Statistical Tables

F. James Rohlf   Robert R. Sokal

44.819	81954.924	81965.029	81975.134	81985.240	81995.345
5.874	82055.980	82066.086	82076.193	82086.299	82096.406
6.941	82157.048	82167.155	82177.263	82187.370	82197.478
18	82258.126	82268.235	82278.343	82288.452	82298.561
07	82359.216	82369.326	82379.435	82389.545	82399.655
.206	82460.317	82470.428	82480.538	82490.649	82500.760
.317	82561.429	82571.541	82581.652	82591.764	82601.877
.439	82662.552	82672.664	82682.777	82692.891	82703.004
.572	82763.685	82773.799	82783.914	82794.028	82804.142
854.715	82864.830	82874.945	82885.061	82895.176	82905.291
955.870	82965.986	82976.102	82986.219	82996.335	83006.452
057.036	83067.153	83077.270	83087.388	83097.505	83107.623

Statistical Tables

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Preface

This set of tables grew out of our dissatisfaction with the customary placement of statistical tables at the end of textbooks of biometry and statistics. Serious users of these books and tables are constantly inconvenienced by having to turn back and forth between the text material on a certain method and the table necessary for the test of significance or for some other computational step. Occasionally, the tables are interspersed throughout a textbook at sites of their initial application; they are then difficult to locate, and turning back and forth in the book is not avoided. Frequent users of statistics, therefore, generally use one or more sets of statistical tables, not only because these usually contain more complete and diverse statistical tables than the textbooks, but also to avoid the constant turning of pages in the latter.

When we first planned to write our textbook of biometry (cited below), we thought to eliminate tables altogether, asking readers to furnish their own statistical tables from those available. However, for pedagogical reasons, it was found desirable to refer to a standard set of tables, and we consequently undertook to furnish such tables to be bound separately from the text. Once embarked upon the task of preparing these tables, we gave considerable thought to making them as useful as we could for statistical work in the biological and social sciences. The following guidelines served us in compiling this collection.

The tables must be as up to date as possible. We have included tables for several statistical techniques developed in the last decade or so. Examples in point are Table **G** of $f \ln f$ for the G-test, or Table **V** of shortest unbiased confidence intervals for the variance. Since the tables are designed for use in the 1960's and 1970's, the availability of at least desk calculators has been assumed. Thus, there are no square root tables as such, but square root and cube root tables for calculating machines are given (Tables **A** and **B**).

Most of the tables are computer-generated. The equations used are given to explain how the tables were prepared. Where library functions were used, these were from the FORTRAN IV compiler for the IBM 7040 and GE 625 computers. Mathematical tables have generally been omitted, except for the bare minimum necessary in ordinary statistical work.

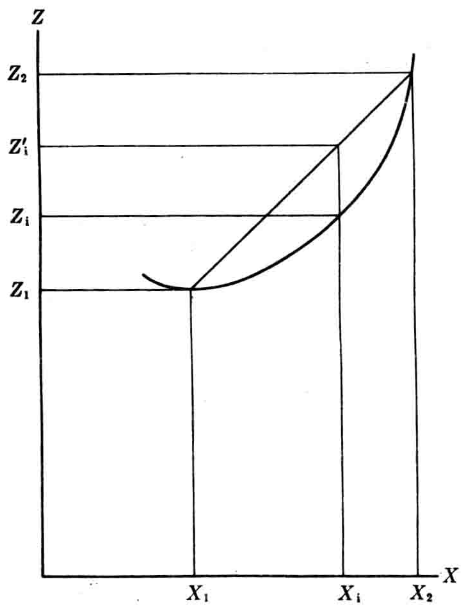
An introductory section on interpolation precedcs the main body of the ιbles. Each table is accompanied by a brief explanation of its nature, a monstration of how to look up a value in it, references to the section (or sections) in our textbook of biometry (R. R. Sokal and F. J. Rohlf, *Biometry*, W. H. Freeman and Company, San Francisco and London, 1969) giving explanations and applications of this table, and by a short note on the method of generation of the table. All references to section, table, or box number unaccompanied by a citation of authors are to this textbook. (Those who use the set of tables but not the textbook should simply disregard these references.)

Several of the tables would have been very complicated and tedious to recompute. These have been copied with permission of authors and publishers, whose courtesy is here acknowledged collectively. We are indebted to the Literary Executor of the late Sir Ronald A. Fisher, F.R.S., Cambridge, to Dr. Frank Yates, F.R.S., Rothamsted, and to Messrs. Oliver and Boyd, Limited, Edinburgh, for permission to reprint tables III and XX from their book *Statistical Tables for Biological, Agricultural and Medical Research*. Other specific acknowledgements are found beneath each table concerned. We appreciate the constructive comments of Professor K. R. Gabriel (Hebrew University), who read a draft of the introductory material.

