

**Mathematics for  
Machine  
Technology**  
Third Edition

**Robert D. Smith**

# **Mathematics for Machine Technology**

## **Third Edition**

**Robert D. Smith**

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# **Mathematics for Machine Technology**

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## PREFACE

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*Mathematics for Machine Technology* is written to overcome the often mechanical “plug in” approach found in many trade-related mathematics textbooks. An understanding of mathematical concepts is stressed in all topics ranging from general arithmetic processes to oblique trigonometry, compound angles, and numerical control.

Both content and method are those used by the author in teaching applied machine technology mathematics classes for apprentices in the machine, tool-and-die, and tool design trades. Each unit is developed as a learning experience based on preceding units—making prerequisites unnecessary.

Presentation of basic concepts is accompanied by realistic industry-related examples and actual industrial applications. The applications progress from the simple to those with solutions which are relatively complex. Many problems require the student to work with illustrations such as are found in machine trade handbooks and engineering drawings.

An analytical approach to problem solving is emphasized in the geometry, trigonometry, compound angle, and numerical control sections. This approach is necessary in actual practice in translating engineering drawing dimensions to machine working dimensions. Integration of algebraic and geometric principles with trigonometry by careful sequence and treatment of material also helps the student in solving industrial applications. The Instructor’s Guide provides answers and solutions for all problems.

Changes from the previous edition have been made to improve the presentation of topics and to update material.

The electronic calculator is introduced in Section 1, Unit 15. Further calculator instruction and examples are given throughout the text wherever calculator applications are appropriate to the material presented. Many of the exercises in the text are well suited to calculator solutions. However, student exercises and problems are not specifically identified as calculator applications. It is felt that decisions regarding calculator usage be left to the discretion of the instructor. Decisions on the degree of emphasis to be placed on calculator applications and the selection of specific exercises and problems for solution by calculator remain with the course instructor.

Robert D. Smith has experience in both the manufacturing industry and in education. He held positions as tool designer, quality control engineer, and chief manufacturing engineer prior to teaching. Mr. Smith taught applied mathematics, physics, and industrial materials and processes on the secondary school level and in Machine Trade Apprentice Programs at the A. I. Prince Vocational-Technical School in Hartford, Connecticut. Mr. Smith is presently Associate Professor of Industrial Technology at Central Connecticut State University, New Britain, Connecticut. He has membership in several professional organizations in his field of interest: the American Technical Education Association and the Society of Manufacturing Engineers.

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# SECTION 1 COMMON FRACTIONS AND DECIMAL FRACTIONS

## UNIT 1 INTRODUCTION TO COMMON FRACTIONS AND MIXED NUMBERS

### OBJECTIVES

After studying this unit you should be able to

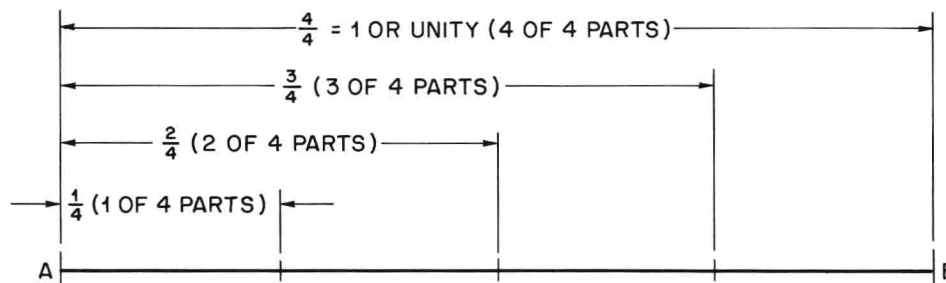
- Express fractions in lowest terms.
- Express fractions as equivalent fractions.
- Express mixed numbers as improper fractions.
- Express improper fractions as mixed numbers.

Most measurements and calculations made by a machinist are not limited to whole numbers. Blueprint dimensions are often given as fractions and certain measuring tools are graduated in fractional units. The machinist must be able to make calculations using fractions and to measure fractional values.

### FRACTIONAL PARTS

A *fraction* is a value which shows the number of equal parts taken of a whole quantity or unit. The symbols used to indicate a fraction are the bar ( — ) and the slash ( / ).

