

WHO IS FOURIER?

A Mathematical Adventure

Written by Transnational College of LEX

Translated by Alan Gleason

Language Research Foundation
BOSTON

Who is Fourier? A Mathematical Adventure

Published by:

Language Research Foundation, 68 Leonard Street, Suite 9, Belmont, MA 02478-2566 USA E-mail: publisher@lexlrf.org, www.lexlrf.org

First Published 1995. Copyright © 1995 by Transnational College of LEX Second Printing 1997.
Third Printing 1998.
Fourth Printing 2000.
Fifth Printing 2001.
Sixth Printing 2002.

Library of Congress Catalog Card Number 94-79684

ISBN 0-9643504-0-8

All rights reserved. No part of this book may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording, or by any information storage and retrieval system, without permission in writing from the Publisher. The Publisher has made every effort to trace the ownership of all copyrighted material and to secure the necessary permissions. In the event of any question arising as to the use of any material, the Publisher, while expressing every regret for any inadvertent error, will be happy to make any necessary corrections.

Printed in the United States of America

Acknowledgments

Writing and producing the original version and the translation of this book has involved many people. It gives us great pleasure to acknowledge those people who have taken time to help and advise us with our projects.

Advisors to the English Version

Yoichiro Nambu, Ph.D., Elementary Particle Physics at Enrico Fermi Institute, University of Chicago

Mayumi Morimoto, (Ph.D. Program), Statistics/Probability, Department of Mathematics, Boston University

Jerry Lemieux, BSEE, MSEE, PhDEE, Massachusetts Institute of Technology Japanese Language Services, Boston

Advisors to the original Japanese Version

Genpei Akasegawa, artist, writer, photographer

Jiro Ohta, Ph.D., Cell Biology, President of Ochanomizu University

Tan Sakakibara, Automatic Control Engineering, Director of LEX Institute

Hiroshi Shimizu, Ph.D., Biological Complexity and Information, The Ba Research Institute, Kanazawa Institute of Technology & International Media Foundation

Yusuke Tsukahara, Technical Research Institute in Toppan Printing Co.

Yao Nakano, Senior Fellow at the LEX Institute

Teru Hayashi, Ph.D., Control and System Engineering, Professor Emeritus of Tokyo Institute of Technology / Professor of Toin University of Yokohama

Toyoo Maeda, Ph.D., Metallurgist, IHI Research Institute Vice-Director

Junichi Miida, Ph.D., Nuclear Engineering, Japan Atomic Energy Research Institute (1956-1981), Deputy Director / Nuclear Science and Engineering, OECD Nuclear Energy Agency, Paris (1976-1979), Deputy Director

Shigeyuki Minami, Ph.D., Electromagnetism, Osaka City University

Kazuo Yamazaki, Ph.D., Theoretical Physics, Professor Emeritus of Kyoto University / Professor of Kobegakuin University

Keiko Nakamura, Ph.D., Biohistory, Biohistory Research Hall Deputy Director General / Professor of Osaka University

Takao Saito, Ph.D., Space Physics, Professor Emeritus of Tohoku University

Hippo Family Club Members who joined in the Fourier Lectures in Tokyo, Nagoya and Osaka

Cover

Design by Rodelinde Albrecht. Illustration by Nataliya Gurshman.

Fourier illustration from, *Portraits et Histoire des Hommes Utiles, Collection de Cinquante Portraits*, (Société Montyon et Franklin, 1839-1840) in Boston Public Library.

FOREWORD TO THE ENGLISH-LANGUAGE EDITION

Fourier mathematics is a powerful means of analyzing phenomena, such as light, sound, vibrations, and heat conduction, that take the form of waves. Sound can be defined as waves of pressure that vibrate the air and are thereby transmitted. Words uttered by human beings are of course a kind of sound, and therefore can be described as waves. A sustained vocal sound, such as a vowel, consists of repetitions of the same wave pattern. Fourier formulas come in very handy when we wish to analyze the structure of these sounds.

From the first stages of its production, the original book was never intended to be sold in regular bookstores. It was conceived and written for the purpose of relating the experiences of the students of the Transnational College of LEX to as many Hippo Family Club members as possible. Those experiences included many surprise discoveries and emotional moments encountered on their adventure into the field of Fourier mathematics. To us, the members of Hippo are the same as relatives. This book was meant for the rest of our family. That the book was produced by amateurs is evident in its appearance. It does not look like a typical "book". In addition to the lack of consistency between chapters and the jagged letters produced by our word processor, the use of jokes and foreign words which could only be understood by Hippo members lined the pages.