We hope that users of statistics will find our tables as useful as we have already found them to be in our work. We shall be grateful for any suggestions about changes, additions, or deletions as well as for any corrections.

F. J. Rohlf

R. R. Sokal


Z
Z_2
Z'_i
Z_i
Z_1
X
X_1
X_i
X_2

Introduction: *Interpolation*

Finding a value of a function for an argument that is intermediate between two arguments in a table requires *interpolation*. In some tables published earlier, aids to mental interpolation (proportional parts) are furnished. Since the present tables are oriented toward use with calculating machines, we furnish several formulas especially adapted for machine interpolation.

We shall employ the following symbolism. The tabled arguments to each side of the desired argument X_i are identified as X_1 and X_2, respectively. Argument X_i must lie between the tabled arguments, $X_1 < X_i < X_2$ or $X_1 > X_i > X_2$. The functions shown in the table are Z_1 and Z_2 corresponding to X_1 and X_2 and the desired function corresponding to argument X_i is labeled Z_i.

The simplest method is *linear interpolation*. It assumes that the function $Z = f(X)$ is approximately linear over the interval from X_1 to X_2. It serves as an adequate method where the interval over which one needs to interpolate is not very wide, or where the function is either truly or approximately linear in that interval. The effect of linear interpolation is seen in the accompanying figure, which illustrates a linear function approximating a curvilinear function over the interval from X_1 to X_2. The true function Z_i corresponding to argument X_i is approximated by the linear interpolate Z_i'.

To carry out a linear interpolation on a desk calculator, first compute $p = (X_i - X_1)/(X_2 - X_1)$. Then substitute the given values of the function and p into the following equation:

$$Z_i' = pZ_2 + (1 - p)Z_1$$

The coefficients p and $1 - p$ represent complementary proportions of the distance from the tabled arguments to the intermediate value. When, as frequently happens, the length of the interval from X_1 to X_2 is 1, the computation is especially simple, since $p = X_i - X_1$. Some examples will show the use of this equation. Suppose we wish to find the value of $-\ln 0.133$. In Table **F** we find arguments $X_1 = 0.13$, $X_2 = 0.14$ and corresponding functions $Z_1 = 2.0402$ and $Z_2 = 1.9661$. We compute $p = (0.133 - 0.13)/(0.14 - 0.13) = 0.003/0.01 = 0.3$.

$$\begin{aligned} Z_i' &= (0.3)(1.9661) + (0.7)(2.0402) \\ &= 2.01797, \text{ which is rounded back to } 2.0180. \end{aligned}$$

This compares with 2.0174 given in more detailed mathematical tables.

When evaluating such an equation on a desk calculator, a partial check is furnished in the counter dials, where, after the accumulative multiplication, $1 = p + (1 - p)$ will be found. The interpolated value Z_i' will be in the long dials of the machine.

Another example is shown in which the length of the interval from X_1 to X_2 is 1. Thus p is simply $X_i - X_1$. As an example interpolate for $f \ln f$ in Table **G** when $f = 103.5$.

$$Z_i' = (0.5)(483.017) + (0.5)(477.377) = 480.197$$

The correct value, shown in Table **G***, is 480.196.

Inverse interpolation is employed to evaluate an argument given a value of a function intermediate between two tabled values. Using the same symbolism as above, one approximates the desired argument X_i by X_i' as follows:

$$X_i' = X_1 + \frac{(Z_i - Z_1)(X_2 - X_1)}{Z_2 - Z_1}$$

By way of an illustration, interpolate for the argument in the earlier example from Table **F**, where 2.01797 was obtained as the interpolated value for $-\ln 0.133$. Bracketing this value Z_i are functions $Z_1 = 2.0402$ and $Z_2 = 1.9661$. The corresponding arguments in the table are $X_1 = 0.13$ and $X_2 = 0.14$. On substitution in the inverse interpolation formula, one obtains

$$\begin{aligned} X_i' &= 0.13 + \frac{(2.01797 - 2.0402)(0.14 - 0.13)}{1.9661 - 2.0402} \\ &= 0.13 + \frac{(-0.02223)(0.01)}{-0.0741} \\ &= 0.13 + 0.003 \\ &= 0.133 \end{aligned}$$

Four-point interpolation may provide more exact results than linear interpolation. The symbolism is as before, except that Z_0 is the tabled function corresponding to X_0, the argument before X_1, and Z_3 is the function corresponding to X_3, the argument after X_2. It is assumed that the X's are equally spaced.

