Teachers Edition

SAXON

Algebra I

An Incremental Development

Algebra I

An Incremental Development

JOHN H. SAXON, JR.

Oscar Rose Junior College

GRASSDALE PUBLISHERS, INC.

Algebra I: An Incremental Development
Teachers Edition

Printed in the United States of America
ISBN: 0-939798-02-6
Third printing, June 1983

Grassdale Publishers, Inc.
1002 Lincoln Green
Norman, Oklahoma 73069

Algebra I

An Incremental Development

To The Student

Algebra is not difficult. Algebra is just different, and time is required in order for different things to become familiar. In this book we provide the necessary time by reviewing all concepts in every Problem Set. Also, the parts of a particular concept are introduced in small units so that they may be practiced for a period of time before the next part of the same concept is introduced. Understanding the first part makes it easier to understand the second part. If you find that a particular problem is troublesome, get help at once because the problem won't go away. It will appear again and again in future Problem Sets.

The Problem Sets contain all the review that is necessary. **Your task is to work all the problems in every Problem Set.** The answers to the Odd-Numbered Problems are in the Appendix. It will be necessary to check the answers to the even problems with a classmate. Don't be discouraged when you continue to make mistakes. Everyone makes these mistakes, and makes them often, and for a long period of time. A large part of learning algebra is devising defense mechanisms to protect you from yourself. If you work at it, you can find ways to prevent these mistakes. Your teacher is an expert because your teacher has made the same mistakes many times and has finally found ways to prevent them. You must do the same. Each person must devise his or her own defense mechanisms.

The repetition is necessary to permit all students to master all of the concepts, and then the application must be practiced for a long time to insure retention. This practice has an element of drudgery to it, but it has been demonstrated that people who are not willing to practice fundamentals often find success elusive. Ask your favorite athletic coach for his opinion on the necessity of practicing fundamental skills.

To The Teacher

The effectiveness of this book was demonstrated during the 1980–1981 school year in twenty Oklahoma public schools. Over 1,360 ninth grade Algebra I students participated. In each school one teacher taught one section from a prototype of this book and one or more sections from the Algebra I book normally used. During the spring, sixteen 10–15 minute tests were given. Each test was on one fundamental skill of beginning algebra and the tests were constructed from problems submitted by the teachers. The topics tested were: signed numbers, evaluation of expressions, solutions of equations in one unknown, adding like terms, number word problems, natural number exponents, factoring, percent word problems, value word problems, addition of rational expressions, simplification of radicals, linear equations, simultaneous equations, and uniform motion word problems. Overall, the students who used this book more than doubled the scores of the students who had the same teacher but used a standard book. For tests on scientific notation and negative exponents, eight classes of Algebra II students were used as controls. The 9th graders who had used this book more than tripled the scores of the Algebra II students on both of these tests. The test program was monitored by the Oklahoma Federation of Teachers, and they have certified the results.

Students should work every problem in every problem set. This book provides only the problems that are necessary and the problems are not in pairs so that either the odd problems or the even problems may be assigned. Experience with this book will demonstrate that no extra problems of the new kind are necessary or desirable. In this book the learning process is spread out and comprehension will come in time. The emphasis is on review and not on an all out attack on the new concept. This development spreads out the learning process, increases the depth of understanding and improves long term retention. Experience with books that use the traditional development of topics has proved that an intensive initial attack on a new concept lends little to long term retention. Remember that this book never drops a topic once it has been introduced and that the student will continue to wrestle with every concept in every homework problem set.

Acknowledgments

I thank the following Oklahoma teachers for their help in conducting the test program: Max Coon in Carnegie; Jim Shields in Cushing; Mickey Yarberry in Del City; Gwen Gaskins in Holdenville; Margaret Spann in Lindsay; Linda Smiley in Madill; Emet Callaway in Marlow; Paula Busking and Elizabeth Drew in Moore; Jack Harding and Annette Gravitt in Oklahoma City; Joyce Fisher in Okemah; Corinne Pitts in Okmulgee; Dennis Lebeda, Anne Boesch, Helen Hull, and Lloyd Gelmers in Ponca City; Hobert Higgs in Purcell; George Beaty in Seminole; David Wade in Stillwater; Virgina Weir in Tecumseh: Calvin Tooley in Wetumka; and Robert Beck in Yukon.

I thank Johnny, Selby, Bruce, and Sarah for their support, suggestions, and contributions. I thank Oscar Rose Junior College for the freedom to experiment to develop new teaching techniques.