The students were divided into groups and each group studied a different aspect of Fourier. The first drafts for each chapter were written by the most mathematically inexperienced first year students of each group. Watching them develop an understanding of the subject and begin to use its jargon bonded everyone together. There were many emotional moments in this process which could never have been seen in a gathering of mathematicians. Every student in the group explained in detail how they came to understand each step. They methodically explored the individual concepts. This book is as much about the process of learning as it is about Fourier mathematics. That may explain why readers are able to readily understand the methods employed.

At first, the students took up the study of Fourier mathematics simply to obtain some basic knowledge to help in their analysis of the voice. But the deeper they plunged in their study of the phenomena of sound, the more they realized that this mathematics and its formulas were a bona fide language for describing phenomena such as the familiar trigonometric functions sine and cosine; differentiation for finding the velocity or acceleration of a moving object; integration for finding the distance moved by an object; the

imaginary number *i*, so useful in calculations; the base *e* of natural logarithms, which has a special significance in differentiation and integration; vectors, which have both magnitude and direction; and Maclaurin's expansion, which allows the conversion of any expression into a single format. All of these concepts, which are normally learned about separately when studying physics or mathematics, appear on-stage together in the drama of Fourier's wave analysis.

We, the student authors, made absolutely no efforts to advertise the book to the public. Its popularity spread by word of mouth. Now, that same word has led to the publication of the English language edition. Thinking back about a book that began with the printing of only 700 copies and now

has sold over 60,000 copies, and seeing how far it has come makes us inexpressibly happy.

We have also been blessed with the chance to have the English version checked for technical errors by Dr. Yoichiro Nambu of the University of Chicago, a world renowned researcher in elementary particle physics, and a number of other prominent scholars. However, any errors in this work are completely our responsibility. Thanks to this book we have met many people and will treasure those ties. We look forward to encountering the many new readers of this book someday in the future.

January 1995 Transnational College of LEX

INTRODUCTION TO THE ENGLISH-LANGUAGE EDITION

"Behind the chaos lurks a simple order"

changing. But behind the eternal fluctuation of natural phenomena lurks an immutable order. This natural order of things is very simple. Seasons repeat, progressing from spring to summer to fall to winter, then back to spring again. Water falls from the sky as rain, flows from high places to low, evaporates, rises to form clouds, then falls to the earth as rain again. Nature is constantly recreating itself, but always in accordance with a set of clear, concise natural laws. This is the concept underlying the ancient Chinese text *I Ching*, or *Book of Changes*.

Since time immemorial human beings have been asking themselves, "What is nature, and what is my place in the natural order?" We continue to ask the same questions today. But we have discovered a language that allows us to precisely describe, to some degree at least, the order that underlies the vicissitudes of nature. That language is what we call natural science.

There is a broad range of possible meanings in an expression as simple as

"You're crazy!" Depending on the speaker and the situation, it could mean "You're an interesting character," "You're really cute," or even "You're a complete idiot!" This broad fluctuation in meaning is what makes everyday language so richly expressive. However, broad ambiguity becomes a liability when we try to use language to describe the order behind the fluctuation. For this purpose we have a language made up of words with a narrow range of meaning — clear, unambiguous words. That language is mathematics. The clarity and brevity of mathematics were critical to the success of the new science of quantum mechanics as well. One of the pioneers of quantum mechanics, Dr. Werner Heisenberg, was heard to have repeatedly told his students the following:

"Use ordinary, everyday language when you analyze or debate a concept. When the image that emerges becomes so clear that anyone can understand it, that is the time to start thinking of an applicable formula for it." Of course, first you must have a full command of the language of mathematics in order to do this.

Mathematics is nothing less than a powerful language devised by human beings for the purpose of describing natural phenomena.

The purpose of the Transnational College of LEX (TCL) is to provide a venue for the scientific study of languages and of the human beings that speak those languages.