Line segment AB as shown is divided into 4 equal parts.



$$1 \text{ part} = \frac{1 \text{ part}}{\text{total parts}} = \frac{1 \text{ part}}{4 \text{ parts}} = \frac{1}{4} \text{ of the length of the line segment.}$$

$$2 \text{ parts} = \frac{2 \text{ parts}}{\text{total parts}} = \frac{2 \text{ parts}}{4 \text{ parts}} = \frac{2}{4} \text{ of the length of the line segment.}$$

$$3 \text{ parts} = \frac{3 \text{ parts}}{\text{total parts}} = \frac{3 \text{ parts}}{4 \text{ parts}} = \frac{3}{4} \text{ of the length of the line segment.}$$

$$4 \text{ parts} = \frac{4 \text{ parts}}{\text{total parts}} = \frac{4 \text{ parts}}{4 \text{ parts}} = \frac{4}{4} = 1, \text{ or unity (4 parts make up the whole).}$$



## 2 Section 1 Common Fractions and Decimal Fractions

Each of the 4 equal parts of the line segment AB is divided into 8 equal parts.  
There is a total of  $4 \times 8$  or 32 parts.

1 part =  $\frac{1}{32}$  of the total length.

7 parts =  $\frac{7}{32}$  of the total length.

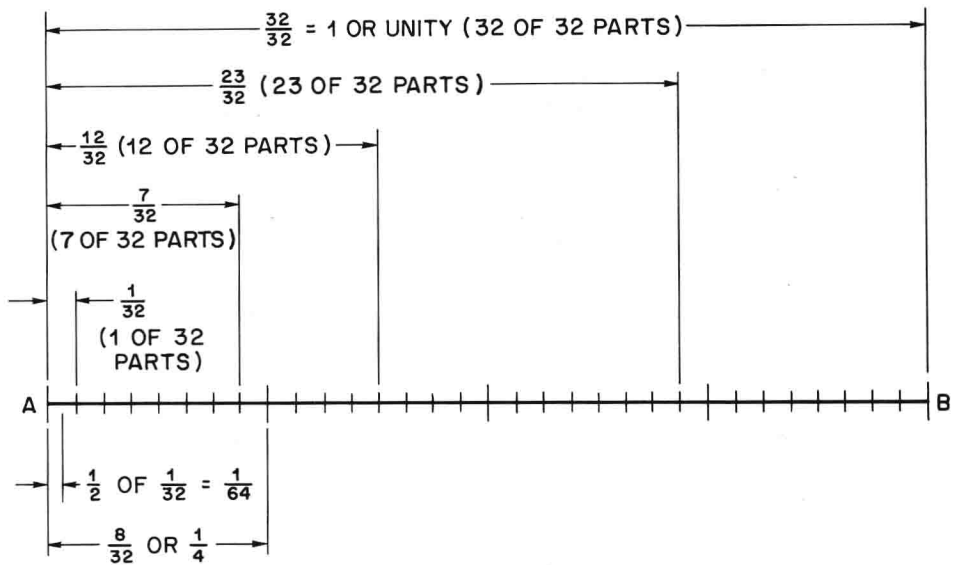
12 parts =  $\frac{12}{32}$  of the total length.

23 parts =  $\frac{23}{32}$  of the total length.

32 parts =  $\frac{32}{32}$  or 1, or unity.

$\frac{1}{2}$  of 1 part =  $\frac{1}{2} \times \frac{1}{32} = \frac{1}{64}$  of the total length.

Note: 8 parts =  $\frac{8}{32}$  of the total length and also  $\frac{1}{4}$  of the total length. Therefore,  $\frac{8}{32} = \frac{1}{4}$ .



### DEFINITIONS OF FRACTIONS

A *fraction* is a value which shows the number of equal parts taken of a whole quantity or unit.

The *denominator* of a fraction is the number that shows how many equal parts are in the whole quantity. The denominator is written below the bar.

The *numerator* of a fraction is the number that shows how many equal parts of the whole are taken. The numerator is written above the bar.

The numerator and denominator are called the *terms* of the fraction.

$\frac{3}{4}$  ← numerator  
 $\frac{3}{4}$  ← denominator

An *improper* fraction is a fraction in which the numerator is larger than or equal to the denominator, as  $\frac{3}{2}$ ,  $\frac{5}{4}$ ,  $\frac{15}{8}$ ,  $\frac{6}{6}$ ,  $\frac{17}{17}$ .

A *mixed number* is a number composed of a whole number and a fraction, as  $3\frac{7}{8}$ ,  $7\frac{1}{2}$ .

Note:  $3\frac{7}{8}$  means  $3 + \frac{7}{8}$ . It is read as three and seven-eighths.  $7\frac{1}{2}$  means  $7 + \frac{1}{2}$ . It is read as seven and one-half.

A *complex fraction* is a fraction in which one or both of the terms are fractions or mixed numbers, as  $\frac{\frac{3}{4}}{6}$ ,  $\frac{32}{\frac{15}{4}}$ ,  $\frac{8\frac{3}{4}}{3}$ ,  $\frac{\frac{7}{16}}{2\frac{2}{5}}$ ,  $4\frac{1}{4}$ .

