GAME THEORY WITH APPLICATIONS TO ECONOMICS

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Preface

In writing this book, I have tried to reach an honors undergraduate or first-year graduate school audience in economics, having a moderate background in mathematics. The book discusses game theory, with examples from economics and sometimes from politics. It is, to quote a source I cannot now recall, "introductory but not elementary." It presumes no prior knowledge of game theory, but many topics are handled in considerable depth. There are two major respects in which the book is aimed at an economics audience. First, the selection of topics is influenced by my views of what is particularly fruitful for economics and, second, the examples are drawn mainly from economics in a way intended to illustrate the breadth and depth of game theoretic influence on that discipline. In addition to serving as a text for courses in game theory, I hope that this book will prove helpful to economists who have not specialized in game theory, and who wish to gain a knowledge of useful developments in the field.

The book can be read on any of several levels, depending on the mathematical background of the reader and on her or his willingness to work hard. A reader with only a working problem-solving knowledge of calculus, and with the ability to accept prose occasionally littered with mathematical symbols, should be able to understand all the examples and to follow everything except the proofs of theorems and lemmas. The proofs themselves vary greatly in their length and difficulty. Undoubtedly, many readers will find some proofs easy and others impenetrable. Although I intended to keep the mathematical depth uniformly modest, I soon found that doing so would force certain topics to be either omitted entirely or treated with insufficient completeness. Consequently, I compromised by including topics of varied technical difficulty. The most mathematically difficult portions are explained in words that give the reader an intuitive grasp.

The coverage of the book is conventional in some respects and particularly up to date in others. Chapters 1, 2, 5, and 6 provide most of the conventional material. Chapter 1 is introductory, Chapter 2 deals with two-person, zero-sum games and with (Nash) noncooperative equilibrium for n-person noncooperative games, touching on both uniqueness of equilibrium and on games of incomplete information as well. Chapter 5 is devoted to two-person cooperative games, and Chapter 6 covers solution concepts for cooperative games having transferable utility. The most recent material occurs in Chapters 3 and 4, which are on noncooperative supergames and include trigger strategy equilibria, cooperation supported by self-enforcing agreements, and refinements of the Nash noncooperative equilibrium such as perfect equilibrium and sequential equilibrium. Chapter 7 is primarily concerned with the generalizations of the core and the Shapley value to nontransferable utility games.

Many writers have commented on their great intellectual debt to others, and my debt must be as large as most. In addition to the influence over the years of many teachers, colleagues, and students, there are two people whom I particularly want to single out: the late William Fellner and Martin Shubik. Willy taught the first-year graduate theory course at Yale when I entered the graduate program there. He was a man of great thoughtfulness and subtlety: in that course came my first introduction to a game-theoretic topic when we studied oligopoly. That provided a glimpse into an interesting area that shortly became a fascination. The following year, Martin Shubik was a visiting professor who taught a game theory/oligopoly course in which his knowledge, point of view, insight, and enthusiasm influenced my main lines of interest in economics.

Several people have read parts of an earlier draft of this book and have given me helpful comments, including Catherine Eckel, Nicholas Economides, Val Lambson, Douglas McManus, John McMillan, David Salant, Patricia Smith, Richard Steinberg, Manolis Tsiritakis, Chang-Chen Yang, and Allan Young. They eliminated typographical errors, improved the clarity of exposition, and corrected my errors. Robert Rosenthal read most of the previous draft and provided comments that helped me eliminate much nonsense and much murky prose. At many points in the text he has saved me from serious error. Some typing was done by Barbara Barker, Irene Dowdy, and Wadine Williams. I am grateful to them for pitching in when there was some time pressure. Most of the typing was done, with great skill, speed, and good humor, by Vickie Carroll and Jay Willard. Much of what is good in the following pages results from the help of all these people. Any blame, of course, for any remaining mistakes and inaccuracies rests with me.

