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Pressure Vessels, Piping, and Components — Design and Analysis

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FOREWORD

Pressure vessels and piping systems constitute a significant portion of the physical plant for chemical, petrochemical, nuclear and other industries. Continued engineering studies which lead to new cost-effective problem solutions and improvements in the general area of pressure vessels, piping and related component design and analysis are essential for future safe and economic plant operation.

This publication consists of several technical papers which present recent works on various topics associated with the design and analysis of pressure vessels, piping and components. While the papers in this publication are separated into two distinct categories, one on pressure vessels and the other on piping, some of the papers are in fact applicable to both pressure vessels and piping.

The papers in this publication were prepared for presentation at the 1986 ASME International Joint Pressure Vessel and Piping Division and Computer Engineering Division Conference in Chicago, Illinois. The technical sessions for these papers were developed under the auspice of the Design and Analysis Committee of the ASME Pressure Vessel and Piping Division.

W. E. Short

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ANALYSIS OF TEMA TUBESHEET DESIGN RULES – COMPARISON WITH UP TO DATE CODE METHODS

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ABSTRACT
TEMA standards appeared for the first time in 1941 in order to give rules for U-tube and floating head heat exchangers. Specific rules for fixed tubesheets were added later based on GARDNER's works. For more than thirty years these rules have been widely used in most countries of the world and have given satisfaction inasmuch as they have not led to failure. The primary purpose of this paper is to analyse which parts of these rules have sound theoretical basis, which are empirical and to show the consequences.
New rules have recently appeared in various pressure vessel codes (specially in the French code-CODAP) or are in preparation (ASME). The other purpose of this paper is to explain why these codes preferred to develop their own rules instead of employing TEMA ones.

NOMENCLATURE (SI units)
All following notations are in accordance with TEMA rules (except d_o here noted d_t) :

d_t = outside tube diameter
D_o = outside shell diameter
D_J = expansion bellows inside diameter
$D^* = \dfrac{E^*.h^3}{12(1-\upsilon^{*2})}$ = flexural rigidity of equivalent plate
E^* = elastic modulus of equivalent plate
E_s , E_t = elastic modulus of shell, tubes
$f_s = 1-N\left(\dfrac{d_t}{G}\right)^2$ = shellside drilling coefficient
$f_t = 1-N\left(\dfrac{d_t-2t_t}{G}\right)^2$ = tubeside drilling coefficient
F, F_q = TEMA coefficients
G = inside shell diameter
h = tubesheet thickness obtained by analytical method.
H_1,H_4 = coefficients depending on parameters X and Z

$k_s = \dfrac{\pi.t_s.(D_o-t_s).E_s}{l}$ = axial rigidity of half-shell
$k_t = \dfrac{\pi.t_t.(d_t-t_t).E_t}{l}$ = axial rigidity of one half-tube
$k_w = \dfrac{N.k_t}{\pi R^2}$ = modulus of equivalent elastic foundation
$K = \dfrac{k_s}{N.k_t}$ = ratio of shell/tube-bundle rigidity
$K_\theta = \dfrac{M_E}{\Theta_E}$ = constant of elastic restrain at tubesheet edges
L = tube length
$l = \dfrac{L}{2}$ = half-tube length
M_E = reactive bending moment at tubesheet periphery
N = number of tubes
p = tube pitch
P = equivalent differential pressure in TEMA
P^* = equivalent differential pressure in analytical method
P_s , P_t = shellside, tubeside pressure
q(r) = effective pressure at radius r
r = radius ($0<r<R$)
R = outer tubesheet radius
S = allowable stress of ASME VIII-1
S_J = axial rigidity of expansion bellows
t_s,t_t = shell, tube thickness
T = tubesheet thickness in TEMA code
V_E = axial reaction at tubesheet periphery
w(r) = tubesheet deflection at radius r
X,Z = characteristic parameters of the exchanger
α_s,α_t = thermal expansion coefficient of shell, tubes
Δ_s = axial displacement of half-shell
υ_s,υ_t = POISSON ratio of shell, tubes
υ^* = POISSON ratio of equivalent tubesheet
$\eta = \dfrac{D^*}{D}$ = tubesheet deflection efficiency

μ = tubesheet ligament efficiency
Θ_E = tubesheet rotation at its periphery
Θ_s, Θ_t = mean temperature less 20°C of shell, tubes
σ = maximum stress in tubesheet, tubes, shell or channel (with appropriate subscripts)
τ = maximum shear stress in tubesheet
subscripts : .t, s, c refer to tubes, shell,channel
.b, l refer to bending, longitudinal
.without subscript refers to tubesheet

INTRODUCTION

Up to the present time most tubesheet heat exchangers have been designed by applying the American rules of the Tubular Exchanger Manufacturers Association [1].
This standard proposes a simple formula to determine the tubesheet thickness for the three classical types of heat exchangers : U-tube, floating head and fixed tubesheets.
This empirical formula is based on the maximum stress in thin circular plates under uniform pressure. It takes no account of the holes which weaken the tubesheets, nor of the tubes which stiffen it. If this second effect prevails, the tubesheet may become unnecessarily thick. MILLER [2] mentioned the striking case of an exchanger erected with a 34 mm tubesheet thickness whereas TEMA would have required a 264 mm thickness !
Conversely, it will be shown that in some cases TEMA thickness may be insufficient.
Nevertheless, TEMA rules have been used, generally with success, in the past fourty years for the construction of thousands of exchangers and have the merit of simplicity and industrial experience.
The first purpose of this paper is not to undertake a systematic and unfruiful criticism of the TEMA method, but to analyse which parts have scientific basis, which ones are empirical and to show the consequences for the heat exchanger design.

HISTORICAL BACKGROUND

About ten years after the first issue of TEMA standards, in 1941, new methods appeared for the design of heat exchangers : in U.S.A in 1948-1952 thanks to GARDNER [3,4] and in U.K in 1952 thanks to MILLER [2].
These methods are similar and take account for the support given by the tubes to the tubesheet in treating them as an elastic foundation on which the tubesheet rests. The weakening effect of the holes is taken into account by replacing the tubesheet by an equivalent solid plate for which the flexural rigidity was empirically determined.
Thanks to many valuable experimental and theoretical works this concept has realized a strong evolution which allows to determine soundly the effective elastic constants E^* and ν^* of the equivalent solid plate. Reference [5] gives details on this topic.

GARDNERS's method has been resumed in 1968 by TEMA for fixed tubesheets heat exchangers and in 1973 by STOOMWEZEN [6], whereas MILLER's method was used by BS 1515 [7] in 1965.
In 1969, GARDNER [8] improved his method for floating head and U-tube heat-exchangers by taking into account the unperforated annulus at the tubesheet periphery and proposing a direct formula for tubesheet thickness. This method has been used by various Pressure Vessel Codes : ISO [9] in 1973, BS 5500 [10] in 1976, CODAP [11] in 1982 and more recently (for U-tube exchangers only) by ASME section VIII [12] with some modifications.
All these code methods have been examined in detail in reference [13]. They present the disadvantage to consider the tubesheet as simply supported or clamped at its edges, which obliges the designer to make an arbitrary choice between these two theorical cases.