### EXPRESSING FRACTIONS AS EQUIVALENT FRACTIONS

The numerator and denominator of a fraction can be multiplied or divided by the same number without changing the value. For example,  $\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$ . Both the numerator and denominator are multiplied by 4. Because  $\frac{1}{2}$  and  $\frac{4}{8}$  have the same value, they are *equivalent*. Also,  $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$ . Both numerator and denominator are divided by 4. Since  $\frac{8}{12}$  and  $\frac{2}{3}$  have the same value, they are *equivalent*.

A fraction is in its *lowest terms* when the numerator and denominator do not contain a common factor, as  $\frac{5}{9}$ ,  $\frac{7}{8}$ ,  $\frac{3}{4}$ ,  $\frac{11}{12}$ ,  $\frac{15}{32}$ ,  $\frac{9}{11}$ . *Factors* are the numbers used in multiplying. For example, 2 and 5 are each factors of 10;  $2 \times 5 = 10$ . Expressing a fraction in lowest terms is often called *reducing* a fraction to lowest terms.

*Procedure:* To reduce a fraction to lowest terms

- Divide both numerator and denominator by the greatest common factor.

*Example:* Reduce  $\frac{12}{42}$  to lowest terms.

Both terms can be divided by 2.

$$\frac{12 \div 2}{42 \div 2} = \frac{6}{21}$$

Note: The fraction is reduced, but not to lowest terms.

Further reduce  $\frac{6}{21}$ .

Both terms can be divided by 3.

$$\frac{6 \div 3}{21 \div 3} = \frac{2}{7} \text{ Ans}$$

Note: The value  $\frac{2}{7}$  may be obtained in one step if each term of  $\frac{12}{42}$  is divided by  $2 \times 3$  or 6.

$$\frac{12 \div 6}{42 \div 6} = \frac{2}{7} \text{ Ans}$$

*Procedure:* To express a fraction as an equivalent fraction with an indicated denominator which is larger than the denominator of the fraction

- Divide the indicated denominator by the denominator of the fraction.
- Multiply both the numerator and denominator of the fraction by the value obtained.

*Example:* Express  $\frac{3}{4}$  as an equivalent fraction with 12 as the denominator.

Divide 12 by 4.

$$12 \div 4 = 3$$

Multiply both 3 and 4 by 3.

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12} \text{ Ans}$$

## EXPRESSING MIXED NUMBERS AS IMPROPER FRACTIONS

*Procedure:* To express a mixed number as an improper fraction

- Multiply the whole number by the denominator.
- Add the numerator to obtain the numerator of the improper fraction.
- The denominator is the same as that of the original fraction.

*Example 1:* Express  $4\frac{1}{2}$  as an improper fraction.

Multiply the whole number by the denominator.

$$4 \times \frac{2 + 1}{2} = \frac{9}{2} \text{ Ans}$$

Add the numerator to obtain numerator for the improper fraction.

The denominator is the same as that of the original fraction.

*Example 2:* Express  $12\frac{3}{16}$  as an improper fraction.

$$\frac{12 \times 16 + 3}{16} = \frac{195}{16} \text{ Ans}$$

## EXPRESSING IMPROPER FRACTIONS AS MIXED NUMBERS

*Procedure:* To express an improper fraction as a mixed number

- Divide the numerator by the denominator.

*Examples:* Express the following improper fractions as mixed numbers.

$$\frac{11}{4} = 11 \div 4 = 2\frac{3}{4} \text{ Ans}$$

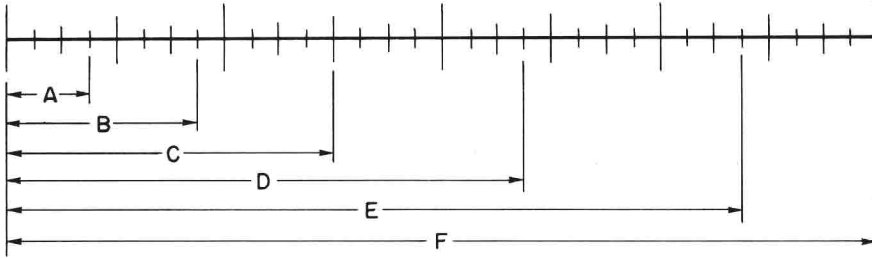
$$\frac{43}{3} = 43 \div 3 = 14\frac{1}{3} \text{ Ans}$$