$$Z_i' = \tfrac{1}{2}\{[2 + (p - p^2)][pZ_2 + (1 - p)Z_1] - \frac{(p - p^2)}{3}[(1 + p)Z_3 + (2 - p)Z_0]\}$$

Applying this formula to the problem of finding Z_i' when $X_i = -\ln 0.133$, handled above by linear interpolation, one obtains from Table **F**: $X_1 = 0.13$, $X_2 = 0.14$, $Z_0 = 2.1203$, $Z_1 = 2.0402$, $Z_2 = 1.9661$, $Z_3 = 1.8971$. Therefore $p = (0.133 - 0.13)/(0.14 - 0.13) = 0.3$. Solving for Z_i', one obtains

$$Z_i' = \tfrac{1}{2}\{[2 + (0.3 - 0.3^2)][(0.3)(1.9661) + (1 - 0.3)(2.0402)]$$
$$- \frac{(0.3 - 0.3^2)}{3}[(1 + 0.3)(1.8971) + (2 - 0.3)(2.1203)]\}$$
$$= \tfrac{1}{2}\{[2.21][2.01797] - \frac{0.21}{3}[6.07074]\}$$
$$= \tfrac{1}{2}\{4.4597137 - 0.4249518\}$$

= 2.0173810, which is rounded back to 2.0174 and agrees with the correct value.

Many tables, such as Table **Q**, are arranged for *harmonic interpolation.* For the upper range of the arguments, functions in these tables will be approximately linearly related to the reciprocal of the arguments. Usually the arguments are degrees of freedom spaced as follows: 30, 40, 60, 120, ∞. For purposes of convenience in interpolation these are changed by dividing them into the last finite value of the argument, yielding 120/30, 120/40, 120/60, 120/120, $120/\infty$, or 4, 3, 2, 1, 0. These integral values are the new arguments; functions for any argument between these tabled values are linearly interpolated between these transformed values. One advantage of this method is that it permits interpolation between a finite and an infinite argument. An example will illustrate the method. Find the value of $t_{.001[200]}$. In Table **Q** can be found $Z_1 = t_{.001[120]} = 3.373$ and $Z_2 = t_{.001[\infty]} = 3.291$. Change the arguments $X_1 = 120$ and $X_2 = \infty$ to $X_1 = 120/120 = 1$ and $X_2 = 120/\infty = 0$, respectively. Evaluate X_i as $120/200 = 0.6$, and apply the formula for linear interpolation:

$$p = (0.6 - 1)/(0 - 1) = 0.4$$

$$Z_i' = (0.4)(3.291) + (0.6)(3.373)$$
$$= 3.3402, \text{ which is rounded back to } 3.340.$$

Contents

Statistical Tables

TABLE A. Computation of square roots on a desk calculator.

This table furnishes constants necessary to obtain square roots on a desk calculator. The number whose square root is to be obtained is entered into the long (accumulating) dials, the appropriate constant is added, and the sum is divided by a second constant. The square root appears in the short dials, correct to five significant figures with a maximal error of 1 in the fifth significant digit.

Numbers with one or two significant digits to the left of the decimal point should be entered into the keyboard, allowing for at least five decimal places to the right of the decimal point. They are transferred into the long dials by means of the enter dividend key. Set the tabs to obtain seven decimal places in the quotient. Thus 1.234 should be entered in the keyboard as 1.23400. Now consult the table. In the first column you will find class limits at varying intervals ranging from 1.000 to 100.000. Find the two limits bracketing the number whose square root you want. Thus, in the case of 1.234, the class limits are 1.230 and 1.259. The values in columns 2 and 3 between these class limits are employed in obtaining the square root. Add the value in the second column (1.24454) to the number 1.234 already in the long dials of the machine to obtain 2.47854. Divide this sum by the divisor in column 3 (2.2312). All numbers must be properly aligned on the keyboard in terms of their decimal points. The quotient 1.1108551 is recorded as the square root 1.1109, correct to five significant figures. In general the fifth digit may be in error by ± 1 (in our example the actual square root is 1.1108555). Numbers with more than two significant digits to the left of the decimal point and those with no significant digits to the left of the decimal point are divided or multiplied, respectively, by even powers of 10 (10^2, 10^4, 10^6, . . .) until either one or two significant digits are obtained to the left of the decimal point. The square root is then computed as shown above and is corrected by multiplying or dividing by $10^{p/2}$, the original number having been divided or multiplied by 10^p. As an example, find the square root of 0.005278. Multiply this by 10^4 to obtain 52.78. In the table this value is bracketed by 52.481 and 53.703, yielding constants in columns 2 and 3 of 53.0893 and 14.5726, respectively. The computation $(52.78 + 53.0893) \div 14.5726$ yields 7.2649562, which is rounded to 7.2650, again with a possible error of ± 1 in the last digit. To obtain the correct square root, divide by $10^{p/2} = 10^{4/2} = 10^2$, which yields 0.072650. As a check, square this number and obtain 0.005278022500. When the number whose square root is sought happens to be exactly one of the class limits in column 1 of the table, you are free to choose the constants above or below the limit for the computation.

