

Edited by

Y Sinai

E Stein

Wolf Prize in Mathematics

Volume 3

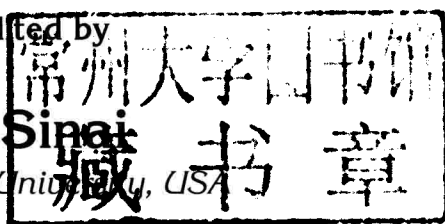
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WOLF PRIZE IN MATHEMATICS — Vol. 3

edited by Y. G. Sinai and E. M. Stein

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List of Wolf Prize Winners

1978	Israel M. Gel'fand	1913–2009
	Carl L. Siegel	1896–1981
1979	Jean Leray	1906–1998
	André Weil	1906–1998
1980	Henri Cartan	1904–2008
	Andrei N. Kolmogorov	1903–1987
1981	Lars V. Ahlfors	1907–1996
	Oscar Zariski	1899–1986
1982	Hassler Whitney	1907–1989
	Mark Grigor'evich Krein	1907–1989
1983/4	Shiing-Shen Chern	1911–2004
	Paul Erdős	1913–1996
1984/5	Kunihiko Kodaira	1915–1997
	Hans Lewy	1904–1988
1986	Samuel Eilenberg	1913–1998
	Atle Selberg	1917–2007
1987	Kiyoshi Itō	1915–2008
	Peter D. Lax	1926
1988	Friedrich Hirzebruch	1927
	Lars Hörmander	1931
1989	Alberto P. Calderón	1920–1998
	John W. Milnor	1931
1990	Ennio de Giorgi	1928–1996
	Ilya Piatetski-Shapiro	1929–2009
1992	Lennart A. E. Carleson	1928
	John G. Thompson	1932

1993	Mikhail Gromov	1943
	Jacques Tits	1930
1994/5	Jürgen K. Moser	1928–1999
1995/6	Robert Langlands	1936
	Andrew J. Wiles	1953
1996/7	Joseph B. Keller	1923
	Yakov G. Sinai	1935
1999	László Lovász	1948
	Elias M. Stein	1931
2000	Raoul Bott	1923–2005
	Jean-Pierre Serre	1926
2001	Vladimir I. Arnold	1937–2010
	Saharon Shelah	1945
2002/3	Mikio Sato	1928
	John T. Tate	1925
2005	Gregory A. Margulis	1946
	Sergei P. Novikov	1938
2006/7	Stephen Smale	1930
	Hillel Fürstenberg	1935
2008	Pierre R. Deligne	1944
	Phillip A. Griffiths	1938
	David Mumford	1937
2010	Shing-Tung Yau	1949
	Dennis P. Sullivan	1941
2012	Michael G. Aschbacher	1944
	Luis A. Caffarelli	1948

Preface

This is the third volume in the series dealing with the winners of the Wolf Prize in mathematics. The present volume covers mostly the period of the first decade of this century. It also includes I. M. Gel'fand, who in 1978 with C. L. Siegel, was a first laureate of the prize, as well as Atle Selberg, J. Thompson, and M. Gromov, each of whom won the prize before 2000.

It may be well to recall some facts about the Wolf Prize. It was founded by Dr. Ricardo Wolf and his wife Francisca Subirana Wolf and is given each year in five of the six areas of Agriculture, Chemistry, Mathematics, Medicine, Physics, and the Arts. Laureates receive their awards from the President of the State of Israel in a ceremony at the Knesset (Israel's parliament) in Jerusalem. About this prize, S. S. Chern and F. Hirzebruch said it well when they wrote: "The Fields medal goes to young people ... The Wolf prize often honors the achievements of a whole life."

As in the previous volumes, the documents collected here are varied in nature but are intended to sketch a picture of the work and scientific life of each of the laureates. As such, these include bibliographies and curricula vitae, particularly noteworthy papers or surveys, as well as articles written about the winners, whenever these were available.

It is our pleasure to record here our appreciation to the Wolf foundation. We also wish to express our thanks to World Scientific for their role in publishing this volume, and in particular to the help we have received from Ms. E. H. Chionh.

Yakov G. Sinai
Elias M. Stein

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Curriculum Vitae

Vladimir I. Arnold

Date of Birth: June 12, 1937, Odessa, USSR

Died: June 3, 2010, Paris, France

Education:

- 1954–1961 Student of Mathematics, Moscow State University, Moscow USSR
- 1959 M.S. Diploma work: “On mappings of circle to itself”; under the supervision of Prof. A. N. Kolmogorov
- 1961 Dr. Thesis: “On the representation of continuous functions of 3 variables by the superpositions of continuous functions of 2 variables”, under the supervision of Prof. A. N. Kolmogorov; Referees: Prof. A. G. Vitushkin and Prof. L. V. Keldysh
- 1963 Ph.D. Thesis: “Small denominators and stability problems in classical and celestial mechanics”; referees: Prof. N. N. Bogoliubov, Prof. V. M. Volosov and Prof. G. N. Duboshin

Employment History:

- 1961–1986 Moscow State University, Mech. Math. faculty: assistant, dozent, professor
- 1986–2010 Steklov Mathematical Institute, Moscow
- 1993–2004 Universite’ Paris 9, France

Member of Academies:

- 1976 Honorary Member of London Math. Soc.
- 1983 French Acad of Sci.
- 1984 Foreign Associate, National Academy of Sciences, USA
- 1986 USSR Academy, corresponding member
- 1987 Academy of Arts and Sciences, USA
- 1988 Royal Soc. of London
- 1988 Acad. Lincei Roma
- 1989 American Philosoph. Soc.
- 1990 Full member of the Soviet Academy of Sciences
- 1991 Russian Acad. of Natural Sciences

Doctor Honoris Causa:

- 1979 Univ. P. et M. Curie Paris
- 1988 Warwick (Coventry)
- 1991 Utrecht
- 1991 Bologna

Awards:

- 1958 Moscow Math. Soc. Prize for Young Mathematicians
- 1965 Lenin Prize (with *A. N. Kolmogorov*)
- 1982 Crafoord Prize of the Swedish Acad. (with *L. Nirenberg*)
- 2001 Wolf Prize in Mathematics
- 2007 State Prize of the Russian Federation
- 2008 Shaw Prize in Mathematical Science

List of Publications

1. V. I. Arnold, Gibbs phenomenon for Fourier series on finite sets, *Funct. Anal. Other Math.* **3** (2010), No. 1, 91–96.
2. V. I. Arnold, Periods and Young's diagram of Fermat–Euler's geometrical progressions of residues, *Funct. Anal. Other Math.* **3** (2010), No. 1, 21–38.
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