DEMANA WAITS FOLEY KENNEDY

# PRECALCULUS

**FUNCTIONS** and **GRAPHS** 

FOURTH EDITION



### **PRECALCULUS**

#### FUNCTIONS AND GRAPHS

**FOURTH EDITION** 

Franklin Demana Ohio State University

Bert K. Waits
Ohio State University

**Gregory D. Foley**Appalachian State University

Daniel Kennedy Baylor School

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Senior Production Supervisor: Peggy McMahon

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**505** Exploration is based on a problem originally posed by Neal Koblitz in March 1988 issue of The American Mathematical Monthly.

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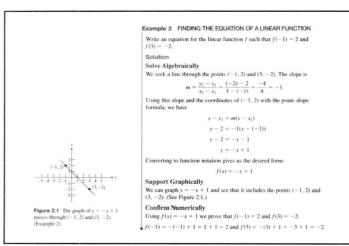
#### **Preface**

Our text combines appropriate use of technology with standard "paper and pencil" analytic techniques to provide a balanced approach to the study and implementation of precalculus. Technology is fully integrated, rather than just added. The text encourages graphical, numerical, and algebraic modeling of functions as well as problem solving, conceptual understanding, and facility with technology.

Our primary objectives are:

- to help students to truly understand the fundamental concepts of algebra, trigonometry, and analytic geometry,
- · to foreshadow important ideas of calculus, and
- to show how algebra and trigonometry can be used to model real-life problems.

In writing this edition, we have followed the guidelines and recommendations published by AMATYC, MAA, and NCTM, and also responded to the many helpful suggestions of both students and instructors. As a result, we believe that the changes made in this edition make this the most effective text available to prepare students for calculus, science, and advanced mathematics courses.



For Example 2, see page 157.

#### The Rule of Four - A Balanced Approach

A principal feature of this edition is the balance among the algebraic, numerical, graphical, and verbal methods of representing problems: the rule of four. For instance, we obtain solutions algebraically when that is the most appropriate technique to use, and we obtain solutions graphically or numerically when algebra is difficult to use. We urge students to solve problems by one method and then support or confirm their solutions by using another method. We believe that students must learn the value of each of these methods of representation and must learn to choose the one most appropriate for solving the particular problem under consideration. This approach reinforces the idea that to understand a problem fully, students need to understand it algebraically as well as graphically and numerically.

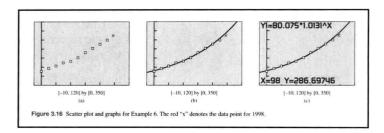
#### Algebraic Skills

- The Prerequisite Chapter includes a review of solving equations and inequalities algebraically and graphically.
- Appendix A gives instructors the option of including algebraic skills review when necessary.
- Quick Review exercises precede each section exercise set so that students can review the algebraic skills needed to solve the exercises in that section.

#### **Applications and Real Data**

The majority of the applications in the text are based on real data from cited sources, and their presentations are self-contained; students will not need any experience in the fields from which the applications are drawn.

As they work through the applications, students are exposed to functions as mechanisms for modeling data and are motivated to learn about how various functions can help model real-life problems. They learn to analyze and



Example 6 MODELING U.S. POPULATION USING EXPONENTIAL REGRESSION

Use the 1900-1990 data in Table 3.9 and exponential regression to predict the U.S. population for 1998. Compare the result with the listed value for 1998.

Solution

Model

Let P(t) be the population (in millions) of United States t years after 1900. Figure 3.16a shows a scatter plot of the data. Using exponential regression, we find a model for the data:

 $P(t) = 80.075 \cdot 1.0131^{t}$ 

Figure 3.16b on the next page shows the scatter plot of the data with a graph of the population model just found. You can see that the curve fits the data fairly well. The coefficient of determination is  $r^2\approx 0.9945$ , indicating a close fit and supporting the visual evidence.

Solve Graphically

To predict the 1998 U.S. population we substitute t=98 into the regression model. Figure 3.16c reports that  $P(98)=80.075 \cdot 1.0131^{98} \approx 286.7$ .

