

4th Edition

PREALGEBRA and ALGEBRA



Daniel D. Benice

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Daniel D. Benice

Montgomery College



PRENTICE HALL, Englewood Cliffs, New Jersey 07632

Editorial/production supervision: Joan L. Stone
Interior design: Jayne Conte
Cover design: Jayne Conte
Manufacturing buyer: Paula Massenaro
Cover photo: "King County Correctional Facility Plaza," Martha Schwartz, artist; office of Peter Walker and Martha Schwartz

This is the fourth edition of the book previously published under the title "Arithmetic and Algebra."



© 1989, 1985, 1979, 1973 by Prentice-Hall, Inc.
A Division of Simon & Schuster
Englewood Cliffs, New Jersey 07632

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Printed in the United States of America

10 9 8 7 6 5 4 3

ISBN 0-13-706327-X

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

PREFACE

This book is intended to be used for any basic mathematics course in which the students are expected to learn elementary algebra. A presentation of essential arithmetic is provided for review or reference. In any case, students are led gently into a study of standard elementary algebra topics. The text is suitable for three types of courses:

1. Prealgebra and algebra
2. Elementary algebra
3. Arithmetic and algebra

FEATURES OF THE TEXT

The book retains many features that have proven successful in previous editions:

1. Theory and discussions have been kept brief and are followed by numerous examples.
2. Because I have found that an extra step or example can make the difference between understanding and confusion, students are led step by step through basic concepts.
3. Many exercises are included to ensure mastery of the material. Chapter review exercises are also included.
4. Answers to more than half of the exercises are provided.
5. Important ideas are boxed for easy reference.
6. Step-by-step procedures are given and boxed where appropriate.

NEW IN THIS EDITION

Several changes have been made to improve the text:

1. The use of *two colors* makes drawings clearer, reference easier, and the text more attractive and easier to use.
2. *Cumulative* review exercises appear in Chapters 11 through 22. They are intended to help students reinforce and remember concepts studied earlier in the course.
3. *Many more word problems* appear throughout the text. *The emphasis is on realism.* In particular, Chapter 13 (“Word Problems”) has been rewritten to include more examples and many more exercises.
4. The *Example/Solution format* makes the examples easier to locate and use.
5. Exercise sets have been examined, refined, and expanded to better suit the needs of both students and instructor.
6. Many clarifying steps, statements, and examples have been added.
7. The appendix topic “ratio and proportion” has been merged into the main text for easier use by those who choose to include it.
8. Section 10.5 on *notation* has been simplified.
9. The presentation of *slope* has been moved forward to Chapter 14. However, the topic of slope (Sections 14.3 and 14.4) can be omitted or taught later without any loss of continuity.
10. Chapters 19 and 20 have been interchanged. As a result, Section 18.7 (“Equations with Fractions”) leads nicely into 19.2, which contains many equations with fractions. Also, fractional exponents (20.5) now leads gently into Chapter 21, “Radicals and Roots.”
11. Chapter 19 on *formula manipulation* now consists of two sections. As a result, the first material (19.1) can be taught earlier, say, as an extension or application of Chapter 12.

TITLE CHANGE

The text *Arithmetic and Algebra* was published originally in 1973 as the first book to offer chapters of arithmetic review (for use in class or as a reference) followed by a complete presentation of elementary algebra. More recently the marketplace has been filled with huge books offering *developmental* approaches to arithmetic via thousands of exercises. Those books assume the students know virtually nothing about basic arithmetic and must learn everything from the beginning. By contrast, I assume the student has forgotten some things and perhaps never really understood certain concepts, especially fractions and percent.

The approach is brief, to-the-point, and stresses understanding. My intent in these early chapters is to present the essence of arithmetic, to prepare students sufficiently so that they have a reasonable chance for success in algebra. This is why I now choose to call the approach *prealgebra* and the text *Prealgebra and Algebra*.

SUPPLEMENTS

Special Instructor's Edition

The publisher has created a special edition of the text for use by instructors. It includes answers to *all* exercises, course outlines, and teaching suggestions.