$$\frac{931}{8} = 931 \div 8 = 116\frac{3}{8} \text{ Ans}$$

**APPLICATION**

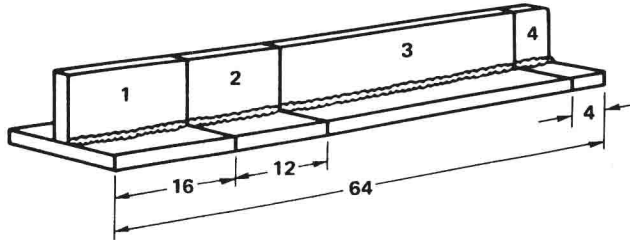
**Fractional Parts**

1. Write the fractional part which each length, A through F, represents of the total shown on the scale.



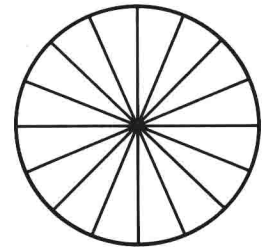
- A = \_\_\_\_\_  
 B = \_\_\_\_\_  
 C = \_\_\_\_\_  
 D = \_\_\_\_\_  
 E = \_\_\_\_\_  
 F = \_\_\_\_\_

2. A welded support base is cut in four pieces. What fractional part of the total length does each of the four pieces represent? All dimensions are in inches.



3. The circle is divided into equal parts. Write the fractional part each of the following represents.

- |                  |                                  |                                   |
|------------------|----------------------------------|-----------------------------------|
| a. 1 part _____  | e. 16 parts _____                | h. $\frac{3}{4}$ of 1 part _____  |
| b. 3 parts _____ | f. $\frac{1}{2}$ of 1 part _____ | i. $\frac{1}{10}$ of 1 part _____ |
| c. 7 parts _____ | g. $\frac{1}{3}$ of 1 part _____ | j. $\frac{1}{16}$ of 1 part _____ |
| d. 5 parts _____ |                                  |                                   |



**Expressing Fractions as Equivalent Fractions**

4. Reduce to halves.

- |                         |                            |                          |                           |
|-------------------------|----------------------------|--------------------------|---------------------------|
| a. $\frac{4}{8}$ _____  | c. $\frac{100}{200}$ _____ | e. $\frac{25}{10}$ _____ | g. $\frac{126}{36}$ _____ |
| b. $\frac{9}{18}$ _____ | d. $\frac{121}{242}$ _____ | f. $\frac{18}{12}$ _____ | h. $\frac{225}{50}$ _____ |

5. Reduce to lowest terms.

- |                         |                          |                          |                           |
|-------------------------|--------------------------|--------------------------|---------------------------|
| a. $\frac{6}{8}$ _____  | d. $\frac{30}{5}$ _____  | g. $\frac{24}{8}$ _____  | i. $\frac{25}{150}$ _____ |
| b. $\frac{12}{4}$ _____ | e. $\frac{11}{44}$ _____ | h. $\frac{65}{15}$ _____ | j. $\frac{14}{105}$ _____ |
| c. $\frac{6}{10}$ _____ | f. $\frac{14}{6}$ _____  |                          |                           |

6. Express as thirty-seconds.

- |                        |                         |                          |                           |
|------------------------|-------------------------|--------------------------|---------------------------|
| a. $\frac{1}{4}$ _____ | c. $\frac{11}{8}$ _____ | e. $\frac{21}{16}$ _____ | g. $\frac{197}{16}$ _____ |
| b. $\frac{3}{4}$ _____ | d. $\frac{7}{16}$ _____ | f. $\frac{19}{2}$ _____  | h. $\frac{21}{8}$ _____   |

7. Express as equivalent fractions as indicated.

a. $\frac{3}{4} = \frac{?}{8}$ _____	d. $\frac{17}{14} = \frac{?}{42}$ _____	g. $\frac{7}{16} = \frac{?}{128}$ _____
b. $\frac{7}{12} = \frac{?}{36}$ _____	e. $\frac{20}{9} = \frac{?}{45}$ _____	h. $\frac{13}{8} = \frac{?}{48}$ _____
c. $\frac{6}{15} = \frac{?}{60}$ _____	f. $\frac{14}{3} = \frac{?}{18}$ _____	i. $\frac{21}{16} = \frac{?}{160}$ _____

### Mixed Numbers and Improper Fractions

8. Express the following mixed numbers as improper fractions.

a. $2\frac{2}{3}$ _____	d. $3\frac{3}{8}$ _____	g. $10\frac{1}{3}$ _____	j. $4\frac{63}{64}$ _____
b. $1\frac{7}{8}$ _____	e. $5\frac{9}{32}$ _____	h. $9\frac{4}{5}$ _____	k. $49\frac{3}{8}$ _____
c. $5\frac{2}{5}$ _____	f. $8\frac{3}{7}$ _____	i. $100\frac{1}{2}$ _____	l. $408\frac{13}{16}$ _____

