendix C: Rational random behavior | 61 |
| References | 64 |

Chapter 2

Statistical Theory and Econometrics

| | |
|--|-----------|
| ARNOLD ZELLNER | 67 |
| 1. Introduction and overview | 68 |
| 2. Elements of probability theory | 69 |
| 2.1. Probability models for observations | 70 |
| 2.2. Definitions of probability | 71 |
| 2.3. Axiom systems for probability theory | 74 |
| 2.4. Random variables and probability models | 82 |
| 2.5. Elements of asymptotic theory | 110 |
| 3. Estimation theory | 117 |
| 3.1. Point estimation | 117 |
| 3.2. Criteria for point estimation | 118 |