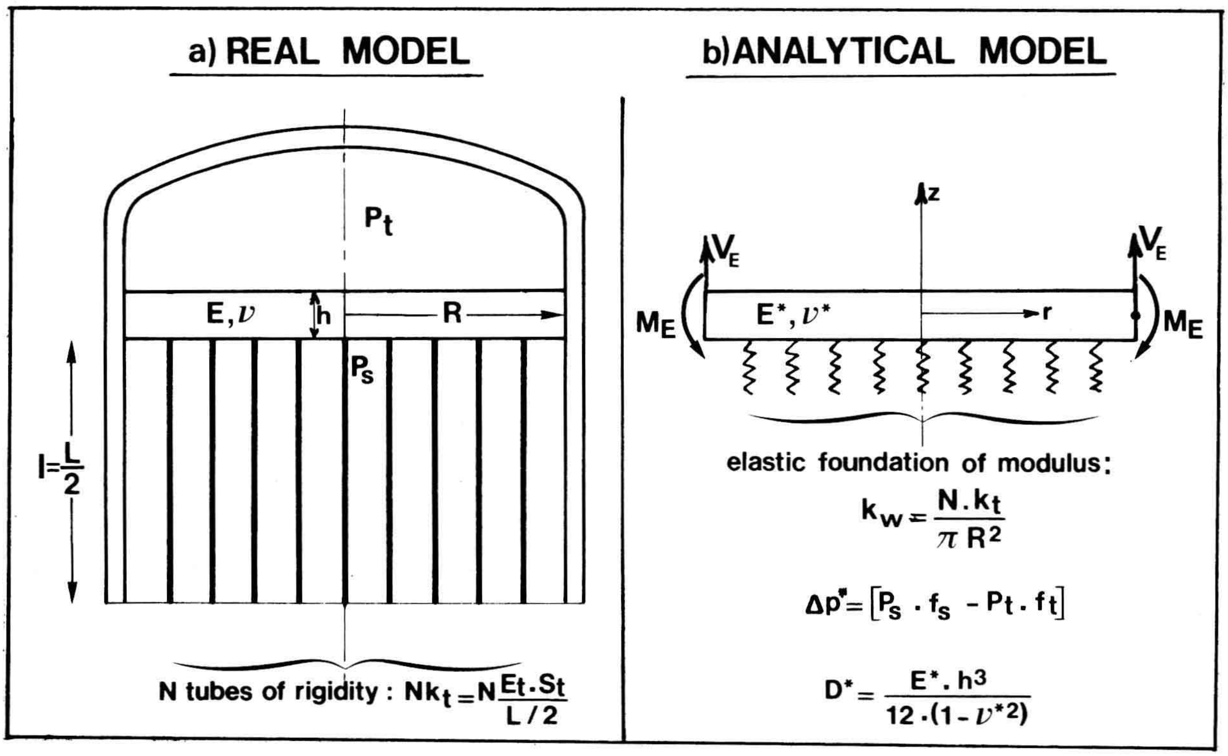

Fig. 1 - Analytical model used in design method

In 1959, GALLETLY [14] improved this approach by treating the more general case of a tubesheet elastically restrained at its edges against rotation. This method has been resumed for the first time in CODAP in 1982 for the design of fixed tubesheet heat exchangers, presented by the author in a recent paper [15].
A similar approach is being developped in ASME [12], based on the works of SINGH and SOLER [16].
An other purpose of this paper is to give the main ideas of these new methods and to explain why these codes preferred to elaborate their own rules instead of resuming TEMA ones.

BASIS OF TUBESHEET ANALYSIS METHODS

All heat exchangers tubesheets design methods mentionned above are based on an elastic analysis of the exchanger allowing to determine the maximum stresses in the various parts of the exchanger, which are then limited to maximum allowable stress values.
The exchanger is assumed to be of revolution and to be symmetric about the plane midway between the tubesheets so as to analyse only a half-structure of length $l=L/2$ (fig. 1a).
The main steps of the analysis, summarized in figure 1, are presented hereafter :

STEP 1 : the tubesheet, provided with its tube-bundle, is deconnected from the remainder of the exchanger by applying at its periphery (fig.2) :

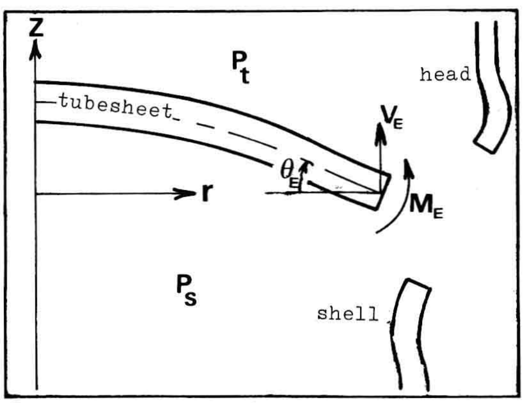

Fig.2 : Connection of tubesheet with shell and head

- an axial reaction V_E :

$$V_E = \frac{P_t \cdot \pi R^2 - k_s \cdot \Delta_s}{2\pi R} \quad (1)$$

due to the end load acting on the head and to the axial displacement Δ_s of the half-shell of axial rigidity :

$$k_s = \frac{E_s \cdot \pi\, t_s \cdot (D_o - t_s)}{L/2} \quad (2)$$

- a reactive bending moment M_E.

In CODAP, this moment is supposed proportional and opposite to the tubesheet rotation at its edges :

$$M_E = - K_\theta \cdot \Theta_E \quad (3)$$

Coefficent K_θ, which depends on the bending rigidities of shell and head, represents the degree of elastic restrain of the tubesheet by shell and head. It may vary between two extreme values : 0 (which corresponds to the simply supported case) and ∞ (which corresponds to the clamped case).

STEP 2 : the tubesheet is replaced by an equivalent solid plate of effective elastic constants E^* and ν^* which depend primarily on ligament efficiency μ (fig.3). Its flexural rigidity is given by :

$$D^* = \frac{E^* . h^3}{12(1-\nu^{*2})} \quad (4)$$

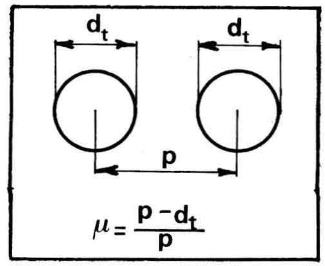

Fig. 3 : ligament efficiency μ

STEP3 : the tubes are assumed uniformly distributed and in sufficient number so as to act as a uniform elastic foundation of modulus :

$$k_w = \frac{N . k_t}{\pi R^2} \quad (5)$$

where k_t represents the axial rigidity of one half-tube :

$$k_t = \frac{E_t \cdot \pi\, t_t \cdot (d_t - t_t)}{L/2} \quad (6)$$

STEP 4 : the equivalent solid plate is submitted to:
- the reactive loads due to the tubes :