Test Item File

There is a Test Item File consisting of many questions (and answers) for each chapter. The file can be used to create review questions, quizzes, or chapter tests.

Computerized Testing System

A Computerized Testing System is available. It generates individual tests from a bank of over 700 items.

ACKNOWLEDGMENTS

I am grateful for the assistance of the following reviewers of *Prealgebra and Algebra* who have helped me to improve the text.

Previous Editions

Margaret G. Aldrich, Montgomery College; Robert Alwin, St. Petersburg Junior College; Dorys J. Barban, Montgomery College; Marty Campbell, College of the Virgin Islands at St. Croix; James W. Gorham, Jr., Harris-Stowe College; Anita Kitchens, Appalachian State University; Janice McFatter, Gulf Coast Community College; Gerhart Moore, Southeastern Louisiana University; Carla B. Oviatt, Montgomery College; Dudley R. Pitt, Northwestern State College of Louisiana; Patricia D. Roecklein, Montgomery College; Patricia H. Rubenstein, Montgomery College; Ronald E. Ruemmler, Middlesex County College; George Schultz, St. Petersburg Junior College; Robin Symonds, Indiana University at Kokomo.

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Karen J. Schroeder, *Bentley College*
Arlene Sherburne, *Montgomery College*
Robin G. Symonds, *Indiana University at Kokomo*
Herbert C. Voltz, *Slippery Rock University*
Richard Walker, *Mansfield University*
James P. Winner, *College of William & Mary*

DEDICATION

This book is dedicated to my wife Sylvia and her mother Sylvania Prince.

Daniel D. Benice
Rockville, Maryland

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REVIEW OF BASIC OPERATIONS

To succeed in your study of algebra, you must have a solid foundation in arithmetic. Why? Because most of the properties and manipulations used in algebra are the same as those used in arithmetic. Algebra is a generalization and extension of arithmetic. You will use a great deal of arithmetic when you study algebra. So if your arithmetic background is weak, you will not be able to master algebra.

The assumption is made that arithmetic is not totally foreign to you. In other words, we assume that you have seen fractions and decimals before, and that you have added, subtracted, multiplied, and divided whole numbers before. On the other hand, the assumption is made that many things have been forgotten, confused, or never really learned.

1.1 NUMBER STRUCTURE

We begin our study of arithmetic with a look at the structure of numbers and the basic operations used to combine numbers. Much of our attention will be on the **whole numbers**—that is, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, and so on. At the very beginning we will be using the numbers called **digits**: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

When you read the number 572, you don't think five-seven-two but rather five hundred seventy-two. In other words, each digit (5, 7, and 2) has an additional meaning that corresponds to where it appears in the number, that is, its *place value*.

$$572 = 500 + 70 + 2$$

$$752 = 700 + 50 + 2$$

$$275 = 200 + 70 + 5$$

A digit placed in the rightmost column of a number contributes that many *ones* to the value of the number. In 572 there are 2 ones.

A digit placed in the next column to the left contributes that many *tens* to the number. In 572 there are 7 tens.

A digit placed in the next column over contributes that many *hundreds* to the number. In 572 there are 5 hundreds.

Thus

$$572 = (5 \times 100) + (7 \times 10) + (2 \times 1)$$

$$894 = (8 \times 100) + (9 \times 10) + (4 \times 1)$$

Similarly,

$$5360 = (5 \times 1000) + (3 \times 100) + (6 \times 10) + (0 \times 1)$$

$$47,815 = (4 \times 10,000) + (7 \times 1000) + (8 \times 100) + (1 \times 10) + (5 \times 1)$$

Here is a chart showing place values.

billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
3 ,	2	1	5 ,	7	8	4 ,	9	1	6

The *word equivalent* of any whole number can be read from the number. Here are a few examples.

$$894 = \text{eight hundred ninety-four}$$

$$47,815 = \text{forty-seven thousand eight hundred fifteen}$$

$$3,050,907 = \text{three million fifty thousand nine hundred seven}$$

Notice the use of commas to separate at every three digits, counting from the right.

EXERCISES 1.1

Answers to starred exercises are given in the back of the book.

