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Lectures on Microeconomic Theory

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LECTURES ON MICROECONOMIC THEORY

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LECTURES ON MICROECONOMIC THEORY

ADVANCED TEXTBOOKS IN ECONOMICS

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Introduction to the series

The aim of the series is to cover topics in economics, mathematical economics and econometrics, at a level suitable for graduate students or final year undergraduates specializing in economics. There is at any time much material that has become well established in journal papers and discussion series which still awaits a clear, self-contained treatment that can easily be mastered by students without considerable preparation or extra reading. Leading specialists will be invited to contribute volumes to fill such gaps. Primary emphasis will be placed on clarity, comprehensive coverage of sensibly defined areas, and insight into fundamentals, but original ideas will not be excluded. Certain volumes will therefore add to existing knowledge, while others will serve as a means of communicating both known and new ideas in a way that will inspire and attract students not already familiar with the subject matter concerned.

The Editors

Preface

The aim of this book is to help towards the understanding of microeconomic theory, particularly where it concerns general economic equilibrium with its implications for prices and resource allocation. I shall deal with the structure of the theory and briefly discuss its motivation. But I shall make only passing remarks about its practical relevance or about the precepts that have been deduced from it for applied economics.

Like the first one, this revised and extended edition is addressed to students who possess a good background in mathematics and have been introduced to economic phenomena and concepts. But their power of abstraction is not considered high enough to allow them to take immediate full advantage of the most rigorous and condensed works in mathematical economics.† On the other hand, they need some introduction to the many extensions that the theory has received during the past thirty years.

The theoretical exposition does not attempt to achieve the greatest generality that is possible today. Most of the results could be strengthened. But a complete catalogue of the known theorems would be tedious and of only secondary interest to the student. Those who wish to specialise in microeconomic theory must refer to the original works for those questions which they want to investigate more deeply.

On the other hand, the various chapters do cover almost completely the different viewpoints that have contributed to our precise understanding of general equilibrium. The scope of these lectures is satisfactorily defined by the table of contents, without the need for further discussion here.

†Debreu, *Theory of Value: an axiomatic analysis of economic equilibrium*, John Wiley and Sons, New York, 1959; Arrow and Hahn, *General Competitive Analysis*, Holden-Day, San Francisco, 1971.

It follows from my purpose that the proofs of the principal results should be given or at least outlined, since they are essential for the understanding of the properties involved. It makes it equally desirable that the level of rigour currently achieved by microeconomic theory should be respected. Therefore the assumptions used in the main proofs have been stated explicitly even when they could have been eliminated by resort to a more powerful argument. In many cases, where simplicity seemed to be advisable, special models with very few agents and commodities have been used rather than general specifications. In short, the accent is placed on the logical structures of the theory rather than on the statement of its results.

As thus described, the text should be useful to those who are solidly equipped in mathematics, are ready to make the effort required to understand existing microeconomic theory and are not prepared to be content with less rigorous presentations, which are naturally easier but also are responsible for some confusion.

The historical development of microeconomic theory has been only occasionally touched on. To trace and describe the origin of each result would have been to overburden the exposition. The few references given in the various chapters do not pretend to do justice to the authors of the most important contributions, but rather to give the student some indications as to how he may follow up certain questions. When the book is to be used for a course, the teacher will be well advised to prepare a reading list appropriate to the specific needs of his students.

It is a pleasure to acknowledge that once again Mrs. Anne Silvey was good enough to prepare the English translation of my work and to make it both fluent and accurate.

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Conceptual framework of microeconomic theory

1. Object of the theory

L. Robbins put forward the following definition: 'economics is the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses'.[†] Such a statement does not make it clear that economics is a social science which studies the activity of men living in organised communities. It also risks failure to make sufficient distinction between economics and political science, since the terms 'ends' and 'means' may be interpreted in a very general sense.

In a work which follows marxist thinking, O. Lange writes: 'Political economy, or social economy, is the study of the social laws governing the production and distribution of the material means of satisfying human needs.'[‡] There is nothing to say about this very compact definition except that the terms 'social laws' and 'material means' are capable of misinterpretation. The social nature lies in the analysed phenomena, production and distribution, rather than in the permanent relations which we establish between them, and which we call laws. 'Material means', also called 'goods', must be interpreted sufficiently widely to include, for example, the provision of services.