$$p_t(r) = - k_w \cdot \Delta_t(r) \quad (7)$$

where $\Delta_t(r)$ is the expansion of the tubes located at the distance "r" of the axis :

$$\Delta_t(r) = [w(r)] - [(\alpha_t \cdot \theta_t - \alpha_s . \theta_s).l] + [\Delta_s] \quad (8)$$

in which the first term represents the tubesheet deflection at radius r, and the second term is the differential axial free expansion between tubes and shell.
- the differential pressure acting on the equivalent solid plate :

$$\Delta p^* = P_s \cdot f_s - P_t \cdot f_t \quad (9)$$

where f_s and f_t are the shellside and tubeside drilling coefficients. The net effective pressure acting on the plate is given by :

$$q(r) = p_t(r) + \Delta p^* \quad (10)$$

STEP 5 : the problem is now reduced (fig. 1b) to the case of a solid circular plate, of effective elastic constants E^* and ν^*, elastically restrained at its edges by a moment M_E and a reaction V_E, resting on an elastic foundation of modulus k_w, and subjected to an effective pressure q(r). The bending deflection of such a plate is governed by a fourth order equation which solution may be written :

$$w(r) = A . \mathrm{ber}x + B . \mathrm{bei}x + \frac{\Delta p^*}{k_w} + (\alpha_t . \theta_t - \alpha_s . \theta_s).l - \Delta_s \quad (11)$$

where : . $x = \sqrt[4]{\frac{k_w}{D^*}} . r$

. berx, beix are Bessel functions of order 0

From w(r) one may determine at radius r the slope, the shear and the radial bending moment.

STEP 6 : The constants of integration A and B are determined by the boundary conditions (1) and (3) which makes appear 2 fundamental dimensionless parameters for he heat-exchanger :

$$.X = \sqrt[4]{\frac{k_w}{D^*}}.R = \sqrt[4]{\frac{24N(1-\upsilon^{*2})E_t \cdot t_t \cdot (d_t - t_t)R^2}{E^* . h^3 . L}} \qquad (14)$$

which represents the relative rigidity of the tube-bundle with respect to the tubesheet : it may vary from 0 (no tube bundle) to about 50.

$$. \ Z = \frac{K_\theta}{\sqrt[4]{k_w . D^{*3}}} \qquad (15)$$

which represents the elastic rotational restrain at the tubesheet edges : it may wary from 0, when the tubesheet is simply supported, to ∞ when clamped. These 2 coefficients depending, through D*, on the tubesheet thickness, the method will be iterative. Their numerical values control the maximum stresses in the tubesheet, the tube-bundle, the shell and the channel.

STEP 7 : the determination of maximum stresses in the various parts of the exchanger are as follows :

Maximum radial bending stress in tubesheet is determined from the radial bending moment and may be written in the classical form of circular plates under pressure :

$$\sigma = \frac{1}{\mu . H_1(X,Z)} . \left[\frac{R}{h}\right]^2 . P^* \qquad (16)$$

in which $H_1(X,Z)$ is given by curves plotted in figure (4) in function of X for various values of Z. It must be pointed out that all curves are not located between those relative to simply supported (Z = 0) and clamped (Z = ∞) cases. Some, relative to Z = 0.6 0.8 1.0, are located outside : conseqently, it is not correct to interpolate between simply supported and clamped cases, as proposed by many authors and by TEMA.

FIG.4 shows that there exists an optimal value Z_{opt} of Z (about 0.5) for which H_1 is maximum and this tubesheet stress minimum (the physical significance is that the maximum stress appears simultaneously inside and at the periphery of the tubesheet with the same value).

Equivalent pressure P*

In equation (16) the equivalent pressure P* acting on the tubesheet is given by :

$$P^* = \frac{P_s\left\{J.K\left[f_s + 2\upsilon_t(1-f_s)\right] + \left[2\upsilon_s J\right] - \left[\frac{D_J^2 - G^2}{G^2} . \frac{J.k_s}{4S_J}\right]\right\}}{1 + J.K.H_4(X,Z)}$$

$$\frac{-P_t\left\{J.K\left[f_t + 2\upsilon_t(1-f_t)\right] + 1\right\} + \left\{(\alpha_t . \theta_t - \alpha_s . \theta_s) . \frac{k_w}{2} . JK\right\}}{1 + J.K.H_4(X,Z)} \qquad (17)$$

in which :

. $K = \frac{k_s}{N.k_t}$ represents the ratio of the axial rigidity of the shell to that of the tube-bundle

. S_J is the axial rigidity of an expansion joint set on the shell.

$$.J = \frac{1}{1 + \frac{k_s}{2S_J}} = \frac{1}{1 + \frac{\pi(D_o - t_s) \cdot t_s \cdot E_s}{S_J . L}} \qquad (18)$$

(J = 1 for shells without expansion joint)

. $H_4(X,Z)$ – like $H_1(X,Z)$ – is given in figure 4 in function of X, for various values of Z.

In formula (17) :

- terms in υ_s and υ_t account for the POISSON effect under pressure on the axial expansion and contraction of the shell and tubes.
- term in $D_J^2 - G^2$ account for the end load due to shell pressure P_s acting on the expansion joint of internal diameter D_J (Fig.5).

