

Formal Specification and Design

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H. B. M. Jonkers

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FORMAL SPECIFICATION AND DESIGN

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Preface

This book is about formal specification and design techniques, including both algebraic specifications and state-based specifications.

The construction and maintenance of complex software systems is a difficult task and although many software projects are started with great expectations and enthusiasm, it is too often the case that they fail to achieve their goals within the planned time and with the given resources. The software often contains errors; attempts to eliminate the errors give rise to new errors, and so on. Moreover, the extension and adaptation of the software to new tasks turns out to be a difficult and tedious task, which seems unsuitable for scientific methods.

This unsatisfactory situation can be improved by introducing precise specifications of the software and its constituent parts. When a piece of software P has a precise specification S say, then ' P satisfies S ' is a clear statement that could be verified by reasoning or that could be falsified by testing; users of P can read S and rely on it and the designer of P has a clearly formulated task. When no precise specifications are available, there are hardly any clear statements at all, for what could one say: 'it works' or more often 'it almost works'? Without precise specifications, it becomes very difficult to analyse the consequences of modifying P into P' , for example, and to make any clear statements about that modification. Therefore it is worthwhile during the software development process to invest in constructing precise specifications of well-chosen parts of the software system under construction. Writing precise specifications turns out to be a considerable task itself. In many situations the use of natural language, pictures and pseudo-code does not yield specifications of the required level of abstractness and precision. *Formal specification* is an approach to writing precise specifications, building on concepts from mathematical logic. During the past decades, much research and development concerning formal specification techniques has been conducted. Well-known results in this field are the techniques of 'abstract data types' and of 'pre- and postconditions'.

What is the role of 'language' in connection with formal specifications? One can say that, in many respects, the practical progress in software engineering is language-driven: it is hard to introduce methodological concepts unless these

are concretely available as constructs in the language in use. This is a major motivation behind the introduction of formal specification languages. In practice it is not enough to have good methodological concepts for writing formal specifications: one needs a language as a vehicle. Throughout this book, the language COLD-K is employed as a vehicle. COLD is an acronym for *Common Object-oriented Language for Design*. This book explains the constructs offered by the language and shows how to use them. The use of formal specification techniques at certain well-chosen points in the design process is one of the key factors – though certainly not the only one – in increasing the quality of the software development process.

A formal specification language is a language whose constructs are derived both from mathematical logic and from programming languages and which has a precise syntax and semantics. If, furthermore the language allows for descriptions at several levels of abstraction, it is called a *wide-spectrum* specification language. The language employed in this book is such a wide-spectrum specification language; other wide-spectrum specification languages are VDM, CIP and RSL. One could also call it a *design language* to emphasise that the language can be used for recording a software system in its intermediate stages of design, ranging from *specification* to *implementation*.

Many techniques such as ‘abstract data types’, ‘abstraction functions’, ‘invariants’, ‘pre- and postconditions’, ‘modular specification’ and ‘information hiding’ can be explained using the constructs offered by COLD-K [1]. The language is in the tradition of VDM [2, 3] and Z [4, 5], but has been influenced by ASL [6], Module Algebra [7], Harel’s dynamic logic [8], Scott’s E-logic [9, 10] and object-oriented languages. Furthermore it contains a novel notion of ‘design’ comparable with the structuring mechanisms provided by e.g. HOOD [11].

The language was developed at the Philips Research Laboratories in Eindhoven within the framework of ESPRIT project 432 (also known as METEOR). It has been designed mainly by H.B.M. Jonkers, with technical contributions from C.P.J. Koymans, G.R. Renardel de Lavalette and L.M.G. Feijs. The fact that its well-formedness and semantics are defined mathematically guarantees that descriptions in the language leave no room for ambiguity and that a high level of tool support can be provided. Actually, COLD-K is one out of a sequence of language versions, in which it plays a special role: it is a *kernel language*, serving as a point of departure in the further development of the language. It is meant to be used as the kernel of user- and application-oriented language versions, to be derived by syntactic extensions. All essential semantic features are contained in this kernel language, as well as high level constructs for modularisation, parameterisation and designs. It is important to realise that this language is the forerunner of versions which are much more user-friendly – at least from a syntactic point of view. Indeed, certain aspects

of the language are somewhat Spartan, but for the purpose of this book this is hardly a disadvantage.

One of the main goals of this book is to treat the basic concepts underlying algebraic specification techniques. The book shows how algebraic specification techniques can be effectively used in the software development process. Yet, the approach of this book goes far beyond algebraic specifications: it shows how algebraic and state-based techniques can be combined in an integrated approach. The main motivation for using COLD-K is as follows. It is a *formal* language, with a well-defined syntax and semantics; it can be used as an *algebraic* specification language; furthermore, it is an *integrated* language, unifying algebraic and state-based techniques.

The book is divided into three parts. The first part is concerned with algebraic specifications, the second part with state-based specifications. The first and the second part have essentially the same structure, beginning with an introduction of the basic concepts followed by methodological guidelines about setting up a specification. After that the topics of *large* specifications as well as *implementation* strategies are treated – each in one chapter. In the third part we have three chapters, each of a different nature. In Chapter 9 a number of existence proofs and theoretical discussions are presented. These are related to the earlier chapters, but are not presented there in order not to distract too much from the main line of these chapters. In Chapter 10 a number of additional language constructs are presented informally. In the last chapter (Chapter 11) a pictorial representation of module structures is explained as well as a systematic approach for putting specifications and implementations together in a top-level language construct called *design*. The structure of the book is as follows.

- I 1. Introducing the basic concepts,
 2. Setting up algebraic specifications,
 3. Structuring algebraic specifications,
 4. Implementing algebraic specifications.
- II 5. From algebras to states,
 6. Setting up state-based specifications,
 7. Structuring state-based specifications,
 8. Implementing state-based specifications.
- III 9. Theoretical topics,
 10. Additional language constructs,
 11. Towards large systems.

Since the goal of the book is not to serve as a language reference manual, we decided not to present *all* language features. Instead of that we restricted ourselves to the most essential aspects of the language. These are covered

in depth by the Chapters 1–9, and additional language features as well as constructs for building systems from components are dealt with briefly and informally in Chapters 10 and 11. The syntax of the full language COLD-K is contained in Appendix A.

This book developed from the course material for a post-graduate course given by the authors at the Technical University of Eindhoven and at the University of Nijmegen. Special thanks go to J.A. Bergstra, R.J. Bril and C.A. Middelburg for their contributions, reviewing and discussions supporting the creation of this text.

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