Here we propose the alternative, more explicit definition: *economics is the science which studies how scarce resources are employed for the satisfaction of the needs of men living in society: on the one hand, it is interested in the essential operations of production, distribution and consumption of goods, and on the other hand, in the institutions and activities whose object it is to facilitate these operations.*

The most cursory observation of economic life under the differing regimes which exist today reveals a juxtaposition of large numbers of individuals,

[†] L. Robbins, *Essay on the Nature and Significance of Economic Science*, Macmillan, London, 1932.

[‡] Lange, *Political Economy* (English translation), Pergamon Press, Oxford, 1963.

each acting with some autonomy but within a complex institutional framework which organises their mutual interdependences.

So, in so far as it is a positive, that is, explanatory science, economics must analyse the behaviour of agents who enjoy some freedom but are subject to the constraints imposed on them by nature and institutions. It must investigate the consequences of such individual behaviour for the state of affairs which is realised in the community.

In so far as it is a normative science, economics must also investigate the best way of organising production, distribution and consumption. It must give the conceptual tools which enable us to assess the comparative advantages of different forms of organisation.

In its pursuit of this double activity, positive and normative, our science has come to attribute a central role to the prices which regulate the exchange of goods among agents. For the individual, these prices reflect more or less exactly the social scarcity of the products which he buys and sells. This is why the study of the price system is just as important as the study of production and consumption.

The main object of the theory in which we are interested is the analysis of the simultaneous determination of prices and the quantities produced, exchanged and consumed. It is called microeconomic because, in its abstract formulations, it respects the individuality of each good and each agent. This seems a necessary condition *a priori* for a logical investigation of the phenomena in question. By contrast, the rest of economic theory is in most cases macroeconomic, reasoning directly on the basis of aggregates of goods and agents.

The theory of prices and resources allocation, somewhat improperly called 'microeconomic theory', has now attained a fairly high level of rigour, in the sense that its main sections are constructed from a consistent set of abstract concepts, which provide a formal representation of the society under study. So the reasoning in these lectures will be based on a single general model to which more specific assumptions will be introduced as we proceed. The first task is to define the elements of this model.

2. Goods, agents, economy

'Goods' and 'agents' are the first two concepts. Bread, coal, electrical power, buses, etc., are considered as goods, the quantity of each being measured in appropriate units. Services such as transport, hairdressing, medical care, etc., are also goods since they satisfy human needs. Labour is a good of particular importance since it is an essential element in all production. In relation to it, we should, properly speaking, distinguish as many goods as there are types of labour. We shall speak of 'commodities' interchangeably with 'goods'. These two terms will be taken as equivalent, at least

up to Chapter 10 where it will be convenient to give them different meanings.

The economic activity of individuals is both professional and private; in most cases, professional activity takes place in the context of firms engaged in production; private activity generally occurs within households and involves the consumption of goods for the satisfaction of widely varying needs. It is convenient for the purposes of theory to distinguish the two types of organised cells in which each activity is carried on. So we shall speak of 'producer agents' and 'consumer agents'.

More generally, 'agents' are the individuals, groups of individuals or organisms which constitute the elementary units of activity. To each agent there corresponds an autonomous centre of decision.

Here we shall assume in most cases that the agents can be divided into two categories: 'producers', who transform certain goods into other goods, and 'consumers' who use certain goods for their own needs. The former are also sometimes called 'enterprises' or 'firms'. The latter may represent either individuals, or cells of united individuals who constitute households, or possibly larger social groups pursuing common aims for the direct satisfaction of their needs.

In the model with which we shall mainly be concerned, there exist l commodities, m consumers and n producers. Certain *resources*, which are available *a priori*, can be used either for production or for consumption. Finally, we shall often add to the model the clause that every good has a *price*. Let us briefly examine these notions in turn.

(a) With each commodity, identified by an appropriate index h ($h = 1, 2, \dots, l$), there is associated a definite unit of quantity. The commodity is characterised by the property that two equal quantities of it are completely equivalent for each consumer and each producer. When taking the normative standpoint, we also assume that two equal quantities of the same good are socially equivalent.

We shall often have to consider 'complexes of goods', a complex being defined as a set of quantities of the l commodities, for example, z_1, z_2, \dots, z_l . It is therefore a vector of R^l , z say.

(b) The social organisation of economic activity generally allows individuals to exchange goods among themselves. One of our main objects in these lectures is to understand how these exchanges are carried out. In most of the following chapters, these exchanges conform to prices given to the different goods.

With each commodity, therefore, we associate a *price* which is a positive or zero number. We say, for example, that the price of the h th good is p_h . For the set of goods, we can define a corresponding vector p , the price vector.