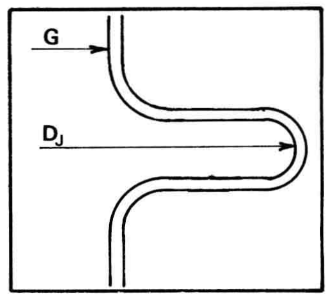

Fig.5 Expansion joint set on the shell

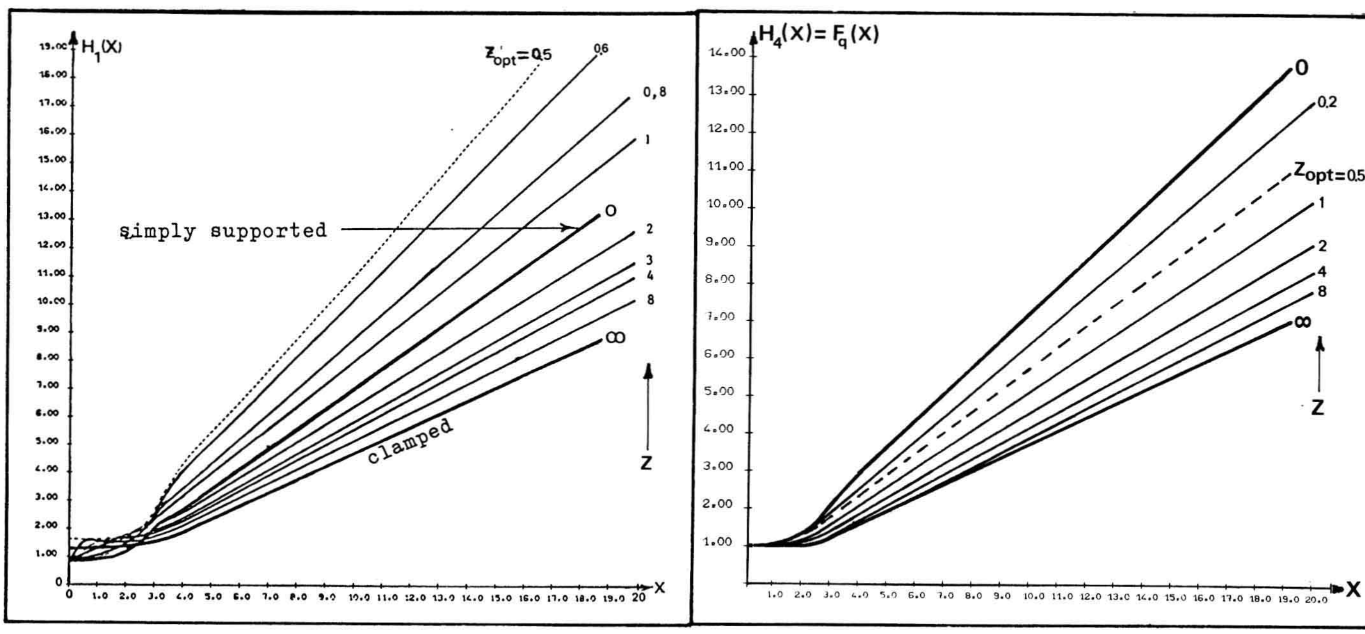

Fig. 4 : Values of coefficients H_1 and H_4 as function of X, for various values of Z

Maximum shear stress in tubesheet is located at its edges and determined from shear :

$$\tau = \frac{P^*.R}{2\mu \cdot h} \qquad (19)$$

Maximum stresses in shell

- Maximum longitudinal stress in shell is determined from its longitudinal displacement Δ_s :

$$\sigma_{s,l} = \frac{(D_o - t_s)}{4\ t_s}\ (P_t + P^*) \qquad (20)$$

- Maximum bending stress in shell is determined from the connection moment M_E at the tubesheet periphery (equ.3)

$$\sigma_{s,b} = \frac{6.M_E}{t_s^2} \qquad (21)$$

Maximum longitudinal stress in tubes is determined from their longitudinal displacement $\Delta_t(r)$ and may appear :

- either at tube-bundle periphery :

$$\sigma_{t,e} = \frac{R^2}{N \cdot t_t \cdot (d_t - t_t)} \left[(P_s . f_s - P_t . f_t) - P^* . H_4 \right] \qquad (22)$$

where H_4 (X,Z) is plotted in Fig.4.

- or inside the tube-bundle :

$$\sigma_{t,i} = \frac{R^2}{N . t_t \cdot (d_t - t_t)} \left[(P_s . f_s - P_t . f_t) - P^* . H_3 \right] \qquad (23)$$

where $H_3(X,Z)$ - like H_1 and H_4 - is given by curves in function of X for various values of Z (see Ref.[17]).

Floating head heat exchangers

The same analysis apply to floating heads in making J=0 (expansion joint of zero axial rigidity) and replacing P_t by : $P_t - P_s$ (differential pressure acting on the head is $P_t - P_s$ instead of P_t). This will lead to :

$$P^* = P_t - P_s \qquad (24)$$

to be used in conjunction with equation (16).

TEMA RULES RELATIVE TO FLOATING HEAD EXCHANGERS

The Tubular Exchanger Manufacturers Association was founded in the 1930's in order to establish standards for shell and tube heat exchangers which should represent the combined experience and judgement of its members in producing reliable equipment at reasonable costs. The first edition of TEMA standards was issued in 1941 and comprised 50 pages (instead of 240 pages now!), treating only floating head and U-Tube heat exchangers. The design portion was very short : 25 lines, instead of 16 pages now, and was only concerned with gasketed tubesheets. Later, in the fourth edition of 1959, integral tubesheets were incorporated and tubesheet thickness formula took the current well-known form (Fig.6) :

$$T = F \cdot \frac{G}{2} \sqrt{\frac{P}{S}}$$

The calculation must be carried out in considering both tubeside and shellside conditions.

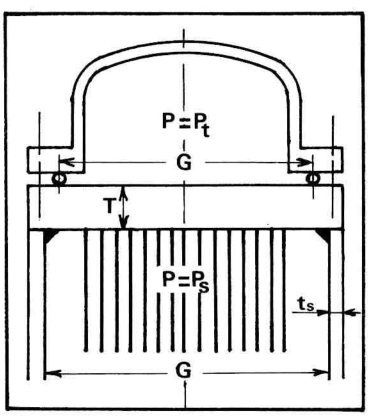

Fig.6 Main TEMA rotations

Coefficent F has a fixed value which varies from 0.8 when the tubesheet is considered as clamped to 1.0 when considered as simply supported. Between these two extreme cases TEMA proposes a linear interpolation in function of t_s / G (Fig.7).

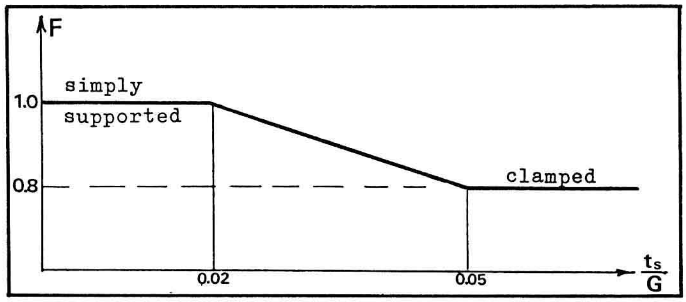

Fig.7: Coefficient F of TEMA

This approach is open to criticism because the parameter t_s/G cannot correctly represents the degree of rotational restraint exerted by the shell and head on the tubesheet ; it depends in fact on the relative bending rigidities of tubesheet, shell and head which are involved in the characteristic coefficient Z described above in equation (15).

Of course for a given tubesheet thickness, if shell thickness t_s is very high,the tubesheet rotation will be prevented and F = 0.8 ; inversely, if the shell is very thin, the tubesheet will be free to rotate at its edges and F = 1. But between these two extreme cases it is difficult to justify that the tubesheet is clamped when $t_s/G \geqslant 0.05$ and simply supported when $t_s/G \leqslant 0.02$. In the intermediate case ($0,02 \leqslant t_s/G \leqslant 0.05$) it has been pointed out previously that it was not correct to interpolate between simply supported and clamped cases (yet, doing so, TEMA remains conservative).

The TEMA formula is derived from the equations of circular flat heads under pressure by limiting the maximum stress to S :

$$h = C \cdot \frac{G}{2} \sqrt{\frac{P}{S}}$$

in which : C = 1,113 when the head is simply supported

C = 0,866 when the head is clamped.