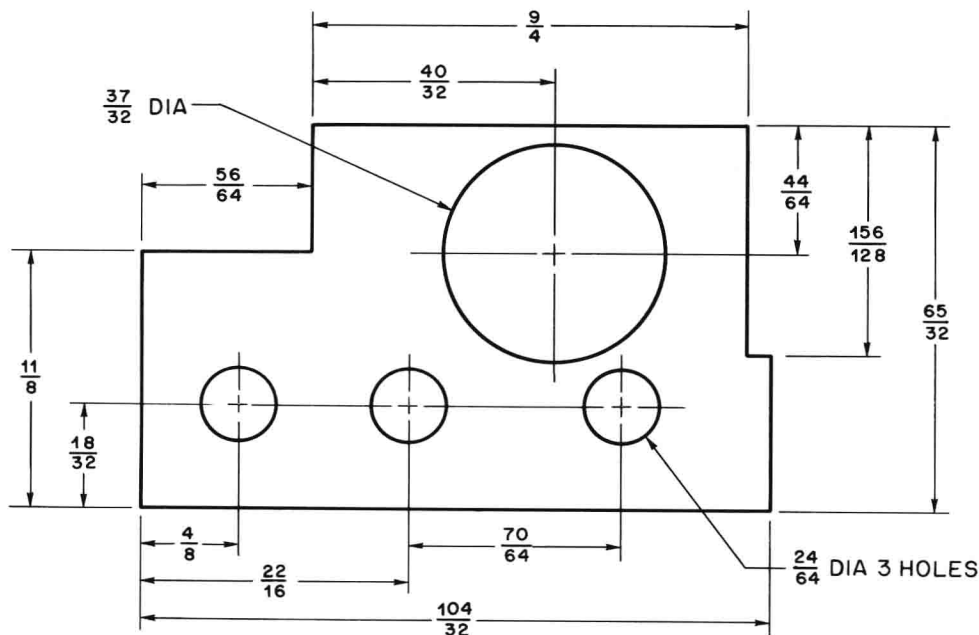
9. Express the following improper fractions as mixed numbers.

a. $\frac{5}{3}$ _____	d. $\frac{87}{4}$ _____	g. $\frac{127}{32}$ _____	j. $\frac{235}{16}$ _____
b. $\frac{21}{2}$ _____	e. $\frac{72}{9}$ _____	h. $\frac{57}{15}$ _____	k. $\frac{514}{4}$ _____
c. $\frac{9}{8}$ _____	f. $\frac{127}{124}$ _____	i. $\frac{150}{9}$ _____	l. $\frac{401}{64}$ _____

10. Express the following mixed numbers as improper fractions. Then express the improper fractions as the equivalent fractions indicated.

a. $2\frac{1}{2} = \frac{?}{8}$ _____	c. $7\frac{4}{5} = \frac{?}{15}$ _____	e. $9\frac{7}{8} = \frac{?}{64}$ _____
b. $3\frac{3}{8} = \frac{?}{16}$ _____	d. $12\frac{2}{3} = \frac{?}{18}$ _____	f. $15\frac{1}{2} = \frac{?}{128}$ _____

11. Sketch and redimension this plate. Reduce all proper fractions to lowest terms. Reduce all improper fractions to lowest terms and express as mixed numbers. All dimensions are in inches.



# UNIT 2 ADDITION OF COMMON FRACTIONS AND MIXED NUMBERS

## OBJECTIVES

After studying this unit you should be able to

- Determine least common denominators.
- Express fractions as equivalent fractions having least common denominators.
- Add fractions and mixed numbers.

A machinist must be able to add fractions and mixed numbers in order to determine the length of stock required for a job, the distances between various parts of a machined piece, and the depth of holes and cutouts in a workpiece.

## LEAST COMMON DENOMINATORS

Fractions cannot be added unless they have a common denominator. *Common denominator* means that the denominators of each of the fractions are the same, as

$$\frac{5}{8}, \frac{7}{8}, \frac{15}{8}.$$

In order to add fractions which do not have common denominators, such as  $\frac{3}{8} + \frac{1}{4} + \frac{7}{16}$ , it is necessary to determine the least common denominator.

The *least common denominator* is the smallest denominator which is evenly divisible by each of the denominators of the fractions being added. Or, stated in another way, the *least common denominator* is the smallest denominator into which each denominator can be divided without leaving a remainder.

*Procedure:* To find the least common denominator

- Determine the smallest number into which all denominators can be divided without leaving a remainder.
- Use this number as a common denominator.