I thank Julie Filer for help in reading proof. I thank Frank Wang for reading proof, checking answers, and for his constructive comments. I thank William F. Buckley, Jr. and Bob Worth for their encouragement and help. Lastly, I thank Detia Roe, graduate physicist and expert typist for her typing, patience, and support.

Norman, Oklahoma — John Saxon
August 1981

Contents

	Basic Course	**1**
LESSON *A*	*Addition and subtraction of fractions*	**3**
LESSON *1*	*Real numbers and the number line • Multiplication and division of fractions*	**4**
LESSON *2*	*Sets and operations • Decimal numbers*	**9**
LESSON *3*	*Absolute value • Addition on the number line*	**13**
LESSON *4*	*Rules for addition • Definition of subtraction*	**18**
LESSON *5*	*Opposites and multiple signs*	**22**
LESSON *6*	*Rules for multiplication of signed numbers*	**25**
LESSON *7*	*Inverse operations • Rules for multiplication and division*	**27**
LESSON *8*	*Division by zero • Exchange property for multiplication*	**30**
LESSON *9*	*Reciprocal and multiplicative inverse • Order of operations*	**32**
LESSON *10*	*Symbols of inclusion • Order of operations*	**36**
LESSON *11*	*Multiple symbols of inclusion*	**38**
LESSON *12*	*More on order of operations • Products of signed numbers*	**40**
LESSON *13*	*Evaluation of algebraic expressions*	**43**
LESSON *14*	*More complicated evaluations*	**47**
LESSON *15*	*Terms and the distributive property*	**48**
LESSON *16*	*Like terms • Addition of like terms*	**52**
LESSON *17*	*Exponential notation • Integers as bases*	**55**
LESSON *18*	*Evaluation of exponentials*	**57**
LESSON *19*	*Product theorem for exponents • Like terms with exponents*	**59**
LESSON *20*	*Statements and sentences • Conditional equations*	**61**

LESSON 21 *Equivalent equations • Additive property of equality* **65**

LESSON 22 *Multiplicative property of equality* **68**

LESSON 23 *Solution of equations* **71**

LESSON 24 *More complicated equations* **74**

LESSON 25 *More on the distributive property • Simplifying decimal equations* **77**

LESSON 26 *Fractional parts of numbers* **79**

LESSON 27 *Negative exponents* **81**

LESSON 28 *Zero exponents • Decimal parts of a number* **84**

LESSON 29 *Algebraic phrases* **86**

LESSON 30 *Equations with parentheses* **88**

LESSON 31 *Word problems* **90**

LESSON 32 *Products of prime factors • Statements about unequal quantities* **93**

LESSON 33 *Greatest common factor* **96**

LESSON 34 *Factoring the greatest common factor* **98**

LESSON 35 *Canceling* **100**

LESSON 36 *Multiplying fractions • Minus signs and negative exponents* **103**

LESSON 37 *Graphing inequalities* **106**

LESSON 38 *Ratio* **109**

LESSON 39 *Trichotomy axiom • Negated inequalities • Advanced ratio problems* **112**