This formula does not account for the staying action of the tubes, nor for the tubeholes which weaken the tubesheet : coefficient F has been empirically chosen in supposing that these two effects are balanced and it reflects the experience and judgement of TEMA manufacturers. A look at the analytical approach presented the preceding section shows that coefficient F of TEMA should involve 20 variables (5 for the tubes, 7 for the tubesheet and 8 for its connection with shell and channel). And this inspired GARDNER [8] to write that : "symbol F must represent a really superb compound of "lumped" parameters".

Comparison with analytical methods

Comparison with analytical methods presented above can easily be done by inversing equation (16) and limiting maximum tubesheet stress to 2S :

$$h = F \cdot \frac{G}{2} \sqrt{\frac{P}{S}} \qquad (25)$$

in which coefficient F is given by :

$$F = \frac{1}{\sqrt{2\mu . H_1(X,Z)}}$$

For sake of comparison, F has been plotted in figure (8) as function of X, for Z=0 (simply supported case), Z=∞ (clamped case) and for a ligament efficiency μ=0.3.

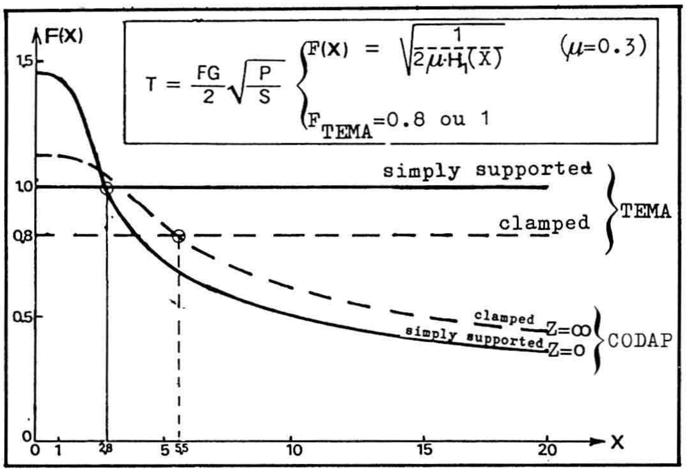

Fig.8: Comparison of coefficient F in CODAP and TEMA codes

This figure shows that TEMA leads to excessive tubesheet thicknesses if X>2.8 for the simply supported case, and X>5.5 for the clamped case. On the other hand, TEMA formula may become unconservative for low values of X (X<1.5) which physically corresponds to a low tube-bundle stiffness with respect to the tubesheet. Nevertheless, it must be pointed out that TEMA F values are remarkably well chosen : they represent approximatively the average values of functions F(X) relative to the simply supported case (F=1) and the clamped case (F=0.8).

It is probably for this reason that, in spite of rough simplifications, the TEMA formula has proven very successful for about 40 years and have been used for building and operating hundreds of thousands of heat exchangers in the world with indeed very few problems.
An other deficiency of TEMA is that the tubes are not verified against overloading. Reason seems to be that the introduction of curves $H_3(X,Z)$ and $H_4(X,Z)$, necessary for tubes design, would have complicated the rules.

U-tube Heat Exchangers

In this type of exchanger the tubes provide no staying action to the tubesheet. TEMA takes this into consideration by increasing the F value by 25 % : F = 1.25 when the tubesheet is simply supported and 1.0 when clamped.
TEMA formula remains empirical and too simplistic because it ignores the presence of holes and of an unpierced annullus at the periphery of the tubesheet.
New modern methods account for these effects. They are based on the 1969 GARDNER paper [8], and, more recently, SOLER and SINGH [18] developped an elastic-plastic method which takes into account the support provided by the shell and the channel.

TEMA METHOD RELATIVE TO FIXED TUBESHEETS EXCHANGERS

A design rule relative to fixed tubesheets heat exchangers appears for the first time in the fifth 1968 TEMA edition. Minor modifications were brought into the next edition of 1978 which is the present one.
These rules were elaborated by GARDNER from his 1952 paper [4] dealing with fixed tubesheets with drastic simplifications due to two TEMA requirements :
- tubesheet formula must be presented in the usual TEMA form (Equ.25).
- the method must not necessitate the use of curves such as H_1 and H_4.

It is a pity that GARDNER was obliged to comply with these requirements which will turn his theoretical method into a semi-empirical one.

Equivalent pressure P in TEMA

Analytical equation (16) giving the maximum stress in the tubesheet may be inversed so as to present it in the TEMA form :

$$h = \frac{1}{\sqrt{\mu . H_1(X,Z)}} \cdot \frac{G}{2} \cdot \sqrt{\frac{P^*}{\sigma}} \qquad (26)$$

In this equation P* is the equivalent pressure acting on the tubesheet, given by equation (17) which can be written, using TEMA sign conventions :

$$P = -P^* = \frac{P_t\left\{1+JK\left[f_t + 2\nu_t(1-f_t)\right]\right\} - P_s\left\{\left[2\nu_s J\right] + JK\left[2\nu_t + f_s(1-2\nu_t)\right]\right\}}{1 + JKF_q}$$

$$\frac{-\left(\frac{1-J}{2}\right)\cdot\left(\frac{D_J^2}{G^2}-1\right)\Bigg\} + \left\{4J.E_s.t_s.(\alpha_t\,\theta_t - \alpha_s\,\theta_s).\frac{D_o - t_s}{(D_o-2t_s)^2}\right\}}{1 + JKF_q}$$

Three simplifications have been made in this equation to give ease for calculation :
1- POISSON ratios ν_s and ν_t are taken equal to 0.3
2- t_s is supposed small as compared to D_o ; thus :

$$\frac{D_o - t_s}{(D_o - 2t_s)^2} \simeq \frac{1}{D_o - 3\,t_s}$$

3-Variation of F_q with X, given by figure (4), is replaced by its asymptotic expression which is a straight line of equation :

$$F_q(X) = 0.25 + \alpha . X \qquad (28)$$

of slope : $\alpha = \sqrt{2}/2$ for simply supported case (Z=0)
$\alpha = \sqrt{2}/4$ for clamped case (Z=∞)
which can be written in function of coefficient F of TEMA :

$$\alpha = (F-0,6)\ \sqrt{2}/0.8 \qquad (29)$$

Replacing X by its expression (14), F writes :

$$F_q = 0.25+(F-0.6)\cdot\frac{\sqrt{2}}{0.8}\left[\frac{6(1-\nu^2)}{\eta}\cdot\frac{E_s}{E}\frac{t_s}{KL}\left(\frac{2R}{h}\right)^3\right]^{\frac{1}{4}} \qquad (30)$$

where $\eta = D^*/D$ is the deflection efficiency, taken by TEMA equal to 0.178 so as to make appear a coefficient 300 inside the bracket of TEMA formula which writes :

$$F_q = 0.25 + (F-0.6)\left[\frac{300\ t_s . E_s}{K.L.E}\left(\frac{G}{T}\right)^3\right]^{\frac{1}{4}} \qquad (31)$$

This expression shows that the method is a trial and error one : the designer must assume a value for T to calculate F_q and iterations must be achieved until the assumed value match the computed one with less than 1.5 %.
Formula giving P may be written in the form :

$$P = P_t' - P_s' + P_d \qquad (32)$$

In order to make appear the three TEMA effective pressures :