*Example 1:* Find the least common denominator of  $\frac{3}{8}$ ,  $\frac{1}{4}$ , and  $\frac{7}{16}$ .

The smallest number into which 8, 4, and 16 can be divided without leaving a remainder is 16.

Write 16 as the least common denominator.

*Example 2:* Find the least common denominator of  $\frac{3}{4}$ ,  $\frac{1}{3}$ ,  $\frac{7}{8}$ , and  $\frac{5}{12}$ .

The smallest number into which 4, 3, 8, and 12 can be divided is 24.

Write 24 as the least common denominator.

*Note:* In this example, denominators such as 48, 72, and 96 are common denominators because 4, 3, 8, and 12 divide evenly into these numbers, but they are not the least common denominators.

Although any common denominator can be used when adding fractions, it is generally easier and faster to use the least common denominator.

### EXPRESSING FRACTIONS AS EQUIVALENT FRACTIONS WITH THE LEAST COMMON DENOMINATOR

*Procedure:* To change fractions into equivalent fractions having the least common denominator

- Divide the least common denominator by each denominator.
- Multiply both the numerator and denominator of each fraction by the value obtained.

*Example 1:* Express  $\frac{2}{3}$ ,  $\frac{7}{15}$ , and  $\frac{1}{2}$  as equivalent fractions having a least common denominator.

The least common denominator is 30.

$$30 \div 3 = 10; \quad \frac{2 \times 10}{3 \times 10} = \frac{20}{30} \text{ Ans}$$

Divide 30 by each denominator.

$$30 \div 15 = 2; \quad \frac{7 \times 2}{15 \times 2} = \frac{14}{30} \text{ Ans}$$

Multiply each term of the fraction by the value obtained.

$$30 \div 2 = 15; \quad \frac{1 \times 15}{2 \times 15} = \frac{15}{30} \text{ Ans}$$

*Example 2:* Change  $\frac{5}{8}$ ,  $\frac{15}{32}$ ,  $\frac{3}{4}$ , and  $\frac{9}{16}$  to equivalent fractions having a least common denominator.

The least common denominator is 32.

$$32 \div 8 = 4; \quad \frac{5 \times 4}{8 \times 4} = \frac{20}{32} \text{ Ans} \quad 32 \div 4 = 8; \quad \frac{3 \times 8}{4 \times 8} = \frac{24}{32} \text{ Ans}$$

$$32 \div 32 = 1; \quad \frac{15 \times 1}{32 \times 1} = \frac{15}{32} \text{ Ans} \quad 32 \div 16 = 2; \quad \frac{9 \times 2}{16 \times 2} = \frac{18}{32} \text{ Ans}$$

### ADDING FRACTIONS

*Procedure:* To add fractions

- Express the fractions as equivalent fractions having the least common denominator.
- Add the numerators and write their sum over the least common denominator.
- Express an improper fraction as a mixed number when necessary and reduce the fractional part to lowest terms.

*Example 1:* Add  $\frac{1}{2} + \frac{3}{5} + \frac{7}{10} + \frac{5}{6}$ .

Express the fractions as equivalent fractions with 30 as the denominator.

$$\frac{1}{2} = \frac{15}{30}$$

Add the numerators and write their sum over the least common denominator, 30.

$$\frac{3}{5} = \frac{18}{30}$$

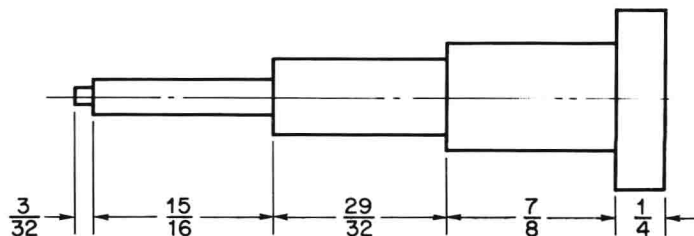
$$\frac{7}{10} = \frac{21}{30}$$

Express the fraction as a mixed number.

$$+ \frac{5}{6} = \frac{25}{30}$$

$$\frac{79}{30} = 2 \frac{19}{30} \text{ Ans}$$

*Example 2:* Determine the total length of the shaft shown. All dimensions are in inches.



## 8 Section 1 Common Fractions and Decimal Fractions

Express the fractions as equivalent fractions with 32 as the denominator.

$$\frac{3}{16} = \frac{6}{32}$$

Add the numerators and write their sum over the least common denominator, 32.

$$\frac{15}{16} = \frac{30}{32}$$

Express  $\frac{98}{32}$  as a mixed number and reduce to lowest terms.

$$\frac{29}{32} = \frac{29}{32}$$

Total Length =  $3\frac{1}{16}$  Ans

$$\frac{7}{8} = \frac{28}{32}$$

$$+ \frac{1}{4} = \frac{8}{32}$$

$$\frac{98}{32} = 3\frac{2}{32} = 3\frac{1}{16}$$

### —ADDING FRACTIONS, MIXED NUMBERS, AND WHOLE NUMBERS

*Procedure:* To add fractions, mixed numbers, and whole numbers

- Add the whole numbers.
- Add the fractions.
- Combine whole number and fraction.

