# STATISTICAL TECHNIQUES IN BUSINESS AND ECONOMICS

**FOURTH EDITION** 

ROBERT D. MASON

IRWIN SERIES IN QUANTITATIVE ANALYSIS FOR BUSINESS

STATISTICAL TECHNIQUES IN BUSINESS AND ECONOMICS



Fourth Edition 1978

RICHARD D. IRWIN, INC. Homewood, Illinois 60430 Irwin-Dorsey Limited Georgetown, Ontario L7G 4B3

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ISBN 0-256-02025-6 Library of Congress Catalog Card No 77-83573 Printed in the United States of America

#### 1234567890K54321098

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## **PREFACE**

As the title implies, this book is designed primarily for a first course in business and economic statistics. Ample material is included for a one-year course, but considerable latitude is possible in the selection of topics for shorter courses. The mathematics involved can be handled by students who do not have an extensive mathematical background.

New material in this fourth edition includes two-way analysis of variance and an expansion of nonparametric tests of hypotheses to include the Mann-Whitney test. The chapters on multiple regression and correlation and on decision making have been revised and expanded. A summary outline has been added to each chapter. It can be used by students to put all of the ideas of the chapter in brief form. The instructor can use it to insure that each of the topics contained in the chapter is covered.

Based on student response, the problems in the chapters (with answers) and the chapter self-review examinations (with answers) introduced in the previous edition have been expanded. By solving these problems, students can immediately evaluate how well they have grasped the preceding concepts. In a sense, this is the powerful tool of programmed learning applied to basic statistics. The number of unsolved problems appearing at the ends of the chapters has been increased.

Several computer outputs using the Statistical Package for the Social Sciences have been included in the text to illustrate the computer's potential for problem solving.

A problems manual is a supplement to the textbook and a solutions manual is available

I wish to thank colleagues William G. Marchal, Douglas A. Lind, and Patsy F. Scott for their many helpful comments in organizing this fourth edition. Special acknowledgment is given to Bruce R. Boals, University of Tennessee in Nashville; Barbara Crafford, California State University in San Jose; Shou-Eng Koo, Indiana Purdue University in Indianapolis; Joseph Longi, St. Louis Community College in Meramec; Harold L. Royer, Miami Dade Community College South; and John C. Shannon, Suffolk University, who made many valuable comments and criticisms in their reviews of the manuscript. I wish to express special thanks to G. Harris Walker of Western Illinois University for his many helpful suggestions. Finally, I am indebted to the Literary Executor of the late Sir Ronald A. Fisher, F.R.S. Cambridge, to Dr. Frank Yates, F.R.S. Rothamsted, and to Messrs. Oliver and Boyd Ltd., for permission to reprint Tables III and IV from their book *Statistical Tables for Biological, Agricultural and Medical Research*.

January 1978

Robert D. Mason

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## INTRODUCTION

## **DEFINITION**

Followers of the Los Angeles Rams, the Cincinnati Reds, the Boston Celtics, and other professional and college teams often visualize a statistician as one who sits in the press box and records the number of yards gained, home runs, and foul shots attempted. Then he releases statistics about the 42 points scored by Dr. J. against the hapless Knicks, and the 67 shot by Jack Nicklaus.

Likewise, reference to *The Wall Street Journal*, local newspapers, *Forbes* and other business magazines, and annual reports of industry revealed such diversified statistics as the \$350,000 annual income of Robert J. Suslow (Saks Fifth Avenue) and the \$1.5 million income of Michel C. Bergerac (Revlon).<sup>3</sup> Further, salaries of teachers in Alaska start at \$18,000, the average annual salary is \$21,000, and earnings go up to \$30,000.<sup>2</sup> And, from a local newspaper,<sup>3</sup> it will cost about \$90,000 to buy a medium-priced new home by 1986 (compared with about \$45,000 today); total outstanding consumer installment debt is \$178.3 billion; 400,000 color television sets and 1,224,000 tons of macaroni were produced in the USSR last year, and for every 100,000 persons, there were 2.3 murders in Toronto last year compared with 14.3 in Los Angeles, 8.0 in U.S. rural areas, and 9.6 for the entire

<sup>&</sup>lt;sup>1</sup> Forbes, April 1, 1977, p. 30.

<sup>&</sup>lt;sup>2</sup> The Wall Street Journal, April 5, 1977, p. 1.

<sup>3</sup> The Blade, Toledo, Ohio, April 1, 1977.

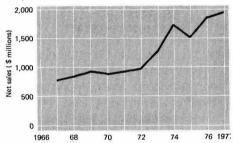
United States. The National Health Education Committee in a statistical report said that "The typical American had 0.9 of a cold or case of flu.... The cold or flu curtailed the patient's normal activities for approximately five days for a total of more than one billion days of restricted activity." Finally a statistic from Kroger: Ground beef is on sale this week for 69¢ a pound.

Statistics has another meaning, however. It is defined as the science of collecting, organizing, presenting, analyzing, and interpreting numerical data for the purpose of making better decisions in the face of uncertainty. It is in this context that statistics will be approached in this book.

## **DESCRIPTIVE STATISTICS**

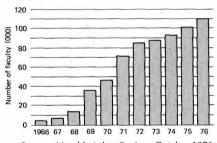
Note in the foregoing definition that one facet of statistics deals with collection, organization, and presentation, that is, with methods used in describing numerical data. As examples, Kaiser Aluminum & Chemical

CHART 1-1 Net sales of the Kaiser Aluminum & Chemical Corporation (1967-1976)



Source: 1976 Annual Report, Kaiser Aluminum & Chemical Corporation, p. 30.

CHART 1–2 Number of unionized faculty in institutions of higher education (1966–1976)



Source: Monthly Labor Review, October 1976, p. 33.

Corporation wanted to show their stockholders the rapid growth in sales since 1967. Sales were portrayed by a descriptive tool called a *line chart* (Chart 1–1). The dramatic increase in the number of unionized faculty in colleges and universities was described by the Bureau of Labor Statistics in the form of a *bar chart* (Chart 1–2).

Several chapters in this book are devoted to descriptive statistics. Various sources of statistical data and graphic presentation are examined in Chapter 2. Organization of data is explored in Chapter 3. Analysis of statistical data may be accomplished by computing one or more measures of central tendency (Chapter 4) and measuring the dispersion (Chapter 5). Further, it may be appropriate to convert the data to indexes (Chapter 6) or to use the statistical techniques described in other chapters.