The heart of all research conducted by LEX is the Hippo Family Club, of which every student at the College is a member (see Note). The Hippo Family Club provides its members with the opportunity to acquire multiple languages by a natural method. When the Club was founded in 1981, its Japanese-speaking members started out by simultaneously learning English, Korean and Spanish. By 1988, when this book was written, the list had grown to include German, French and Chinese, for a total of seven languages. Russian and Italian were added in 1990, and Malaysian and Thai in 1991. Thus Hippo members now practice speaking eleven languages at the same time. At first glance this may seem unrealistic from the standpoint of conventional thinking. But the number of languages is insignificant. What the Hippo experience has done is radically alter the way people view languages, and with it the way they comprehend them.

n infant raised in an environment that Lexposes him to a spoken language will. in time, learn to speak that language on his own. This is an absolute given, perhaps even a defining characteristic of human beings. If a child is exposed to a new linguistic environment at any point up to about five or six years of age, it will only be a matter of months or a year before he can speak the new language as well. We have all noticed that people who come to Japan from places like Africa or India, where many languages are spoken, seem to pick Japanese up almost instantly. This is true of adults as well as children. If the language is spoken by human beings somewhere, it can be acquired naturally by anyone.

"Language spoken by human beings" may sound redundant, but it is a key point. For all of their superficial differences, languages share the common definition of being spoken by humans. They share certain characteristics precisely because they are all a form of human language.

Consider another kind of language, or form of self-expression, if you will — music. However strange the melody or rhythm might be, people intuitively recognize it as music, made by human beings just like themselves. Just as people from multilingual environments are quick to learn new

languages, people already familiar with a variety of melodies and rhythms learn new musical forms with ease.

In multilingual regions of the world, that is, regions not dominated by one or even two languages, people are constantly exposed to speech that may be unfamiliar to them. People who grow up in such an environment are instinctively curious about new languages. To them, it is perfectly natural to want to figure out and understand what the unfamiliar words mean. By contrast, people raised in regions where one language dominates tend to write off foreign tongues as something beyond their comprehension. They unconsciously shut their ears to new sounds.

"How did you learn to speak Korean so well? You sound just like a native!"

Hippo Family Club member Mrs. Ogura was overjoyed when her homestay guest from Korea, Mrs. Kim, complimented her in this manner. This is what Mrs. Ogura later told her fellow club members:

"When Mrs. Kim said I sounded just like a native Korean, I guess that meant she thought my pronunciation was perfect. But I've never once studied how to pronounce Korean! When I first listened to the Hippo language tapes, I would just hum the sounds I heard as if they were a song. Gradually I was able to distinguish individual sounds, and then I got to the point where I could mimic them. Even now, if someone told me to enunciate the vowels and consonants of Korean one at a time, I probably couldn't do it!"

"There are still so many Korean words whose meanings I don't exactly know. A year ago, I could only repeat most of the sounds I heard on the Korean tapes without knowing what any of them meant. When I met some Korean friends back then, I would speak to them in their language even when I wasn't sure of the meaning of the words I was using. But they understood! I was so happy! From then on I spoke Korean as often as I could, and gradually I figured out what the different phrases meant. The more I used them, the clearer they became."

"Now, if I really sounded just like a native to Mrs. Kim, that means my grammar must have been correct too. But I've never studied Korean grammar either..."

"During the one week she stayed with us, Mrs. Kim kept drawing Korean sounds out of my mouth like a magician pulling rabbits from a hat. At some point, what were at first just sounds to me turned into words I could understand. And even more mysteriously, I found that Mrs. Kim could utter a Korean word or expression I'd never heard before, and after hearing it only once or twice, I'd

start using it too, without even thinking about it! Once I'd used a phrase myself, I felt I understood not only its basic meaning, but its more subtle nuances as well. It was as if I'd somehow acquired a magic box in my head for catching and storing Korean. Even after Mrs. Kim left, I could still hear in my head the sounds of the Korean we spoke together."

Listening to Mrs. Ogura tell her story was like hearing a little child describe her first encounter with language. Indeed, it was a glimpse of the child that lives on inside every adult.

We take it for granted that a baby in a natural environment where language is spoken will acquire that language on his own. But what are the characteristics and conditions that define this environment, this learning process? These are questions that need to be answered.

If we take Mrs. Ogura's experience and apply it to the way a baby learns a language, we can interpret it in the following way: All human languages, despite their superficial differences in sound and structure, share an elegant and universal order. The set of sounds that make up the Korean language has its own distinctively Korean melodies and rhythms. But Korean is a human language. Consequently any human should be able to speak Korean. If you repeatedly

listen to it and try speaking it, you will be able to distinguish and mimic smaller and smaller components of what started out as just a vaguely Korean-like sound to you. Among the sounds that make up human language, there is none that the human ear cannot, through repeated listening, eventually isolate from the whole and identify as a discrete sound. And once you have identified a sound, learning how to reproduce it with your own mouth will come easily. If the sound is part of a natural language, it should not require any undue effort, such as muscle strain or the like, to reproduce. It is the same with music; if you listen to any song often enough, you will eventually be able to sing it. The individual sounds that make up a language are never separate from that language; they are always embedded within it. You cannot simply string a bunch of discrete sounds together and hope to make sense. The whole is always greater than the sum of its parts.