- the effective tubeside pressure :

$$P_s' = P_s \frac{0.4J\left[1.5 + K\,(1.5 + f_s)\right] - \left[\left(\frac{1-J}{2}\right)\left(\frac{D_J^2}{G^2} - 1\right)\right]}{(1 + JKF_q)} \qquad (33)$$

- the effective shellside pressure :

$$P_t' = P_t \frac{1 + 0.4JK\cdot(1.5 + f_t)}{(1 + JKF_q)} \qquad (34)$$

- the equivalent differential expansion pressure :

$$P_d = \frac{4J\cdot E_s\cdot t_s(\alpha_s\cdot\Theta_s - \alpha_t\cdot\Theta_t)}{(D_o - 3t_s)\cdot(1 + JKF_q)} \qquad (35)$$

For practical use TEMA considers the 7 combined design loading cases where pressure acts on shellside ($P_t = 0$), on tubeside ($P_s = 0$), on both sides, and in each case with and without thermal expansion ; seventh case is obtained for thermal expansion acting alone ($P_s = 0$, $P_t = 0$). This is implicitly realised in using for the equivalent pressure the greatest value of P obtained for these different loadings.
One inconvenience of this procedure is that the controlling service loading may be not a real one ; in such a case the calculated thickness will be unnecessarily high. Nevertheless, it presents the advantage to account for all intermediate loadings, in particular starting-up and shutt-off operating conditions.

Tubesheet bending formula

In equation (26) σ represents the maximum allowable stress in the tubesheet established according to the stress category system : primary stresses (i.e. those due to pressure loads) are limited to the allowable stress S of ASME VIII - Division 1 and primary + secondary stresses (i.e those due to pressure + thermal expansion loads) are limited to 2S. This is automatically done by TEMA in dividing the equivalent pressure P by 2 when the equivalent thermal expansion P_d appears in P formula ; doing so allows to present the tubesheet formula in the classical TEMA form in which coefficient F should write :

$$F = \frac{1}{\sqrt{\frac{\sigma}{S}\cdot\mu\cdot H_1(X,Z)}} \qquad (36)$$

and takes into account the tubesheet edge fixing through coefficient Z, the weakening effect of tube holes through ligament efficiency μ and the staying action of tubes through coefficient X. As for floating head exchangers TEMA ignores these two effects and F is taken equal to 1 when tubesheet is simply supported and 0.8 when clamped.

A look at figure (8) shows that TEMA thickness will be excessive when X is high ($X>5$) and sometimes insufficient when X is low ($X<2$), which occurres for big heat exchangers submitted to high pressures. TEMA, aware of this problem, has limited in its last edition pressures to 20 MPa, diameters to 1500 mm and product of both to 10500 mm MPa.

Tubesheet shear formula

Maximum shear stress in TEMA, given by equation (19), is limited to 0.8S which leads to shear thickness :

$$h = \frac{P^*(2R)}{4\mu\,(0.8S)}$$

or using TEMA notations :

$$T = \frac{0.31.\,D_L}{1 - \frac{d_t}{p}}\cdot\frac{P}{S} \qquad (37)$$

Where D_L is the equivalent diameter of the perforated part of the tubesheet.

Maximum stresses in shell and channel

- Maximum longitudinal stress in shell is derived from equation (20) which writes with TEMA notations

$$\sigma_s = \frac{D_o - t_s}{4\,t_s}\,C_s\,.\,P_s^* \qquad (38)$$

where : $P_s^* = P_t + P^* = (P_t - P_t') + P_s' - P_d$ (39)
as for tubesheet design :

. 7 loading cases are examined which lead to 7 values of P_s^*.
. σ_s is divided by 2 when P_d appears in P_s^* formula (thanks to coefficient C_s, taken equal to 0.5) which allows to limit primary stresses to S_s and primary + secondary stresses to $2S_s$. Compressive stresses, obtained for negative value of σ_s, are limited to the ASME code allowable compressive stress.

- Maximum bending stresses in shell and channel are ignored in TEMA rules.

Maximum stresses in tubes

- Maximum longitudinal stress at the periphery of bundle is derived from equation (22) which writes with TEMA notations :

$$\sigma_{t,e} = \frac{F_q\cdot G^2}{4\,N\cdot t_t\cdot(d_t - t_t)}\,C_t\,.\,P_t^* \qquad (40)$$

$$\text{where : } P_t^* = (P_t' - P_t\cdot\frac{f_t}{F_q}) - (P_s' - P_s\cdot\frac{f_s}{F_q}) + P_d \qquad (41)$$

As for the shell :

. 7 loading cases are examined which lead to 7 values of P_t^*
. Primary stresses are limited to S_t and primary + secondary stresses to $2.S_t$ (C_t is taken equal to 0.5 when P_d appears in P_t^* formula)
. Compressive stresses (negative value of $\sigma_{t,e}$) are limited to the allowable compressive stress S_c given by a specific formula and based on the critical Euler stress :

$$\sigma_{cr} = \frac{\pi^2\,.\,E_t}{\lambda^2}$$

where λ is the slenderness ratio of the tubes. Compressive stresses must also be limited to the yield stress S_y in order to avoid plastification.

In figure 9 forbidden range for compressive stress is shown by cross-hatched area, above S_yMN boundary. In fact TEMA preferred to limit this range by a more continuous and conservative boundary : $S_y M_o N$ where M_o ordinate is taken equal to $S_y/2$ and abscissa is :

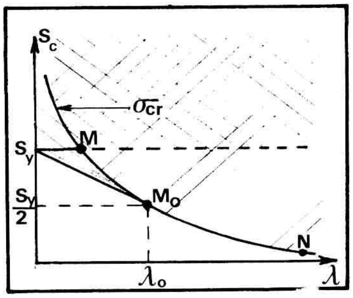

Fig.9 : Determination of allowable compressive stress in TEMA

$$\lambda_o = \sqrt{\frac{2\pi^2 \cdot E_t}{S_y}}$$

noted C_c in TEMA.

The $S_y M_o$ straight line equation writes :

$$\sigma_o = S_y\left(1 - \frac{\lambda}{2\lambda_o}\right)$$

Finally, using a factor of security of 2 on σ_{cr} and σ_o, the TEMA allowable tube compressive stress writes :

$$S_c = \frac{\pi^2 \cdot E_t}{2\,\lambda^2} \text{ when } \lambda \geqslant \lambda_o \text{ or } S_c = \frac{S_y}{2}\left(1 - \frac{\lambda}{2\lambda_o}\right) \text{ when } \lambda < \lambda_o$$

- Maximum longitudinal stress at the interior of the bundle is not considered by TEMA. These tubes may be overloaded, both in tension and compression, and in such cases it is recommended to verify them by using equation (23).

<u>In conclusion,</u> TEMA takes into account the staying action of the tubes and the weakening effect of the holes for the determination of equivalent pressure P, but does not for the determination of coefficient F which controls the bending tubesheet thickness.