Interpret

The model predicts the U.S. population was 286.7 million in 1998. See Figure 3.16c. The actual population was 273.8 million. We overestimated by 12.9 million, slightly less than a 5% error.

For Example 6, see pages 276-277.

Year	Population
1900	76.2
1910	92.2
1920	106.0
1930	123.2
1940	132.2
1950	151.3
1960	179.3
1970	203.3
1980	226.5
1990	248.7
1998	273.8

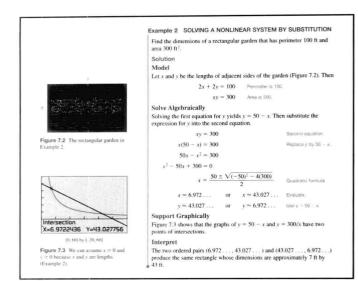
model data, represent data graphically, interpret from graphs, and fit curves. Additionally, the tabular representation of data presented in this text highlights the concept that a function is a correspondence between numerical variables. This helps students build the connection between the numbers and graphs and recognize the importance of a full graphical, numerical, and algebraic understanding of the problem. For a complete listing of applications, please see the Applications Index on page 945.

#### Chapter Openers

The answer to the chapter opening application is given at the end of the section in which the appropriate concepts are covered. Frequently, real data is presented in these problems to enable students to explore realistic situations using graphical, numerical and algebraic methods. In other cases, students are asked to model problem situations using the functions studied in the chapter.



For Chapter Opener, see page 60.



#### **Problem Solving Approach**

Systematic problem solving is emphasized in the examples throughout the text, using the following variation of Polya's problem-solving process:

understand the problem;

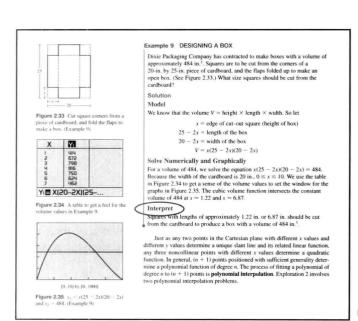
develop a mathematical model;

*solve* the mathematical model and support or confirm the solutions;

and interpret the solution.

Students are encouraged to use this process throughout the text.

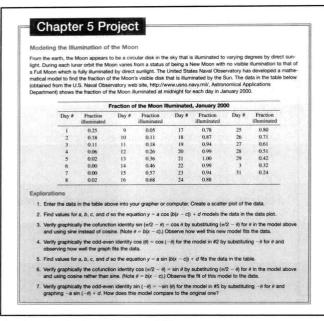
For Example 2, see page 544.



#### **Graphing Utilities**

Students are expected to use a graphing utility (grapher) to visualize and solve problems. Throughout the text, the student is prompted to *recognize* that a graph is reasonable; *identify* all the important characteristics of a graph, *interpret* those characteristics, and *confirm* them using analytic techniques. By using this method to analyze the numerous figures presented in the examples and exercises, students develop excellent graph-viewing skills.

For Example 9, see page 194.



#### **Chapter Projects**

Each chapter concludes with a project that requires students to analyze data, and can be assigned as either individual or group work. Each project expands upon concepts and ideas taught in the chapter, and many projects refer students to the Web for further investigation of real data.

For Chapter Project, see page 477.

#### **Tips and Historical Notes**

**Tips** throughout the text offer practical advice to students on using their grapher to obtain the best, most accurate results. **Margin Notes** include historical information, hints about examples, and provide additional insight to help students avoid common pitfalls and errors.

Some Words of Warning

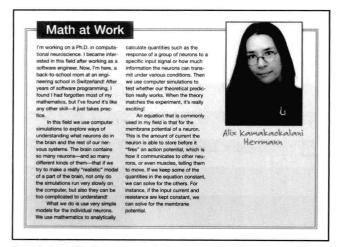
In Figure 3.21, notice we used "10^Ans" instead of "10^1.537819095" to check log (34.5). This is because graphers generally store more digits than they display and so we can obtain a more accurate check. Even so, because log (34.5) is an irrational number, a grapher cannot produce its exact value, so checks like those shown in Figure 3.21 may not always work out so perfectly.