By definition, the *value* of a complex z of goods is

$$\sum_{h=1}^l p_h z_h,$$

which can obviously be denoted by pz . Two complexes with the same value are considered to be mutually exchangeable. Thus, z^1 and z^2 are exchangeable if $pz^1 = pz^2$.

Suppose that in particular we have the following two complexes:

$$z^1 = (0, 0, \dots, 0, 1, 0, \dots, 0), \quad z^2 = (0, 0, \dots, 0, x),$$

where the component 1 has the h th position in z^1 . The complexes are exchangeable if

$$p_h = p_l x.$$

So the ratio between p_h and p_l defines the quantity of the good l which must be given in exchange for one unit of h .

In what follows we shall be concerned only with the ratios of the values of different complexes. In fact, in our formulations, the vector p will be defined only up to a multiplicative constant, λp representing the same price vector as p , whatever the positive number λ . We shall verify this in each of the following chapters.

It is sometimes convenient to eliminate this indeterminacy by demanding that p satisfy a conventionally chosen condition. Thus, the price of one commodity is often fixed at 1, and the commodity in question is then called the 'numéraire'. For the purposes of theory, there is no necessity to choose a numéraire; we shall not do this except where explicitly mentioned.

(c) With each *consumer* there is associated an index i ($i = 1, 2, \dots, m$). The activity of the i th consumer is represented by the complex x_i whose components x_{ih} define the quantities consumed of the different goods. The x_{ih} are not necessarily positive; for example, we shall often assume that the i th consumer provides labour of a certain description. This will be represented by negative consumption which appears in x_i as a negative component for the good corresponding to labour of this kind.

(d) With each *producer* there is associated an index j ($j = 1, 2, \dots, n$). The j th producer transforms certain goods, called his 'inputs', into other goods, his 'outputs'. Let a_j and b_j be the vectors which represent respectively the complex of inputs (the a_{jh}) and the complex of outputs (the b_{jh}). The j th producer's *net production* of the good h is, by definition, $y_{jh} = b_{jh} - a_{jh}$. It is positive if h is one of his outputs, negative if it is an input. We shall later consider often the complex of net productions and the vector y_j , without involving inputs and outputs explicitly.

(e) *A priori*, the community has at its disposal certain quantities ω_h of the different goods. These are the *initial resources*, the vector ω of which is one of the data of the situation under study.

Like the notions previously introduced, that of initial resources has some flexibility. Thus, we might conceivably represent the labour provided by the

individuals of the community in two ways. As we have just said, this can be considered as negative consumption by consumers. It can also be considered as an initial resource available to the economy. According to the latter point of view, if h represents labour of a certain kind, x_{ih} is zero while ω_h represents the total quantity of that labour provided by the individuals of the community.

We shall have to introduce many variants of the general model. For example, we shall sometimes assume that the initial resources are privately owned and are therefore in the possession of individual consumers. We shall often simplify our theoretical study by considering a model with no producers, where only the distribution or exchange of goods among consumers is analysed.

Having introduced these initial ideas, we can define formally what we mean by the 'economy'. In fact the definition will vary according to the particular model. Obviously we shall come to elaborate our representation of consumers and producers and to add new concepts. But at this very early stage, we can say that an *economy* is defined by a list of goods, a list of consumers, a list of producers, and a vector ω of initial resources. A *state of the economy* is then defined when particular values are given for the m vectors x_i and the n vectors y_j . In positive theory, where the aim is also to explain how prices are determined, we shall have to introduce a vector p (specified up to a multiplicative constant) when we define a state of the economy.

In this general conceptual context, there are two types of objective for microeconomic theory. In the first place, it must *describe* the activity of agents, that is, it must provide models which explain in abstract terms how each consumer i determines x_i and how each producer j determines y_j , and it must also describe how all the x_{ih} and all the y_{jh} , and possibly also prices p_h , are simultaneously determined. (It must therefore place itself at the level of the individual agent in a partial perspective as well as at the level of the whole economy). This is the objective of *equilibrium theory*, first partial, then general equilibrium.

In the second place, it must look for an *optimal organisation* of production, consumption and exchange, and then study the properties of a state of the economy in which this optimal organisation is realised. This is the objective of *optimum theory*, also called welfare theory.

These are the questions which we shall be discussing in the course of these lectures. Our immediate task is to examine the validity of the general conceptual framework on which all later analysis will be based.

3. Possible interpretations of the concept of a good

What kind of picture of economic reality can we derive from these general concepts?