Behavioral Social Choice

Probabilistic Models, Statistical Inference, and Applications

MICHEL REGENWETTER
BERNARD GROFMAN
A. A. J. MARLEY
ILIA M. TSETLIN

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MICHEL REGENWETTER
University of Illinois at Urbana-Champaign

BERNARD GROFMAN
University of California, Irvine

A. A. J. MARLEY
University of Victoria

ILIA TSETLIN
INSEAD

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
40 West 20th Street, New York, NY 10011-4211, USA

www.cambridge.org
Information on this title: www.cambridge.org/9780521829687

First published 2006

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data

Behavioral social choice : probabilistic models, statistical inference, and applications / Michel Regenwetter ... [et al.].
p. cm.
Includes bibliographical references and indexes.
ISBN-13: 978-0-521-82968-7 (hardback)
ISBN-10: 0-521-82968-2 (hardback)
ISBN-13: 978-0-521-53666-0 (pbk.)
ISBN-10: 0-521-53666-9 (pbk.)
1. Social choice – Mathematical models. 2. Decision making – Mathematical models. 3. Voting – Mathematical models. 4. Probabilities – Mathematical models. I. Regenwetter, Michel. II. Title.
HB846.8.B44 2006
302'.13'01 – dc22 2005023291

ISBN-13 978-0-521-82968-7 hardback
ISBN-10 0-521-82968-2 hardback

ISBN-13 978-0-521-53666-0 paperback
ISBN-10 0-521-53666-9 paperback

Behavioral Social Choice

Behavioral Social Choice looks at the probabilistic foundations of collective decision making rules. The authors challenge much of the existing theoretical wisdom about social choice processes, and seek to restore faith in the possibility of democratic decision making. In particular, they argue that worries about the supposed prevalence of majority rule cycles that would preclude groups from reaching a final decision about what alternative they prefer have been greatly overstated. In practice, majority rule can be expected to work well in most real-world settings. Furthermore, if there is a problem, they show that the problem is more likely to be one of sample estimates missing the majority winner in a close contest (e.g., Bush–Gore) than a problem about cycling. The authors also provide new mathematical tools to estimate the prevalence of cycles as a function of sample size. They provide new insights into how alternative model specifications can change our estimates of social orderings.

Michel Regenwetter is Associate Professor of Psychology and Political Science at the University of Illinois at Urbana-Champaign (UIUC). Dr. Regenwetter has published over 20 scholarly articles in leading academic journals in his field, including *Journal of Experimental Psychology: Learning, Memory and Cognition, Journal of Mathematical Psychology, Management Science, Mathematical Social Sciences, Psychological Review, Psychometrika, Social Choice and Welfare*, and *Theory and Decision*. Dr. Regenwetter has served as guest associate editor for *Management Science*, and since 2003, he has been a permanent member of the editorial board of *Journal of Mathematical Psychology*.

Bernard Grofman is Professor of Political Science at the University of California, Irvine. He is coauthor of *A Unified Theory of Party Competition* (with James F. Adams and Samuel Merrill III, Cambridge University Press, 2005), *A Unified Theory of Voting* (with Samuel Merrill, III, Cambridge University Press, 1999), and *Minority Representation and the Quest for Voting Equality* (with Lisa Handley and Richard G. Niemi, Cambridge University Press, 1994).

Professor A. A. J. Marley, a Fellow of the American Psychological Society, was Chair of the Department of Psychology at McGill University from 1992 to 2001 and is now Adjunct Professor at the University of Victoria and Professor Emeritus of McGill University. He was Editor of the *Journal of Mathematical Psychology* and Section Editor for *Mathematics and Computer Sciences* of the *International Encyclopedia of the Social and Behavioral Sciences* (2001). He is the editor of *Choice, Decision and Measurement: Essays in Honor of R. Duncan Luce* (1997).

Ilia M. Tsetlin is an Assistant Professor of Decision Sciences at INSEAD. His research has appeared in academic journals such as *Management Science, Operations Research, Journal of Risk and Uncertainty*, and *Social Choice and Welfare*.