For tip, see page 285.

History of Conic Sections

Parabolas, ellipses, and hyperbolas had been studied for many years when Apollonius (c. 262–190 B.C.) wrote his eight-volume Conic Sections. Apollonius, who was born in Perga in northwestern Asia Minor, was the first to unify these three curves as cross sections of a cone and was the first to view the hyperbola as having two branches. Interest in these curves was renewed in the 17th century when Galileo proved that projectiles follow parabolic paths and Kepler discovered that planets travel in elliptical orbits.

For historical note, see page 603.



For Math at Work, see page 249.

#### Math at Work

The Math at Work feature introduces the student to individuals who are using mathematics in their jobs. Students see how the ideas they are studying are used in real life.

#### **Chapter Review**

At the end of each chapter are sections dedicated to helping students review the chapter concepts and material. "Key Ideas" has four parts: Concepts; Properties, Theorems and Formulas; Procedures; and Gallery of Functions. The "Review Exercises" represent the full range of exercises covered in the chapter and give students additional practice with the ideas developed in the chapter. The exercises with colored numbers indicate problems that we feel would make up a good practice test.

#### Changes to this edition

#### Chapter P

The new Prerequisites Chapter contains two new sections on solving equations and inequalities graphically and algebraically. The algebra review material has been moved to the new Appendix A. This new appendix will help students review, sharpen, and hone their algebra skills.

#### Chapter 1

The basic functions are presented early in this chapter, enabling students to work with a richer variety of functions when learning the basic concepts—a benefit of grapher technology that gives coherence to the rest of the course. The idea of the limit is introduced conceptually along with limit notation to help describe end behavior and asymptotes. Graphical, algebraic, and numerical modeling with functions is emphasized from the beginning.

#### Chapter 2

This chapter covers polynomial, rational, and power functions. The treatment of rational functions has been streamlined, but coverage of power functions has been extended. We introduce average rate of change of a function to show how the rate of change of a linear function can be related to slope. Students will encounter average rate of change again in calculus, and its inclusion in this chapter allows for the investigation of more interesting applications and models.

#### Chapter 3

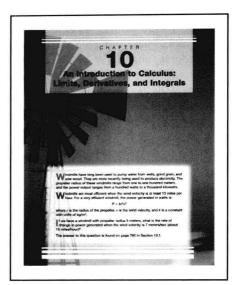
The material in this chapter has been reorganized. We include more on mathematical modeling with special emphasis on this technique in Sections 3.2 and 3.5. We include more material on logistic functions and the natural base e, and also address the concept of "orders of magnitude."

#### Chapter 4

In this chapter we introduce the trigonometric functions using balanced algebraic and graphical approaches. This allows the students to better understand how the functions behave. We have moved the discussion of vectors to Chapter 6 to allow for a more extended treatment of this concept.

#### Chapter 5

This is the second of our new trigonometry chapters. Here, we use trigonometric identities to teach mathematical proof. We also use word ladders to illustrate the strategy for proving an identity. The inclusion of more "explorations" allows students to discover trigonometric relationships rather than just memorize them.



For Chapter 10, see page 752.

#### **Looking Ahead to Calculus**

One of the goals of this text is to help build an intuitive foundation for calculus.

- Throughout the text, many examples and topics are marked with an icon, to point out concepts that students will encounter again in calculus. Ideas that foreshadow calculus are highlighted, such as limits, maximum and minimum, asymptotes, and continuity.
- Early in the text, students are introduced to the idea of the limit using an
  intuitive and conceptual approach. Some calculus notation and language
  is introduced in the beginning chapters and used throughout the text to
  establish familiarity.
- Chapter 10, A Preview of Calculus, is new to this edition. The chapter
  first provides an historical perspective to calculus by presenting the classical studies of motion through the tangent line and area problems.
  Limits are then investigated further, and the chapter concludes with
  graphical and numerical examinations of derivatives and integrals.