LESSON 40 *Quotient theorem for exponents* **115**

LESSON 41 *Distributive property and negative exponents* **118**

LESSON 42 *Like terms and negative exponents* **120**

LESSON 43 *Solving multivariable equations* **122**

LESSON 44 *Least common multiple* **125**

LESSON 45 *Least common multiples of algebraic expressions* **127**

LESSON 46 *Addition of rational expressions* **129**

LESSON 47 *Addition of abstract fractions* **131**

LESSON 48 *Conjunctions* **134**

LESSON 49 *Percents less than 100* **136**

LESSON 50 *Polynomials • Addition of polynomials* **139**

LESSON 51 *Multiplication of polynomials* **143**

LESSON 52 *Percents greater than 100* **146**

LESSON 53 *Rectangular coordinates* **149**

LESSON 54 *Graphs of linear equations* **153**

LESSON 55 *Vertical and horizontal lines* 157

LESSON 56 *Addition of rational expressions* 160

LESSON 57 *Power theorem for exponents* 163

LESSON 58 *Substitution axiom* 165

LESSON 59 *Dividing fractions* 170

LESSON 60 *Set notation • Rearranging before graphing* 173

LESSON 61 *Percent word problems* 177

LESSON 62 *Rearranging before substitution* 179

LESSON 63 *Subsets* 183

LESSON 64 *Square roots* 186

LESSON 65 *Product of square roots theorem* 189

LESSON 66 *Domain* 191

LESSON 67 *Additive property of inequality* 194

LESSON 68 *Addition of radical expressions* 196

LESSON 69 *Simplification of radical expressions* 198

LESSON 70 *Elimination* 200

LESSON 71 *More about complex fractions* 204

LESSON 72 *Factoring trinomials* 207

LESSON 73 *Trinomials with common factors • Subscripted variables* 210

LESSON 74 *Factors that are sums* 213

LESSON 75 *Difference of two squares* 214

LESSON 76 *Scientific notation* 216

LESSON 77 *Consecutive integers* 219

LESSON 78 *Consecutive odd and consecutive even integers • Fraction and decimal word problems* 221

LESSON 79 *Rational equations* 225

LESSON 80 *Systems of equations with subscripted variables* 227

LESSON 81 *Operations with scientific notation* 230

LESSON 82 *Graphical solutions* 233

LESSON 83 *Writing the equation of a line* 235

LESSON 84 *Coin problems* 241

LESSON 85 *Multiplication of radicals* 244

LESSON 86 *Division of polynomials* 247

LESSON 87 *More on systems of equations* 250

LESSON 88 *More on division of polynomials* 252

LESSON 89 *Solution of quadratic equations by factoring* 255

LESSON 90 *Value problems* 259

LESSON 91 *Intercept-slope method of graphing* 261

LESSON 92 *Word problems with two statements of equality* 264

LESSON 93 *Multiplicative property of inequality* 266

LESSON 94 *Uniform motion problems about equal distances* 269

LESSON 95 *Rational expressions* 273

LESSON 96 *Uniform motion problems of the form* $D_1 + D_2 = N$ 276

LESSON 97 *Difference of two squares* 279

LESSON 98 *Pythagorean theorem* 282

LESSON 99 *Distance between two points* 286

LESSON 100 *Uniform motion—unequal distances* 289

LESSON 101 *Square roots of large numbers* 292

LESSON 102 *Rounding off* 294

LESSON 103 *Square root tables* 298

LESSON 104 *Factorable denominators* 302

LESSON 105 *Absolute value inequalities* 305

LESSON 106 *Rational equations* 309

LESSON 107 *Abstract rational equations* 313

LESSON 108 *Equation of a line through two points* 315

LESSON 109 *Functions* 319

LESSON 110 *Functional notation* 325

Additional Topics 331

LESSON 111 *Line parallel to a given line* 333

LESSON 112 *Equation of a line with a given slope* 336

LESSON 113 *Radical equations* 339

LESSON 114 *Slope formula* 343

LESSON 115 *Consistent, inconsistent, and dependent equations* 346

LESSON 116 *Conjunctions and disjunctions* 351

LESSON 117 *More on multiplication of radical expressions* 355

LESSON 118 *Direct variation* 357

LESSON 119 *Inverse variation* 361

LESSON 120 *Linear inequalities* 364

LESSON 121 *Quotient theorem for square roots* 368

LESSON 122 *Advanced trinomial factoring* 371

LESSON 123 *Factoring by grouping* 373

LESSON 124 *Direct and inverse variation squared* 375

LESSON 125 *Completing the square* 377

LESSON 126 *The quadratic formula* 381

Appendix 387

ENRICHMENT LESSON 1 *Closure* 389

ENRICHMENT LESSON 2 *Properties of the real numbers* 393

ENRICHMENT LESSON 3 *Properties of a field* 397

ENRICHMENT LESSON 4 *Algebraic proofs* 403

Enrichment problems 407

Tables of squares and square roots 410

Index 414

Answers 418

Basic

1 Review Lesson
110 Algebra Lessons

Course

REVIEW LESSON A *Addition and subtraction of fractions*

A.A addition and subtraction of fractions

In order to add or subtract fractions that have the same denominators, the numerators are added or subtracted and the result is recorded over a single denominator, as shown here.

$$\frac{5}{11}+\frac{2}{11}=\mathbf{\frac{7}{11}} \qquad \frac{5}{11}-\frac{2}{11}=\mathbf{\frac{3}{11}}$$

If the denominators are not the same, it is necessary to rewrite the fractions so that they have the same denominators.