Shear tubesheet thickness and longitudinal stresses in shell and peripherical tubes are correctly calculated, in spite of some simplifications, but bending stresses in shell and channel and longitudinal stress in tubes interior to the tube-bundle are not considered.

Comparison with New Modern Methods

Due to these simplifications TEMA does not insure an overall security for all exchangers. This is the main reason why studies have been undertaken since 1974 in France by the author [15,17] in order to establish design rules for the CODAP. The same reasoning has prompted ASME to set up a Special Working Group on Heat Transfer Equipment who is preparing new rules for fixed tubesheet exchangers, based on SOEHRENS [19] and SOLER [14] works. The approach is similar to that of CODAP but the method intend to account for the untubed rim of the tubesheet.

Numerical comparison

A fixed tubesheet heat-exchanger manufactured by SPEICHIM (SPIE-BATIGNOLLES Group) has been designed by TEMA and CODAP rules. Tubesheets are integral with shell and head and main dimensions are represented in figure 10, other data are as follows :

P_t = 0.3 MPa	P_s = 0.5 MPa
Θ_t = 195-20=175°C	Θ_s = 220-20=200°C
α_t = 0.121x10^{-4}	α_s = 0.123 x 10^{-4}
E = 187500 MPa	E_t = 187544 MPa
E_s = 186000 MPa	E_c = 186000 MPa
S = 132.3 MPa	S_t = 92 MPa
S_s = 127.3 Mpa	S_c = 127.3 MPa

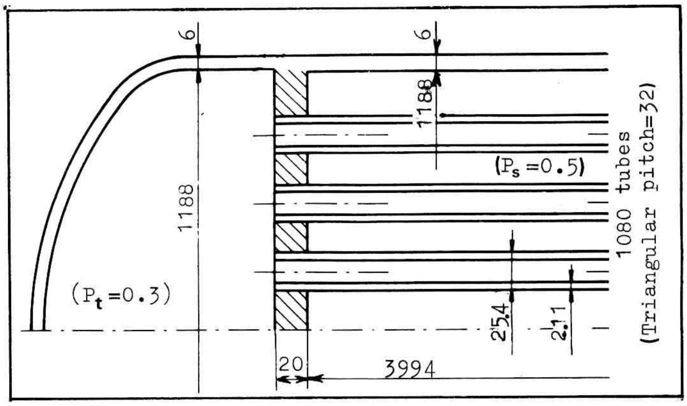

Fig.10 : Fixed tubesheet exchanger (S.I. units)

The 7 implicit loading cases of TEMA have been studied ; the controlling one is obtained for : P_s=0

Comparative results for this loading case are presented in table 1.

		SYMBOL	CODAP	TEMA
INTERMEDIARY RESULTS		η	0.253	0.178
		K	0.134	0.134
		X	15.0	16.4
		Z	0.36	0.0
		H_4, F_q	9.1	11.8
		F	0.425	1.0
EQUIVALENT PRESSURES		P_d	0.583	0.501
		P_s'	0.159	0.137
		P_t'	0.151	0.130
		P	0.367	0.315
STRESSES	TUBE SHEET	σ	176<397 (3S)	556>265 (2S)
		τ	45<106 (0.8S)	43<212 (1.6S)
	TUBES INTERIOR	$\sigma_{t,i}$	4<138 (1.5S)	non-calculated
		$\sigma_{t,i}$ (compression)	$\|-4\|$ < 33	non calculated
	TUBES PERIPHERAL	$\sigma_{t,e}$	43<138 (1.5S)	48<184 (2S)
		$\sigma_{t,e}$ (compression)	$\|-8\|$ < 35	- 9 < 25
	SHELL	$\sigma_{s,l}$	29<191 (1.5S)	25<127 (S)
		$\sigma_{s,l}$ (compression)	$\|-29\|$ <117	-25 <117
		$\sigma_{s,b}$	195<382 (3S)	non calculated
	CHANNEL	$\sigma_{c,b}$	188<382 (3S)	non calculated
OPTIMIZED THICKNESS		h , T	h_{opt} = 12 mm	T_{opt} = 32 mm

TABLE 1 : TEMA-CODAP comparisons (S.I. units)

This comparison confirms (see Fig.8) that for high values of X (here X = 15) TEMA tubesheet stress is too high (556 MPa) as compared to CODAP one (176 MPa) due to TEMA simplifications : this is the main reason why optimized thickness by TEMA is 32 mm, instead of 12 mm by CODAP.
Longitudinal stresses in tubes and shell are of same order in both codes, but bending stresses in shell and channel, which may be excessive, are ignored by TEMA.

CONCLUSIONS

The TEMA method for determining tubesheet thickness is empirical and based on the maximum stress formula for thin circular plates under uniform pressure. It supposes that the stiffening effect supplied by the tubes is conterbalanced by the weakening effect of the holes in the tubesheet.
This allows TEMA to propose a very simple formulation for the coefficient F : 1.0 if tubesheet is simply supported and 0.8 if clamped. In U-Tube exchangers the first effect no longer exists, and F is increased by 25 %.

This simplistic approach may be acceptable for exchangers of normal size subjected to normal pressures for which TEMA rules were originally dedicated. And throughout 40 years of industrial experience theses rules have been proven valid for such exchangers.
But chemical and power industries need increasingly bigger exchangers, operated in increasingly severe conditions. Quite often the stiffening effect of the tube-bundle prevails on weakening effect of holes which leads to a TEMA thickness unnecessarily high and the tubes may become overloaded. In the inverse case, TEMA thickness may becomes unconservative, specially when pressures are high. Aware of this problem, TEMA has limited, in its last edition, pressures and diameters.

Obviously a more analytical approach is now needed which takes into account the axial rigidity of the tubes, the presence of tubeholes, the connection of the tubesheet with shell and head and the untubed rim. Such methods have been recently developed and resumed in new modern codes, such as BS 5500 and CODAP, or are in the course of preparation in ASME. Thanks to a better representation of the tubesheet behaviour, they supply a more overall security.

TEMA simplifications were carried out so as to propose simple rules easily operated by hand : this approach could be understandable in a time when computers were not very spread out.
Some will think that these new methods lead to fairly complicated calculations but this is not a problem because the hand calculation period is now over : computer processing of such sophisticated methods will be easier and more confident than hand-processing thanks to the widespread of micro, mini and super computers. And the 1986 Joint Pressure Vessels and Computers Engineers Conference might be one of the best examples.

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BEHAVIOR OF PRESSURE INDUCED DISCONTINUITY STRESSES AT ELEVATED TEMPERATURE

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ABSTRACT

Conventional wisdom has been that discontinuity stresses at temperatures within the creep regime are self-limiting and therefore secondary. A series of creep relaxation analyses were performed on typical discontinuities (junctions). The analyses show that, in general, discontinuity bending moment and stress due to pressure do not relax, but remain more or less constant with time. These stresses should therefore be evaluated as primary. A creep follow-up mechanism was identified that sustains the discontinuity stresses. The implications for design are discussed.