- Write each number as the sum of ones, tens, hundreds, and so on, as in the example below.

Example: $1563 = (1 \times 1000) + (5 \times 100) + (6 \times 10) + (3 \times 1)$

*(a) 143

(b) 159

*(c) 50

*(d) 1603

*(e) 777

(f) 490

*(g) 600

*(h) 19,734

*(i) 34,851

(j) 65,127

*(k) 648,713

(l) 952,006

*(m) 4,980,617

(n) 1,187,543

2. Write the word equivalent of each whole number.

- *(a) 754 (b) 879 *(c) 1341
 (d) 8694 *(e) 54,617 (f) 71,955
 *(g) 563,432 (h) 782,996 *(i) 1,897,400
 (j) 7,734,015 *(k) 78,800,167 (l) 44,608,500
 *(m) 654,456,546 (n) 780,078,009 *(o) 8,098,764,123
 (p) 9,017,658,432

*3. Which of the following numbers are digits?

- (a) 9 (b) 0 (c) 15 (d) 3 (e) 2
 (f) 10 (g) 11 (h) 167 (i) 23 (j) 32

*4. Which of the following numbers are whole numbers?

- (a) 6 (b) 16 (c) 160 (d) $4\frac{1}{2}$
 (e) $\frac{1}{4}$ (f) 6.4 (g) .78 (h) 10,000,000

1.2 ADDITION

Hopefully you will not need the basic addition table presented next. But if you don't already know it, memorize it. The entry 8 is circled to demonstrate how to read the table: $3 + 5 = 8$.

						↓				
+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
→ 3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

You should be able to add 59 and 25 and get 84 as the sum.

$$\begin{array}{r}
 59 \\
 25 \\
 \hline
 84
 \end{array}$$

Your reasoning may be as follows.

1. $9 + 5 = 14$.
2. Put down 4 and carry 1.
3. $5 + 2 = 7$ plus 1 (carried) gives 8.

But do you *understand* what you are doing? Does the procedure “put down 4 and carry 1” make sense to you or is it something that you memorized in grade school without any real understanding? Let’s look at the problem differently. The number 59 has 5 tens and 9 ones; 25 contains 2 tens and 5 ones. Thus

$$\begin{array}{r} 59 \\ 25 \\ \hline \end{array} \longrightarrow \begin{array}{r} \text{tens} \quad \text{ones} \\ 5 \quad 9 \\ 2 \quad 5 \\ \hline \end{array}$$

Add the ones to get

$$\begin{array}{r} \text{tens} \quad \text{ones} \\ 5 \quad 9 \\ 2 \quad 5 \\ \hline 14 \end{array}$$

Since 14 ones are the same as 1 ten and 4 ones, we can rewrite the problem as

$$\begin{array}{r} \text{tens} \quad \text{ones} \\ \mathbf{1} \quad 9 \\ 5 \quad 5 \\ 2 \quad 5 \\ \hline 4 \end{array}$$

where the boldface 1 indicates the “1 ten” part of the 14 ones. This is the “carry.” The 4 is the “4 ones” part of the 14 ones. The final sum is obtained by adding the 5, 2, and carried 1. Thus

$$\begin{array}{r} \mathbf{1} \\ 59 \\ 25 \\ \hline 84 \end{array}$$

The process is similar for addition of numbers having more than two digits; for example,

$$\begin{array}{r} 395 \\ 280 \\ 155 \\ \hline \end{array} \longrightarrow \begin{array}{r} \mathbf{1} \\ 395 \\ 280 \\ 155 \\ \hline 0 \end{array} \longrightarrow \begin{array}{r} \mathbf{2} \mathbf{1} \\ 395 \\ 280 \\ 155 \\ \hline 30 \end{array} \longrightarrow \begin{array}{r} \mathbf{2} \mathbf{1} \\ 395 \\ 280 \\ 155 \\ \hline 830 \quad \checkmark \end{array}$$

Addition may involve decimal numbers as well as whole numbers. *We will postpone until Chapter 6 any attempt to explain the meaning of decimal numbers. But here we will present the mechanics of adding decimal numbers, since we know that you have used decimals before.*