Other methods of obtaining square roots are by means of Table **C**, common logarithms (where $\sqrt{X}$ is found as antilog $\frac{1}{2}$ log X), or by the direct method for calculating a square root given as exercise 5 of the basic mathematical operations in Appendix A2. The square root can be read off directly in Table **I** for integers between 0 and 999 and in Table **F** for numbers between 0 and 1.

The table was computed using a Tschebyscheff approximation and the spacing of the arguments was adjusted so that the maximum relative error, which is $\leq 10^{-5}$, would be constant for the entire range of the table.

TABLE A. Computation of square roots on a desk calculator.

(1)	(2)	(3)	(1)	(2)	(3)
1.000			1.950		
	1.01162	2.0116		1.97245	2.8089
1.023			1.995		
	1.03519	2.0349		2.01836	2.8414
1.047			2.042		
	1.05924	2.0584		2.06537	2.8743
1.072			2.089		
	1.08398	2.0823		2.11350	2.9076
1.096			2.138		
	1.10921	2.1064		2.16278	2.9413
1.122			2.188		
	1.13506	2.1308		2.21307	2.9753
1.148			2.239		
	1.16142	2.1554		2.26469	3.0098
1.175			2.291		
	1.18852	2.1804		2.31736	3.0446
1.202			2.344		
	1.21626	2.2057		2.37141	3.0799
1.230			2.399		
	1.24454	2.2312		2.42670	3.1156
1.259			2.455		
	1.27349	2.2570		2.48310	3.1516
1.288			2.512		
	1.30323	2.2832		2.54095	3.1881
1.318			2.570		
	1.33354	2.3096		2.60027	3.2251
1.349			2.630		
	1.36455	2.3363		2.66077	3.2624
1.380			2.692		
	1.39639	2.3634		2.72278	3.3002
1.413			2.754		
	1.42896	2.3908		2.78618	3.3384
1.445			2.818		
	1.46226	2.4185		2.85099	3.3770
1.479			2.884		
	1.49632	2.4465		2.91756	3.4162
1.514			2.951		
	1.53113	2.4748		2.98542	3.4557
1.549			3.020		
	1.56673	2.5034		3.05493	3.4957
1.585			3.090		
	1.60324	2.5324		3.12613	3.5362
1.622			3.162		
	1.64068	2.5618		3.19904	3.5772
1.660			3.236		
	1.67881	2.5914		3.27351	3.6186
1.698			3.311		
	1.71791	2.6214		3.34976	3.6605
1.738			3.388		
	1.75798	2.6518		3.42781	3.7029
1.778			3.467		
	1.79892	2.6825		3.50751	3.7457
1.820			3.548		
	1.84074	2.7135		3.58926	3.7891
1.862			3.631		
	1.88372	2.7450		3.67291	3.8330
1.905			3.715		
	1.92762	2.7768		3.75850	3.8774
1.950			3.802		

(1)	(2)	(3)
3.802		
	3.84605	3.9223
3.890		
	3.93560	3.9677
3.981		
	4.02718	4.0136
4.074		
	4.12103	4.0601
4.169		
	4.21700	4.1071
4.266		
	4.31531	4.1547
4.365		
	4.41581	4.2028
4.467		
	4.51874	4.2515
4.571		
	4.62393	4.3007
4.677		
	4.73163	4.3505
4.786		
	4.84190	4.4009
4.898		
	4.95455	4.4518
5.012		
	5.07007	4.5034
5.129		
	5.18806	4.5555
5.248		
	5.30902	4.6083
5.370		
	5.43254	4.6616
5.495		
	5.55913	4.7156
5.623		
	5.68861	4.7702
5.754		
	5.82103	4.8254
5.888		
	5.95667	4.8813
6.026		
	6.09537	4.9378
6.166		
	6.23740	4.9950
6.310		
	6.38284	5.0529
6.457		
	6.53149	5.1114
6.607		
	6.68366	5.1706
6.761		
	6.83916	5.2304
6.918		
	6.99855	5.2910
7.079		
	7.16166	5.3523
7.244		
	7.32827	5.4142
7.413		