BACKGROUND

Secondary stresses are defined as strain controlled (self-equilibrating). That is, a strain rather than a stress is imposed upon the structure. The principle source of secondary stress is differential thermal expansion. Discontinuity stresses due to pressure are also classified as secondary stresses because they are due to compatibility; they are not necessary for equilibrium.

At temperatures below the creep regime, secondary stresses are typically limited so that the structure will shake down to elastic cycling(1). That is, although the stresses may exceed yield on the initial cycle, a residual stress state will be developed that results in subsequent elastic cycling. In design for shakedown, the stress range is limited to less than twice the yield stress. The classification of discontinuity stresses as secondary was carried into the high temperature nuclear rules of ASME Code Case N-47(2).

An important characteristic of secondary stresses at elevated temperatures is that they relax with time. The presence of stress in the structure, whether it is primary or secondary, results in the accumulation of creep strain with time. Because a secondary stress is strain controlled, there can be no net increase in strain. Therefore, the creep strain replaces elastic strain and thus reduces the stress. This is termed stress or creep relaxation. Because of this phenomenon, relatively high secondary stresses (compared to primary stresses which remain constant with time) may be accepted in elevated temperature design. The stress may start high, but will relax to lower stress levels. In general, because creep damage is highly sensitive to stress, most of the damage due to secondary stress occurs during the first hours of exposure to high temperature, while the stresses are rapidly relaxing. Eventually, the stresses relax to the point where relatively minor further creep strain or damage will continue to accumulate.

In design for primary stress, one must limit the stress to a level that will not result in creep rupture during the design life. However, in design for secondary stress, one may consider the effect of relaxation. Due to creep relaxation, the stress during most of the life will be much lower than the initial stress. It is very conservative to assume that the secondary stress remains constant and limit the initial stress to a value that, if it remained constant during the life, would not result in creep rupture. Some codes, such as the ANSI/ASME B31 piping codes, recognize that secondary (simple thermal expansion stress in the piping code) stresses relax and permit design to higher stress levels than primary limits(3).

In the ANSI/ASME Pressure Piping Codes, the combination of primary and secondary stress is limited to result in shakedown to elastic cycling in the creep regime. That is, they are limited, with the addition of a safety factor, to the hot relaxation stress plus the cold yield stress(3). Design of other components has also been to similar criteria (4,5,6). If a stress is limited to such an allowable, one must be sure that it is truly secondary, that is, it will relax.

Elastic followup is when an increase in creep strain does not result in an equal reduction in elastic strain. Stresses do not relax as quickly as in the case of a pure secondary stress and thus more creep damage will accumulate during a period of relaxation. Various codes (2, 8) require that cases with significant elastic follow-up be evaluated as primary stress. In the limit, where the stress does not relax at all with time as creep strain accumulates, it is behaving as purely primary.

With some Codes and design criteria, the classification is not essential. For example, in Section VIII, Div 1 (7), discontinuity stresses are not calculated at all, but are limited by conservative primary stress allowables and attention to design details. In Code Case N-47, there are two options and proper classification is not critical to either. In the inelastic analysis option, the true stress behavior with time is calculated and no assumption need be made as to whether or not the stress will relax. In the elastic analysis option using Appendix T, no credit can be taken for stress relaxation in any case, so improperly classifying a nonrelaxing (primary) stress as a relaxing (secondary) stress has no effect. The rules already assume the worst case. However, improper classification could lead to misinterpretation of failure data which could lead to undue conservatism in design rules.

The work presented in this paper was performed to determine if discontinuity stresses due to pressure behaved as primary or secondary stress. In design of high temperature pressure vessel internals for the process industry, it is necessary, due to the severity of the service, to take credit for relaxation of secondary stress. That is, use criteria similar to the ANSI/ASME B31.3 Pressure Piping Code (8). Thus, proper classification of the stress (primary versus secondary) is essential to proper design.

METHOD OF EVALUATION

Creep analyses were performed on various discontinuities under internal pressure. The relaxation of the discontinuity stress and bending moment were compared to known cases of primary and secondary stress. The degree of primary versus secondary stress for each discontinuity was determined from the relative rate of stress relaxation.

The evaluations were based on a power creep law (secondary creep only) for Type 304 Stainless Steel (SS). Two types of evaluations were performed: 1) A pressure to give a high initial discontinuity stress (69 MPa effective stress with a 815C temperature) was imposed and 2) the allowable pressure per Section VIII, Div 1 (7) was imposed. The first cases were run based on the assumption that primary stresses would not relax and secondary stresses would relax, regardless of the initial stress or creep rate. The second type of case was run to check the assumption (i.e. make sure that the results were not stress and creep rate dependent). Most of the cases were run with a creep law representative of Type 304 SS at 815C. Some cases were run with a 650C creep law to check for creep law dependence.

The analyses were performed with the BOSOR5 computer code.[1] All were axisymmetric analyses based on thin shell theory. Large displacement effects are included in the formulation of the program. The case of a beam in bending was run on the ABAQUS computer code.

Two variables were used to evaluate the rate of relaxation. These were the stress intensity at the discontinuity (maximum of either inner or outer surface) and the discontinuity bending moment. The stress intensity is best for evaluating the relative rate of relaxation. If the initial stress for each case is the same, they can all be readily compared. This is the reason for the first type of cases run (all with the same initial stress). However, the effect of stress redistribution through the wall thickness must be discounted. This was done by comparing stress relaxation to the case of the outer fiber stress in a beam under pure bending (a cantilever beam with a constant end moment) and a cylinder with axisymmetric meridional end moment.

Evaluating the bending moment was best in clear cut situations. If the bending moment did not decrease at all with time, the case was obviously behaving as primary. However, bending moment was not very good for evaluating the rate of relaxation since the relationship between stress and moment differed from case to case. Bending moment data are presented herein normalized to give an initial value of one. If the value remains essentially one with time, the bending moment is not relaxing and the discontinuity bending stress must be considered primary.

The base cases to which others were compared were as follows:

1) pure uniaxial relaxation (closed form solution) - secondary membrane
2) linear through the wall radial temperature gradient - secondary bending
3) axisymmetric meridional bending moment imposed on the end of a long cylinder - a case of mixed bending and circumferential membrane due to primary load
4) a cantilever beam - pure primary bending

Various discontinuity cases were run. The following are presented herein as they are good, relatively simple examples that show the general behavior that was found:

5) a pressurized cylinder built-in on one end
6) a cylinder to sphere junction where both are designed to the same membrane stress
7) Case 6 but with the sphere thickness tripled to yield one-third the membrane stress of the cylinder

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[1] The BOSOR5 computer program has an internal low creep strain rate cutoff that neglects creep once the creep rate falls below the cutoff. As this cutoff value is greater than the rate associated with primary stresses, it is necessary for the Code allowable pressure cases to remove the cutoff from the program or multiply creep rate by a factor (in the creep equation) and divide time steps by the same factor.