	Problem	Rewritten with equal denominators	Answer
(a)	$\frac{1}{3}+\frac{2}{5}$	$\frac{5}{15}+\frac{6}{15}$	$\mathbf{\frac{11}{15}}$
(b)	$\frac{2}{3}-\frac{1}{8}$	$\frac{16}{24}-\frac{3}{24}$	$\mathbf{\frac{13}{24}}$

A mixed number is the sum of a whole number and a fraction. Thus the notation

$$13\frac{3}{5}$$

does not mean 13 multiplied by $\frac{3}{5}$ but instead 13 plus $\frac{3}{5}$.

$$13+\frac{3}{5}$$

When we add and subtract mixed numbers, we handle the fractions and the whole numbers separately. In some subtraction problems it is necessary to borrow, as shown in (e).

	Problem	Rewritten with equal denominators	Answer
(c)	$13\frac{3}{5}+2\frac{1}{8}$	$13\frac{24}{40}+2\frac{5}{40}$	$\mathbf{15\frac{29}{40}}$
(d)	$13\frac{3}{5}-2\frac{1}{8}$	$13\frac{24}{40}-2\frac{5}{40}$	$\mathbf{11\frac{19}{40}}$
		BORROWING	
(e)	$13\frac{3}{5}-2\frac{7}{8}$	$13\frac{24}{40}-2\frac{35}{40}=12\frac{64}{40}-2\frac{35}{40}$	$\mathbf{10\frac{29}{40}}$

problem set A Add or subtract as indicated. Write answers as proper fractions reduced to lowest terms or as mixed numbers.

1. $\frac{1}{5} + \frac{2}{5}$
2. $\frac{3}{8} - \frac{2}{8}$
3. $\frac{4}{3} - \frac{1}{3} + \frac{8}{3}$
4. $\frac{2}{7} + \frac{13}{7} - \frac{5}{7}$
5. $\frac{18}{11} - \frac{4}{11} + \frac{1}{11}$

Different denominators:

6. $\frac{1}{3} + \frac{1}{5}$
7. $\frac{3}{8} - \frac{1}{5}$
8. $\frac{2}{3} - \frac{1}{8}$
9. $\frac{1}{13} + \frac{1}{5}$
10. $\frac{17}{15} - \frac{2}{3}$
11. $\frac{5}{9} + \frac{2}{5}$
12. $\frac{7}{8} - \frac{4}{5}$
13. $\frac{12}{13} - \frac{3}{4}$
14. $\frac{17}{20} - \frac{4}{5}$
15. $\frac{14}{17} - \frac{6}{34}$
16. $\frac{5}{13} + \frac{1}{26}$
17. $\frac{4}{7} - \frac{2}{5}$
18. $\frac{4}{7} + \frac{1}{8} + \frac{1}{2}$
19. $\frac{3}{5} + \frac{1}{8} + \frac{1}{8}$
20. $\frac{5}{11} - \frac{1}{6} + \frac{2}{3}$
21. $\frac{3}{8} + \frac{1}{2} - \frac{2}{5}$
22. $\frac{3}{5} + \frac{15}{3} + \frac{17}{10}$

Addition and subtraction of mixed numbers:

23. $2\frac{1}{2} + 3\frac{1}{5}$
24. $7\frac{3}{8} + 4\frac{7}{3}$
25. $1\frac{1}{8} + 7\frac{2}{5}$
26. $6\frac{1}{3} + 1\frac{2}{5}$
27. $8\frac{13}{3} - 2\frac{2}{5}$
28. $411\frac{1}{3} - 24\frac{2}{15}$

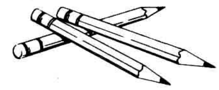
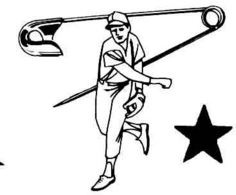
Subtraction with borrowing:

29. $15\frac{1}{3} - 7\frac{4}{5}$
30. $42\frac{3}{8} - 21\frac{3}{4}$
31. $22\frac{2}{5} - 13\frac{7}{15}$
32. $421\frac{1}{11} - 17\frac{4}{3}$
33. $78\frac{2}{5} - 14\frac{7}{10}$
34. $43\frac{1}{13} - 6\frac{5}{8}$
35. $21\frac{1}{5} - 15\frac{7}{13}$
36. $21\frac{2}{19} - 7\frac{7}{10}$
37. $43\frac{3}{17} - 21\frac{9}{10}$

LESSON 1 Real numbers and the number line · Multiplication and division of fractions

1.A numerals and numbers

A number is an idea. A numeral is a single symbol or a collection of symbols that we use to express the idea of a particular number.