(1)	(2)	(3)
7.413		
	7.49898	5.4769
7.586		
	7.67360	5.5403
7.762		
	7.85248	5.6045
7.943		
	8.03539	5.6694
8.128		
	8.22242	5.7350
8.318		
	8.41421	5.8015
8.511		
	8.60997	5.8686
8.710		
	8.81066	5.9366
8.913		
	9.01576	6.0053
9.120		
	9.22595	6.0749
9.333		
	9.44071	6.1452
9.550		
	9.66075	6.2164
9.772		
	9.88583	6.2884
10.000		
	10.1160	6.3612
10.233		
	10.3515	6.4348
10.471		
	10.5926	6.5093
10.715		
	10.8394	6.5847
10.965		
	11.0920	6.6610
11.220		
	11.3503	6.7381
11.482		
	11.6146	6.8161
11.749		
	11.8851	6.8950
12.023		
	12.1621	6.9749
12.303		
	12.4452	7.0556
12.589		
	12.7354	7.1374
12.882		
	13.0319	7.2200
13.183		
	13.3354	7.3036
13.490		
	13.6461	7.3882
13.804		
	13.9638	7.4737
14.125		
	14.2893	7.5603
14.454		

TABLE A. Computation of square roots on a desk calculator.

(1)	(2)	(3)	(1)	(2)	(3)
14.454			28.184		
	14.6220	7.6478		28.5109	10.6792
14.791			28.840		
	14.9627	7.7364		29.1746	10.8028
15.136			29.512		
	15.3109	7.8259		29.8543	10.9279
15.488			30.200		
	15.6679	7.9166		30.5500	11.0545
15.849			30.903		
	16.0326	8.0082		31.2616	11.1825
16.218			31.623		
	16.4063	8.1010		31.9892	11.3119
16.596			32.359		
	16.7884	8.1948		32.7344	11.4429
16.982			33.113		
	17.1795	8.2897		33.4969	11.5754
17.378			33.884		
	17.5797	8.3857		34.2775	11.7095
17.783			34.674		
	17.9892	8.4828		35.0760	11.8451
18.197			35.481		
	18.4081	8.5810		35.8927	11.9822
18.621			36.308		
	18.8370	8.6804		36.7291	12.1210
19.055			37.154		
	19.2757	8.7809		37.5842	12.2613
19.498			38.019		
	19.7244	8.8825		38.4598	12.4033
19.953			38.905		
	20.1840	8.9854		39.3555	12.5469
20.417			39.811		
	20.6540	9.0894		40.2723	12.6922
20.893			40.738		
	21.1353	9.1947		41.2106	12.8392
21.380			41.687		
	21.6277	9.3012		42.1700	12.9878
21.878			42.658		
	22.1315	9.4089		43.1524	13.1382
22.387			43.652		
	22.6468	9.5178		44.1580	13.2904
22.909			44.668		
	23.1742	9.6280		45.1866	13.4443
23.442			45.709		
	23.7141	9.7395		46.2386	13.5999
23.988			46.774		
	24.2666	9.8523		47.3157	13.7574
24.547			47.863		
	24.8319	9.9664		48.4178	13.9167
25.119			48.978		
	25.4103	10.0818		49.5460	14.0779
25.704			50.119		
	26.0019	10.1985		50.7000	14.2409
26.303			51.286		
	26.6076	10.3166		51.8809	14.4058
26.915			52.481		
	27.2276	10.4361		53.0893	14.5726
27.542			53.703		
	27.8616	10.5569		54.3256	14.7413
28.184			54.954		

(1)	(2)	(3)
54.954		
	55.5910	14.9120
56.234		
	56.8861	15.0847
57.544		
	58.2114	15.2594
58.884		
	59.5673	15.4361
60.256		
	60.9545	15.6148
61.659		
	62.3742	15.7956
63.096		
	63.8271	15.9785
64.565		
	65.3144	16.1636
66.069		
	66.8352	16.3507
67.608		
	68.3926	16.5401
69.183		
	69.9854	16.7316
70.795		
	71.6153	16.9253
72.444		
	73.2835	17.1213
74.131		
	74.9909	17.3196
75.858		
	76.7372	17.5201
77.625		
	78.5249	17.7230
79.433		
	80.3538	17.9282
81.283		
	82.2254	18.1358
83.176		
	84.1407	18.3458
85.114		
	86.1012	18.5583
87.096		
	88.1059	18.7731
89.125		
	90.1583	18.9905
91.201		
	92.2583	19.2104
93.325		
	94.4078	19.4329
95.499		
	96.6067	19.6579
97.724		
	98.8566	19.8855
100.000		