If our analysis is correct, then the pronunciation drills of conventional foreign language study reflect woeful misunderstanding of the natural process of human cognition. Rather, they appear to be an unnatural, artificial form of pronunciation exercise. How can you form your mouth in the right shape to produce a sound that you haven't or can't even hear yet? Whatever

language you acquire, you must first start by listening to the whole. As you gradually break this down into ever-smaller parts, pronunciation will come naturally.

language initiate an anticipated response. So the sounds of a language by their very nature must have meaning. Yet at the start of the language learning process, we may articulate sounds without knowing what they mean. Babies provide a good example of this. Adults, however, may even be met with surprise or ridicule, with others asking, "Do you actually know what you're saying?" The temptation is to run to the dictionary for help...But babies don't use dictionaries! The meanings to be found in dictionaries are obviously derivative, secondary to the primary meaning a baby understands.

Without a dictionary, we have no choice but to guess at a word's meaning and try it out in conversation. If we aren't understood, then we guess and try again. Oh, the joy when we're understood! This is the natural way for human beings to learn any language. We unconsciously attempt to find connections between the sounds of language and the environment in which we hear them. In time, we acquire the urge to try speaking those sounds ourselves.

Out of the mouths of two- and three-yearold toddlers fly expressions that sometimes astonish grown-ups. The child may not actually understand what he's saying, but if he tries it out and it gets the proper response, he begins to acquire a hazy sense of the meaning of the sound.

Suppose we were to ask this infant, "Do you understand the meaning of everything you are saying? He would undoubtedly answer, "No...what's meaning?" If we continued to ask, "Are the words you are now using the sum total of everything you can say?" He would undoubtedly reply, "Absolutely not! There are plenty of sounds I possess that will come forth from my mouth when the situations arise."

By the time he turns three years of age, words will be pouring forth from his mouth.

Mrs. Ogura elaborates from the perspective of an adult: "In language, there is no such thing as a meaningless sound. When you try using an unfamiliar sound or phrase, you're just taking a guess at its meaning. If it meets with a good response, that gives you an inkling, at least, of what it means. So you try using it again. But 'taking a guess' is actually an unconscious process. It's as if the environment or situation, the context in which you're using the language, draws the phrase out of you. This is possible because you already have the

sound itself inside you. If you learn a language by this natural process, you won't find yourself uttering sounds that are just meaningless imitations of language."

Like the process by which we master the sounds of language, the process by which we discover their meaning begins with a vague, overall impression. From there we move forward little by little, fitting pieces into the puzzle until a clear picture emerges from the connections we have established between these sounds and their environment. Indeed, this is the function languages are supposed to fulfill for human beings, both young and old, that exist in a world of language.

Tumanity did not recognize that there were rules we subsequently called grammar until after it had invented letters. and had begun to write down the language that until then had existed only in speech. Only when words could be written and read over and over again could people have noticed that language has its own remarkable order. It must have been a shocking discovery, that the words we speak so unthinkingly adhere to a strict set of rules. These rules were not consciously invented by us. Somehow, humanity, which we can perhaps define as a phenomenon of nature that speaks language, created language with a structure ideally suited to the cognitive

capabilities bestowed upon our species by nature. Once our ancestors discovered the rules of so called 'classical' grammar and defined them, they must have had a notion that languages concealed an even deeper, more subtle structure. When people — even a child or an adult who knows nothing of grammar — learn language by the natural process, they basically do not make mistakes. Indeed, most of us are blissfully ignorant of the rules of our mother tongue. When we write something in our native language, all we have to do is first try speaking the words, and a suitable expression spontaneously emerges from within the natural flow of sounds and language.

When we acquire a language by natural means, we do not learn to read or write until we have reached a certain stage in our verbal development. We can only create letters once we have speech. This is because the written letters themselves do not convey the sounds of language. Depending on the written word prematurely tends to make us view language from a distance. First we must feel comfortable with the spoken word. Then, when the time is right, we are ready to work with letters.