Chapel Hill, North Carolina
June 1985

J. W. F.

A note on other books in game theory

There are many other books from which a reader might gain greatly, a few of which are Luce and Raiffa (1957), Owen (1982), Shubik (1982, 1984), Roth (1979), and van Damme (1983). Luce and Raiffa have written the sort of classic book to which many of us must aspire. Despite being much out of date, due to the developments of nearly 30 active years, it is a superb source for much of the central material of the field. The writing is extremely lucid, the intuition supporting various models and the criticisms of them is insightful and illuminating, and the technical demands are never more than the necessary minimum for the subject. Owen has a fine modern text that is particularly strong on cooperative game theory. Shubik's two volumes, totaling over 1200 pages between them, cover immense ground in game theory proper and on applications of game theory to social science disciplines. Shubik surely provides the most comprehensive coverage of game theory and applications by a single hand and will be an invaluable source to serious students of the subject. Roth and van Damme's books are specialized in scope, each treating in a unified way a topic of great importance that has developed in the recent past. Roth's monograph is on axiomatic bargaining models; van Damme's is on refinements of the noncooperative equilibrium.

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Introduction to games

In Samuel Johnson's (1755) dictionary, the first definition of the noun *game* is "sport of any kind," and our modern notion typically would add that games usually have some particular rules associated with them. Examples include athletic games such as soccer, golf, basketball, and tennis; card games such as bridge, poker, and cribbage; and board games such as chess, backgammon, and Go. Most of these games share an interactive and competitive element. That is, a player strives to outdo the other players in the game, and her success and effectiveness depend on the actions of the remaining players, as well as her own actions. For instance, in playing tennis, one does not merely try to hit the ball back to the other player, one tries to hit it back so that the other player cannot return it, so where the ball is aimed will depend on where the other player is placed. An exception to the interactive and competitive element in the preceding list of games is golf, where each player struggles against an absolute standard, and the actual performance of a player does not depend on the actions chosen by other players.

There are several features that are typical of most games. First, games have rules that govern the order in which actions are taken, describe the array of allowed actions, and define how the outcome of the game is related to the actions taken. Second, there are two or more players, each of whom is struggling consciously to do the best he can for himself. Third, the outcome to a player depends on the actions of the other players. The player knows this, and knows that choosing the best action requires making an intelligent assessment of the actions likely to be taken by the other players.

These general characteristics of games typify many situations in life that are not *games* in the sense of being *sport*. For example, when the management of a company and the leadership of a union face each other over the bargaining table to work out a new contract, they are in a gamelike situation. The rules are not as formal and detailed as for chess, but there are rules. Offers and counteroffers are made with a view on each side to making the final settlement as favorable as possible, and what offer one side should make to best further its interest depends on just how that offer will be received and responded to by the other side. In 1979, the new head of the International Harvester Company bargained so hard with the United Auto Workers Union that he went far beyond the bounds of what the union found reasonable. Union members had the impression that he wished to kill their organization. After a long and bitter strike, there was a settlement; however, the union became so intransigent at the subsequent contract negotiations three years later that they did not settle with the company until (coincidentally) that same head resigned from his job.

Many people face bargaining situations at one time or another, for example, when buying or selling a house or an automobile. These are games in quite the same sense that the labor-management situation is a game.

The first important theorem in game theory, the *saddle point theorem* for two-person, zero-sum games, was published by von Neumann (1928). This was followed by the rich collaboration culminating in von Neumann and Morgenstern (1944), which contains approaches to the treatment of many kinds of games along with much discussion of the potential applications of game theory. The early history of game theory, going back to von Neumann's precursors, is discussed in Rives (1975).

1 Examples of games

Several illustrative games are described below. The first is an oligopolistic market with three firms, and the two following are political science models of the election process. All of these are *noncooperative games*, which means that the players are unable to make contractual agreements with one another. The final example is a labor-management dispute whose outcome should be a contract signed by the two players.

1.1 A three-firm computer market

Suppose a market in which the number of active firms is not very large, but is more than one. Imagine firms in the rapidly changing computer industry. Each firm must decide how to direct its efforts in developing and marketing new equipment, and the best plan for one to follow is dependent on the plans adopted by all of the others. A small- to middle-sized firm might be able to seriously tackle the market for just one size of computer, and, to survive, it must select a size that will not have many competing firms. Meanwhile, problems of product development require that it commit itself to a course of action years in advance and before it can possibly have a clear idea of what the others have chosen. As a simple numerical illustration, imagine that there are three firms, each of which can choose to make large (L) or small (S) computers. The choice of firm 1 is denoted S_1 or L_1, and, similarly, the choices of firms 2 and 3 are denoted S_i or L_i where $i=2$ or 3 indicates the firm. Table 1.1 shows the profit each firm would receive according to the choices which the three firms could make. For