*Example 1:* Add  $\frac{1}{3} + 7 + 3\frac{1}{2} + \frac{5}{12} + 2\frac{19}{24}$ .

Express the fractional parts as equivalent fractions with 24 as the denominator.

$$\frac{1}{3} = \frac{8}{24}$$

Add the whole numbers.

$$7 = 7$$

Add the fractions.

$$3\frac{1}{2} = 3\frac{12}{24}$$

Combine the whole number and the fraction. Express the answer in lowest terms.

$$\frac{5}{12} = \frac{10}{24}$$

$$+ 2\frac{19}{24} = 2\frac{19}{24}$$

$$12\frac{49}{24} = 14\frac{1}{24} \text{ Ans}$$

*Example 2:* Find the distance between the two  $\frac{1}{2}$ -inch diameter holes in the plate shown. All dimensions are in inches.

$$1 = 1$$

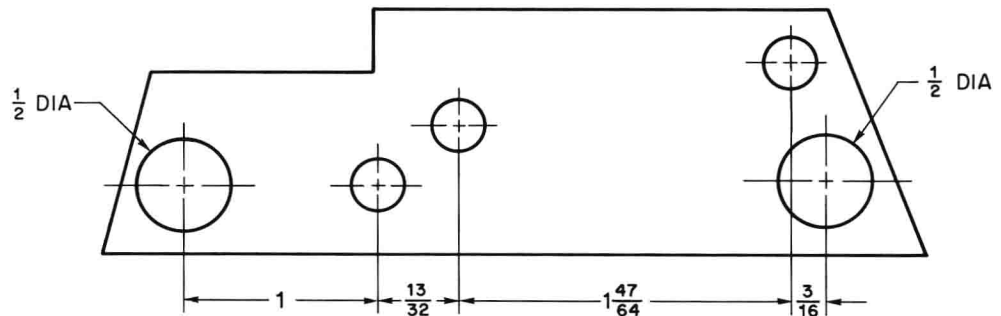
$$\frac{13}{32} = \frac{26}{64}$$

$$1\frac{47}{64} = 1\frac{47}{64}$$

$$+ \frac{3}{16} = \frac{12}{64}$$

$$2\frac{85}{64} = 3\frac{21}{64}$$

$$\text{Distance} = 3\frac{21}{64} \text{ Ans}$$



**APPLICATION**

**Least Common Denominators**

Determine the least common denominators of the following sets of fractions.

- |   |   |
|---|---|
| 1. $\frac{2}{3}, \frac{1}{6}, \frac{5}{12}$ _____ | 3. $\frac{5}{6}, \frac{7}{12}, \frac{3}{16}, \frac{19}{24}$ _____ |
| 2. $\frac{3}{5}, \frac{9}{10}, \frac{5}{6}$ _____ | 4. $\frac{4}{5}, \frac{3}{4}, \frac{7}{10}, \frac{1}{2}$ _____    |

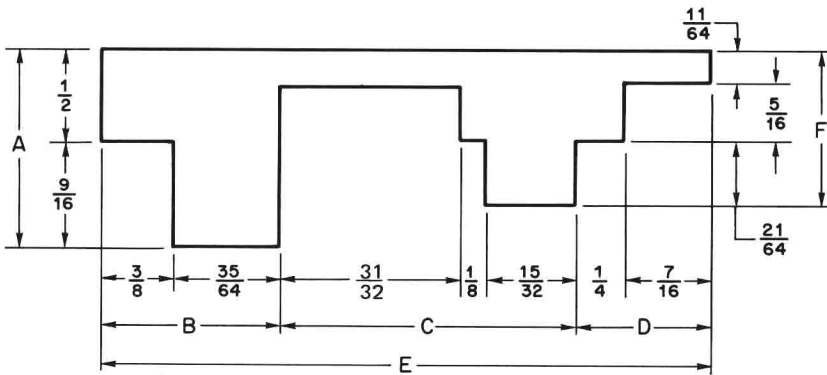
**Equivalent Fractions with Least Common Denominators**

Express these fractions as equivalent fractions having the least common denominator.