We have suggested that an order common to all human language lies behind the apparently random sounds of different spoken languages. Surely a similar order can be found in the seemingly disparate grammars of all languages. Grammar, the structure of the patterns we detect in the repetition of sounds in every language, must share certain universal traits as well.

Today's concept of grammar resembles the approach to natural science in the days of classical biology, every part is classified and isolated from the organism as a whole. Likening grammatical structure to the human skeletal form, modern grammar isolates and names each bone or joint, then sets about assembling the adult human skeleton like so many building blocks. This is how contemporary grammarians regard language. But this is not how language is put together. The entire skeleton already lies latent in the structure of a single fertilized egg. As the embryo grows, so do the various parts. together forming a whole which eventually grows into an adult.

Unlike modern grammar, modern biology has long since moved beyond the confines of natural science's classic building block approach. In such fields as molecular biology, scientists are now beginning to seek universal definitions that will answer such questions as "What is life?" and "What does it mean to be alive?"

Tippo Family Club members today ■ speak of creating a fertile environment for acquiring languages. We call it a "language place (言語場)." The language place is created by the people that participate in this "place." Consider a newborn baby. We tend to think of the infant as an incomplete person, an empty vessel yet to be filled, but this is far from the truth. A new language place immediately arises around a newborn child. The mother tries to discern the meaning of her baby's every glance. gesture, murmur or cry. And these sounds and expressions do have meaning; they are already a form of language. Human languages can be defined as a means of being able to create and carry meaning.

From the day the baby is born, he becomes an active and central participant in the new language place. The adults around him find themselves making adoring, richly expressive utterances they would never use if the baby were not there. The baby draws this language from the mouths of the adults, all the while actively taking in their facial expressions and gestures, and of course the sounds they make. The baby murmurs these sounds to himself, instinctively seeking out the meaning of the speech he hears, and eventually finding it. In this sense, babies are all very talkative participants in this language place.

This language place cannot exist by itself, but only as the creation of the people who engage in it. The place is built by the cooperative process of people seeking, finding and creating language together. Many Hippo members participate in club activities as entire families. The family unit is, after all, the first and closest language place available to people.

Five-year-old Taro moved with his family from Japan to the United States when his father was transferred overseas by his employer. On his first visit to the neighborhood park, Taro met an American boy who was playing and shouting in a totally unfamiliar language. Another Japanese boy who had arrived in America a year or so earlier was shouting back in the same strange tongue. Young Hanako, who had been in the country three or four months, was also playing with the boys and saying something too. The park provided no beginner's lessons, no instructors. There was nothing for Taro to do but jump in and start playing with the other children. In a short time, he too would be a central member of this language place. Within a year he would become a fluent speaker of the dominant language of this place. A language place is created by the participation of people in real life, everyday activities.

When our colleague Mrs. Ogura began speaking Korean with her friend Mrs. Kim, it was not because Mrs. Kim was setting an example for her. Rather, it was because her visit to Japan provided the opportunity for Hippo club members to form a new language place with Mrs. Kim at its center. In the process of creating that place, the Korean that had only been a set of sounds to Mrs. Ogura formed a link with her environment and began to flow freely from her mouth.

Not everyone who participates in Hippo club activities starts speaking new languages this quickly. Some find the process easier than others. The reason for this has gradually become clear, however. Those who immediately try to speak the sounds they hear, just as a baby does, improve rapidly. The ones who make little progress are those who succumb to an adult's selfconsciousness and are afraid they will make mistakes or sound foolish. It is important to be surrounded by kindred spirits who will listen attentively to the sounds you make. Conversely, it is listening to the sounds they make that stimulates your own speech. We acquire our language from others around us.

A three-year-old child is capable of understanding and saying just about anything. All the words he has learned at this point were picked up from the people around him. Using the sounds he was given, he goes on to create his own unique world with them. The ability to speak a language occurs as a result of living in a rich language place.

The Hippo language group is unique. It ranges from first-day newcomers to veterans who have been members for over ten years and speak a number of languages quite well. The continuing practice of Hippo group participants is simple: to produce as fertile and creative a language place as possible together. It must be a place grounded in everyday reality where multiple languages fly through the air unrestrained. This is the natural learning environment for languages that we seek.

Choosing to work on ten or more languages at the same time in the Hippo Family Club activity has proven to be an excellent strategy. It effectively prevents anyone from actually *studying* the languages. If you study only one or two languages, you tend to become distracted by the superficial differences among them. With three or more languages, people unconsciously become aware of the similarities rather than the differences.