The three drawings above all have the quality of *threeness*. The three children and the three pencils both bring to mind the idea of *three*. The drawing at the right also brings to mind the idea of *three*, although all the things in the drawing are not of the same kind.

If we wish to use a symbol to designate the idea of three, we could write any of the following:

$$\text{III}, \quad 3, \quad \frac{30}{10}, \quad \frac{27}{9}, \quad \frac{33}{11}, \quad 2 + 1, \quad 6 \div 2, \quad 11 - 8$$

Each of these is a symbolic representation of the idea of 3. Throughout the book when we use the word *number*, we are describing the idea; and we will use numerals to designate the numbers. But we will remember that none of the marks we make on paper are numbers because

A number is an idea!

Since the symbols

$$3 \quad \text{and} \quad \frac{30}{10}$$

are both numerals that represent the same number, we say that they have the same value. **Thus, the value of a numeral is the number represented by the numeral, and we see that the words value and number have the same meaning.**

1.B natural or counting numbers

The system of numeration that we use to designate numbers is called the decimal system. It was invented by the Hindus of India, passed to their Arab neighbors, and finally transmitted to Europe circa 1200 A.D. The decimal system uses 10 symbols that we call **digits.** These digits are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

We use these digits by themselves or in combination with one another to form the numerals that we use to designate decimal numbers.

We call the numbers that we use to count objects or things the natural numbers or the counting numbers. When we begin counting, we always begin with the number 1 and follow it with the number 2, etc.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, . . .

It would not be natural to try to count by using numbers such as $\frac{1}{2}$ or 0 or $\frac{3}{4}$, so these numbers are not called natural or counting numbers. We designate the natural or counting numbers with the listing above. The three dots after the number 13 indicate that this listing continues without end.

1.C real numbers

The numbers of arithmetic are zero and the positive real numbers. **We say that a positive real number is any number that can be used to describe a physical distance greater than zero.**

Thus, all of the numbers shown here

$$\frac{3}{4} \qquad .000163 \qquad 363 \qquad 3\frac{3}{8} \qquad 46 \qquad \frac{11}{7} \qquad 400.1623232323$$

are positive real numbers, for all of them can be used to describe physical distances when used with descriptive units such as feet, yards, etc.

$$\frac{3}{4}\text{ mile} \qquad .000163\text{ yard} \qquad 363\text{ feet} \qquad 3\frac{3}{8}\text{ meters}$$

$$46\text{ inches} \qquad \frac{11}{7}\text{ kilometers} \qquad 400.16232323\text{ centimeters}$$

The number zero is not a positive number, but it can be used to describe a physical distance of no magnitude, and we say that zero is also a real number. In addition to the positive numbers and zero, in algebra we use numbers that we call **negative numbers** and these numbers are also called real numbers. The ancients did not understand or use negative numbers. A man could not own negative 10 sheep. If he owned any sheep at all, the number of sheep had to be designated by a number greater than zero. The ancients could subtract 4 from 6 and get 2, but they felt that it was impossible to subtract 6 from 4 because that would result in a number that was less than zero itself. To their way of thinking, this was clearly impossible.

While some might tend to agree with the ancients, to the modern mathematician, physicist, or chemist, the idea or concept of negative numbers does exist, and it is a useful concept. **We say that every positive real number has a negative counterpart, and we call these numbers the negative real numbers.** We must always use a minus sign when we designate a negative number as we see here by writing negative seven.

$$-7$$

We may use a plus sign to designate a positive number as we see by writing positive seven,

$$+7$$

or we may leave off the plus sign as we did in arithmetic and just write the numerical part with no sign.

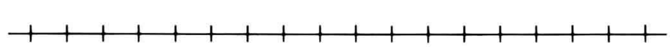
$$7$$

We must remember that when we write a numeral with no sign, we designate a positive number. When we are talking about negative numbers as well as positive numbers, we say that we are talking about **signed numbers.** As we shall see later, the use of signed numbers will enable us to lump the operations of addition and subtraction into a single operation which we will call algebraic addition.

1.D number lines

In the 1950s the so-called new math appeared, and among other things it introduced the **number line** at the elementary algebra level. The number line can be used as a graphic aid when discussing signed numbers, and it is especially useful when discussing the addition of signed numbers.

To construct a number line, we first draw a line and divide it into equal units of length. The units may be any length as long as they are all the same length.



Many books show small arrows on the ends of number lines to emphasize that the lines continue without end in both directions. The arrowheads are not necessary and may be omitted.