- |   |   |
|---|---|
| 5. $\frac{1}{2}, \frac{3}{4}, \frac{5}{12}$ _____ | 7. $\frac{9}{10}, \frac{1}{4}, \frac{3}{5}, \frac{1}{5}$ _____    |
| 6. $\frac{7}{16}, \frac{3}{8}, \frac{1}{2}$ _____ | 8. $\frac{3}{16}, \frac{7}{32}, \frac{17}{64}, \frac{3}{4}$ _____ |

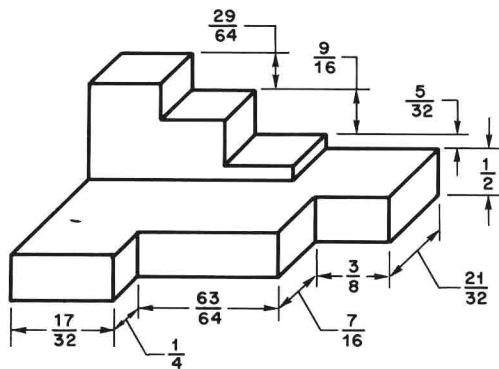
**Adding Fractions**

9. Determine the dimensions A, B, C, D, E, and F of this profile gage. All dimensions are in inches.



- A = \_\_\_\_\_  
 B = \_\_\_\_\_  
 C = \_\_\_\_\_  
 D = \_\_\_\_\_  
 E = \_\_\_\_\_  
 F = \_\_\_\_\_

10. Determine the length, width, and height of this casting. All dimensions are in inches.



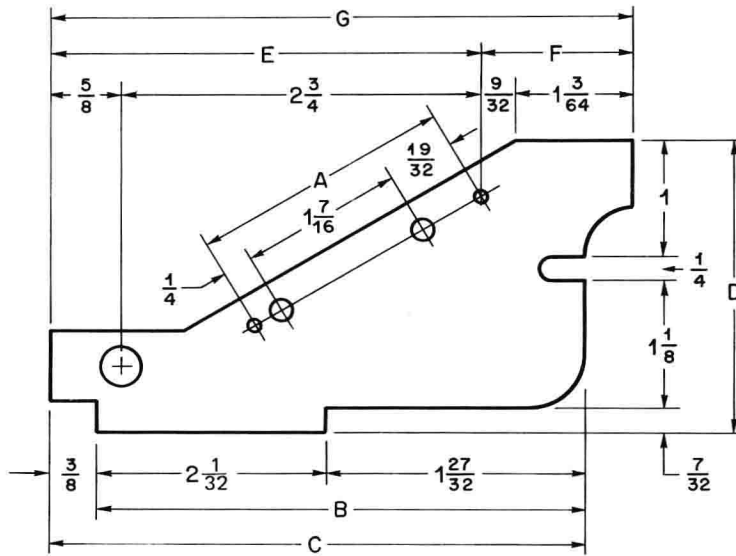
- length = \_\_\_\_\_  
 width = \_\_\_\_\_  
 height = \_\_\_\_\_



## 10 Section 1 Common Fractions and Decimal Fractions

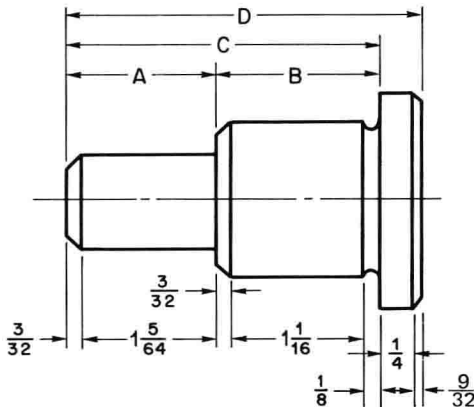
### Adding Fractions, Mixed Numbers, and Whole Numbers

11. Determine dimensions A, B, C, D, E, F, and G of this plate. Reduce to lowest terms where necessary. All dimensions are in inches.



A = \_\_\_\_\_  
 B = \_\_\_\_\_  
 C = \_\_\_\_\_  
 D = \_\_\_\_\_  
 E = \_\_\_\_\_  
 F = \_\_\_\_\_  
 G = \_\_\_\_\_

12. Determine dimensions A, B, C, and D of this pin. All dimensions are in inches.



A = \_\_\_\_\_  
 B = \_\_\_\_\_  
 C = \_\_\_\_\_  
 D = \_\_\_\_\_

13. The operation sheet for machining an aluminum housing specifies 1 hour for facing,  $2\frac{3}{4}$  hours for milling,  $\frac{5}{6}$  hour for drilling,  $\frac{3}{10}$  hour for tapping, and  $\frac{2}{5}$  hour for setting up. What is the total time allotted for this job?

\_\_\_\_\_