#### **Exercise Sets**

The exercise sets have been extensively revised for this edition. There are over 6000 exercises including 680 Quick Review Exercises. These Quick Review Exercises appear at the beginning of each exercise set and help students review skills that will be needed in the rest of the exercise set. Following the Quick Review are exercises that allow students to practice the algebraic skills that they have learned in that section. These have been carefully graded from routine to challenging. The following types of skills are tested in each exercise set:

- · Algebraic and analytic manipulation
- Connecting algebra to geometry
- Interpretation of graphs
- · Graphical and numerical representations of functions
- · Data analysis

Also included in the exercise sets are thought-provoking exercises:

- Explorations are opportunities for students to discover mathematics on their own or in groups. These exercises often require the use of critical thinking to explore the ideas.
- Writing to Learn exercises give students practice at communicating about mathematics and opportunities to demonstrate their understanding of important ideas.
- Extending the Ideas exercises go beyond what is presented in the textbook. These exercises are challenging extensions of the book's material. Students may be asked to work on these problems in groups or to solve them as individual or group projects.

This variety provides sufficient flexibility to emphasize the skills most needed for each student or class. Answers to odd numbered problems appear at the back of the text.

#### Chapter 6

This chapter is dedicated to the discussion of vectors and parametric and polar equations. Coverage of vectors has been extended to two sections, rather than one in this edition. The material on Parametric Equations has been consolidated into its own section, and is followed by sections on polar coordinates and polar equations.

#### Chapter 7

This chapter is a reorganization of Chapter 10 of the previous edition. We have included a new section on matrix algebra, and delayed our discussion of partial fractions until this chapter.

#### Chapter 8

The material in this chapter has also been reorganized. Coverage of conics has been expanded to three separate sections on parabolas, ellipses, and hyperbolas for a more thorough and complete treatment. We have revised our approach to translations and rotations and have added a new section on three-dimensional analytic geometry. We feature a variety of applications of conics, including a great deal on reflective properties and applications to celestial mechanics.

#### Chapter 9

This is our chapter on Discrete Mathematics. The chapter is organized so that coverage of combinatorics appears before probability and the binomial theorem. The section on probability includes a discussion of conditional probability. Data analysis is emphasized throughout the chapter, especially in the statistics sections where we use real and up-to-date data in our examples and exercises.

#### Chapter 10

This introduction to calculus chapter is new to this text. The chapter first provides an historical perspective to calculus by presenting the classical studies of motion through the tangent line and area problems. Limits are then investigated further, and the chapter concludes with graphical and numerical examinations of derivatives and integrals.

#### **Supplements**

#### For the Instructor

#### **Resource Manual** (0-201-70049-2)

- · Major Concepts review
- Group Activity Worksheet
- Sample Chapter Tests
- Standardized Test Preparation Questions
- Contest Problems

#### **Complete Solutions Manual** (0-201-69973-7)

• Complete solutions to all exercises, including chapter review exercises

#### **Tests and Quizzes** (0-201-70067-0)

- · Two parallel test per chapter
- Two quizzes for every 3–4 sections
- Two parallel mid-term tests, covering Chapters P-5
- Two parallel end-of-year tests, covering Chapters 6-10

#### **TestGen: Computerized Test Generator** (0-201-69972-9)

- The instructor can easily view, edit, and add questions, transfer questions to tests, and print tests in a variety of fonts and formats.
- Built-in question editor gives the ability to create graphs, import graphics, insert mathematical symbols and templates, and insert variable numbers or text.
- The instructor can create and save tests using TestGen-EQ so students can take them for practice or a grade on a computer network using QuizMaster-EQ.
- The instructor can set preferences for how and when tests are administered with QuizMaster-EQ.
- All tests can be saved as HTML for use on the Web.

#### For the Student

#### Student's Solution Manual (0-201-69975-3)

Contains detailed, worked-out solutions to odd-numbered exercises

#### **Graphing Calculator Manual** (0-201-70068-9)

- Grapher Workshop provides detailed instruction on important grapher features.
- Features TI-83 and -85, Casio and HP.