Those participants in the Hippo experiment who have become proficient in a number of languages all say more or less the same thing: "Languages may appear different, but they're ultimately the same. Now, when I hear a new language, it's as if I've gotten a head start from all the other languages I've learned. The new ones come easier and easier. In fact, I can't wait to learn more."

There are many human languages. From the outside they appear completely different, both phonetically and grammatically, but at their core they are the same. This is what babies have been telling us from the beginning and now adults are beginning to realize it too.

We label as "foreign" those languages we approach from the outside. Those we learn from the inside, by the natural method, we call our mother tongues. This is what we meant when we said at the beginning that the Hippo experience radically alters the way we understand languages. The practical, reallife approach to language employed by Hippo makes every language our mother tongue.

As the Hippo Family Club language learning process has evolved, so has the research conducted by TCL.

It all began when we were discussing the sounds that make up a language. Suddenly someone said, "I wonder what the ideal distribution of the five Japanese vowels

would be?" (The Japanese vowels are AH, EE, UU, EH, and OH.) "Ideal distribution" means the placement of vowels that would most clearly distinguish them from one another, that is, make them easy for people to tell apart. We tentatively concluded that the ideal distribution would take the form of a regular pentagon. The reasoning was that a pentagon represents the most balanced configuration of five vowels placed as far apart from each other as possible. That was as far as our discussion went at the time. To paraphrase Dr. Heisenberg, we were using everyday language to try to come up with a clear image of order in language.

Then one day, a member of another Hippo group came rushing in to announce breathlessly, "There's a way we can see the sounds of language with our own eyes!" See the sounds of language? How? As a series of complex waves it turns out. If sounds could be viewed as waveforms, that meant they could be measured as physical quantities. Someone else announced that they had heard of a type of mathematics that analyzed waves of this sort, something called Fourier analysis. From this point there was no stopping the TCL students.

"Math is a language for accurately describing nature, isn't it? Let's just treat it like any other language and add it to our Hippo multilingual activity list!"

Most of the TCL students had given up on mathematics when they were in middle school or high school. But now they were Hippo club members, comfortable with the Hippo process, which was to start learning a new language by listening to the language tapes over and over. Applying this same principle to the language of math, they started reading mathematics books over and over, turning the pages even when they didn't understand a word on them. And just as the sounds of a new language start to flow from a person's mouth as soon as his or her ear becomes used to them, the TCL students started to chat about math as they began to familiarize themselves with the new concepts. If when having tried to explain a concept to others, they were unable to follow your explanation well enough to understand it themselves, it meant your own understanding was still incomplete. It meant you were still speaking the math equivalent of baby talk. It took less than a year before everyone was speaking the language of math, or to be specific, Fourier analysis. The fruit of our labors is this book — Who is Fourier? A Mathematical Adventure.

Once again, let us paraphrase the words of Heisenberg: "Use simple, everyday language when you present an argument, and when the image becomes so clear that anyone can understand it, that is the time to start thinking of an applicable formula to express your concept."

At long last we were ready to put our newly acquired mathematical language, Fourier analysis, to the test. We began analyzing data on the five Japanese vowels, AH-EE-UU-EH-OH, as spoken by many different individuals (see Chapter Four). No two people's voices displayed the same vowel distribution, yet a very clear pattern began to emerge. We tried adding up the individual data and taking an average, increasing the number of subjects from five to ten, to twenty and more. As we randomly added and averaged more and more data, the average distribution of the five Japanese vowels began to assume a graceful symmetry. It was very close to the pentagon we had predicted in the first place! Behind the variation in individual values, there indeed lay a simple and elegant order. This was our first glimpse of the natural structure of the sounds of language. We knew we had barely scratched the surface of the phenomenon of vowels. But we also knew our quest was headed in the right direction.

Our discovery about vowels suggests that the consonants of Japanese can also be described as physical quantities, and that we will find an elegant symmetry among them as well. We can also predict that if this symmetry exists among the sounds of Japanese, it must be found in other languages as well.

But what happens when we introduce the indeterminate quantity of "meaning" to the equation? Suddenly the development of a comprehensive description of the order in natural languages seems more problematic. Yet the formation of meaning is an integral part of the formation of language. We cannot ignore the importance of the role played by sounds as an expression of meaning.

The TCL researchers have only just begun to address this question in their effort to develop a scientific approach to the understanding of language and the human beings that speak those languages.

> January 1995 Transnational College of LEX