When the numbers being added have decimals, arrange the numbers so that all the decimal points are aligned. (This will align all the place values, so you will be adding tenths to tenths, and so on.) Then add as in previous examples. To add 342.6, 17.93, and 4.65, arrange the numbers and add as

$$\begin{array}{r} 342.6 \\ 17.93 \\ \underline{4.65} \\ 365.18 \quad \checkmark \end{array}$$

A whole number, such as 173, can be written as the decimal number 173. or 173.0. Thus to add the numbers 154.7, 173, and 23.02, set up the problem as

$$\begin{array}{r} 154.7 \\ 173.0 \\ \underline{23.02} \end{array}$$

and then add. You may prefer to supply enough zeros after the last decimal digit to make the numbers themselves align on the right side, as

$$\begin{array}{r} 154.70 \\ 173.00 \\ \underline{23.02} \end{array}$$

EXERCISES 1.2

*1. Perform each basic addition.

(a) $\begin{array}{r} 5 \\ 4 \\ \hline \end{array}$	(b) $\begin{array}{r} 0 \\ 6 \\ \hline \end{array}$	(c) $\begin{array}{r} 6 \\ 2 \\ \hline \end{array}$	(d) $\begin{array}{r} 7 \\ 6 \\ \hline \end{array}$	(e) $\begin{array}{r} 8 \\ 0 \\ \hline \end{array}$	(f) $\begin{array}{r} 5 \\ 9 \\ \hline \end{array}$
(g) $\begin{array}{r} 9 \\ 9 \\ \hline \end{array}$	(h) $\begin{array}{r} 7 \\ 8 \\ \hline \end{array}$	(i) $\begin{array}{r} 1 \\ 8 \\ \hline \end{array}$	(j) $\begin{array}{r} 9 \\ 1 \\ \hline \end{array}$	(k) $\begin{array}{r} 8 \\ 3 \\ \hline \end{array}$	(l) $\begin{array}{r} 6 \\ 7 \\ \hline \end{array}$
(m) $\begin{array}{r} 6 \\ 4 \\ \hline \end{array}$	(n) $\begin{array}{r} 8 \\ 8 \\ \hline \end{array}$	(o) $\begin{array}{r} 3 \\ 7 \\ \hline \end{array}$	(p) $\begin{array}{r} 5 \\ 8 \\ \hline \end{array}$	(q) $\begin{array}{r} 6 \\ 6 \\ \hline \end{array}$	(r) $\begin{array}{r} 7 \\ 7 \\ \hline \end{array}$

2. Perform each addition.

*(a) $\begin{array}{r} 57 \\ 12 \\ \hline \end{array}$	*(b) $\begin{array}{r} 36 \\ 53 \\ \hline \end{array}$	*(c) $\begin{array}{r} 49 \\ 26 \\ \hline \end{array}$	(d) $\begin{array}{r} 58 \\ 37 \\ \hline \end{array}$
(e) $\begin{array}{r} 67 \\ 79 \\ \hline \end{array}$	*(f) $\begin{array}{r} 99 \\ 54 \\ \hline \end{array}$	(g) $\begin{array}{r} 428 \\ 351 \\ \hline \end{array}$	*(h) $\begin{array}{r} 785 \\ 517 \\ \hline \end{array}$
*(i) $\begin{array}{r} 670 \\ 920 \\ \hline \end{array}$	(j) $\begin{array}{r} 906 \\ 809 \\ \hline \end{array}$	*(k) $\begin{array}{r} 6520 \\ 1349 \\ \hline \end{array}$	(l) $\begin{array}{r} 4503 \\ 2917 \\ \hline \end{array}$
*(m) $\begin{array}{r} 9743 \\ 1059 \\ \hline \end{array}$	*(n) $\begin{array}{r} 94 \\ 37 \\ 15 \\ \hline \end{array}$	*(o) $\begin{array}{r} 691 \\ 473 \\ 120 \\ \hline \end{array}$	(p) $\begin{array}{r} 157 \\ 845 \\ 419 \\ \hline \end{array}$