#### InterAct Math (0-201-69976-1)

- CD version: Available bundled with the text, InterAct Math on CD-ROM provides extra practice and interactive tutorials with algorithmically generated exercises correlated to this text. Each practice exercise is accompanied by an example and guided solution designed to involve students in the solution process. The software recognizes common student errors and provides appropriate feedback. Student performance history is recorded with each use.
- Website version: On the Web, http://www.awl.com/demana, you can
  use our InterAct Math tutorials correlated to this text free of charge.
- LAN version: InterAct Math Plus is available for installation on your school's LAN. InterAct Math Plus enables instructors to create and administer online tests, summarize students' results, and monitor students' progress in the tutorial software, providing an invaluable teaching and tracking resource.
- Full Web System: Delivered over the Web, InterAct MathXL issues a diagnostic test correlated to this textbook, individualized study plans based on those test results, targeted practice and tutorial help in InterAct Math, and student tracking. After practicing exercises from their study plan, students can take another test and compare their results. A free InterAct MathXL registration coupon is available for bundle with this text.

#### The AWL Math Tutor Center

The AWL Math Tutor Center is staffed by qualified mathematics instructors who provide students with tutoring on examples and exercises from their text-book. Tutoring is available via toll-free telephone, fax, or e-mail five days a week, seven hours a day. For qualified adoptors only.

#### Website

Found at http://www.awl.com/demana, our free dedicated Web site provides dynamic resources for instructors and students. Some of the resources include: a syllabus manager, TI graphing calculator downloads, online quizzing, teaching tips, study tips, Explorations and end-of-chapter projects, InterAct Math tutorials, "Looking Ahead to Calculus" hints, and more.

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Bruce Hoelter Raritan Valley Community College
Dwight H. Horan Wentworth Institute of Technology

Margaret Hovde Grossmont College

Miles Hubbard Saint Cloud State University

Sally Jackman Richland College

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Ron Marshall Western Carolina University
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Cynthia M. Piez
University of Idaho
Debra Poese
Jack Porter
University of Kansas
Antonio R. Quesada
University of Kansas
The University of Akron

Thomas H. Rousseau Siena College

David K. Ruch
Mary Margaret Shoaf-Grubbs
Sandy Spears
Shirley R. Stavros
Stuart Thomas
Sam Houston State University
College of New Rochelle
Jefferson Community College
Saint Cloud State University
University of Oregon

Stuart Thomas University of Oregon
Janina Udrys Schoolcraft College
Mary Voxman University of Idaho

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# Prerequisites

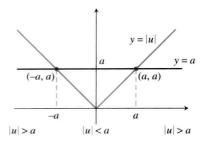
Nothing can travel faster than the speed of light. Engineers planning space missions use the speed of light to measure large distances in units called light years. Using the speed of light, we can use the length of time it takes light to go from one object in space to another to determine the distance between the objects.

The Moon is about 237,000 miles from Earth and light travels from the Earth to the Moon in approximately 1.27 seconds. The Earth is about 93 million miles from the Sun and light travels from the Earth to the Sun in 8.32 minutes. Given this information, how can we determine the speed of light? What is the distance from the Sun to Pluto if light can travel the distance in 5 hours and 29 minutes? We can answer these questions using methods from Section P.4. The solution for this problem is given on page 34.

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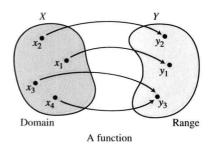


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CHAPTER

2

## Polynomial, Power and Rational Functions

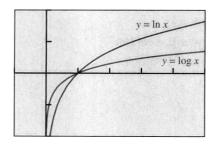
		(1, 1)
		$A_{x}$
	/	$\int \int \int d^2x$
_	///	$x^3$
- /	/////	x <sup>5</sup>
		x <sup>6</sup>
(0, 0)	[0, 1] by [0, 1]	

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CHAPTER

3

## Exponential, Logistic, and Logarithmic Functions



[-1, 5] by [-2, 2]

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