To our families

Acknowledgments

Regenwetter performed the research and writing that went into this book while he was a graduate student in the program in Mathematical Behavioral Sciences in the School of Social Sciences at the University of California, Irvine, as a post-doctoral fellow in the Department of Psychology at McGill University, as an Assistant Professor of Business Administration at the Fuqua School of Business, Duke University, briefly as a scholar in residence at the Center for the Study of Democracy at the University of California, Irvine, and as an Assistant Professor in the Departments of Psychology and Political Science, University of Illinois at Urbana-Champaign. He completed his work on the book while an Associate Professor in the Departments of Psychology and Political Science, University of Illinois at Urbana-Champaign.

Grofman carried out the research for this book while a Professor in the Department of Political Science and a member of the Institute for Mathematical Behavioral Sciences at the University of California, Irvine; and while he was a scholar in residence in the Department of Political Science, and a Fellow of the Institute for Advanced Study, at the University of Bologna (Italy). He completed his work on the book while a Professor in the Department of Political Science and a member of the Institute for Mathematical Behavioral Sciences at the University of California, Irvine.

Marley carried out the research and writing for this book while he was a Professor of Psychology at McGill University (Canada), a Fellow of the Hanse-Wissenschaftskolleg (Delmenhorst, Germany), and a visiting researcher in the Department of Economics at the University of Groningen

(The Netherlands); he completed his work on the book while a Professor Emeritus at McGill University (Canada) and an Adjunct Professor in the Department of Psychology, University of Victoria (Canada).

Tsetlin performed the research and writing for this book while he was a Ph.D. student in Decision Sciences at the Fuqua School of Business, Duke University. He completed his work on the book while an Assistant Professor of Decision Sciences at INSEAD (France and Singapore).

We are pleased to acknowledge the financial support of the following: The National Science Foundation, Grant # SBR-97-30076 (to Michel Regenwetter), Grant # SBR-97-30578 (to Bernard Grofman and A. A. J. Marley), and Grant # SBR-98-18756 (to Michel Regenwetter and Aleksandar Pekeč); NSERC Canada Collaborative Grant # CGP-0164211 (to A. A. J. Marley with J. Aczel, H. Joe, C. Genest, I. J. Myung, H. Colonius); NSERC Canada Discovery Grant # RGPIN 8124-98 (to A. A. J. Marley); the Center for Decision Making and Risk Analysis at INSEAD (to Tsetlin); the INSEAD Alumni Fund (to Tsetlin); The Fuqua School of Business, Duke University (to Regenwetter and Tsetlin); the Institute for Mathematical Behavioral Sciences and the Center for the Study of Democracy, the University of California, Irvine (Grofman, Marley and Regenwetter); the Department of Psychology and the Faculty of Science, McGill University, Montreal, Canada (to Marley and Regenwetter); the Hanse-Wissenschaftskolleg, Delmenhorst, Germany (to Marley); and the Netherlands' Organization for Scientific Research (to Marley).

Various aspects of this work have benefited from helpful comments and suggestions at various times by the following individuals (listed alphabetically): James Adams, William Batchelder, François Bavaud, Steven Brams, Martin Chabot, Robert Clemen, Eric Cosyn, Xinyuan Dai, Jean-Paul Doignon, Jean-Claude Falmagne, Scott Feld, Peter Fishburn, Wulf Gärtner, William Gehrlein, Yung-Fong Hsu, Harry Joe, Craig Leth-Steensen, Duncan Luce, Michael Munger, Louis Narens, Robert Nau, Reinhard Niederée, Prasanta Pattanaik, Aleksandar Pekeč, Donald Saari, James Smith, Reinhard Suck, Robert Winkler, Hsiu-Ting Yu, and various anonymous referees and journal editors. We would like to thank Mark Berger for help in data extraction, Peter Karcher for help in computer programming and computer simulations, Clover Behrend for secretarial and bibliographic assistance, and Chantale Bousquet for her long-distance secretarial support. Hsiu-Ting Yu provided us with extensive assistance with LaTeX, and performed most of the work in generating the subject and author indices.

For access to data which we have used in this book, or drawn on for closely related projects, we wish to thank the Interuniversity Consortium for Political and Social Research (University of Michigan), as well as Steven Brams, Peter Fishburn, John Little, Jack Nagel, Helmut Norpoth, and Nicholas Tideman. Some of these data sets were available on CD-ROM (ICPSR), some were provided to us directly, and some were available through being reprinted in the author(s)' publications.

The material in this book represents a synthesis of previous work by the authors (some of it joint with other scholars). The following nine papers are the most important of the papers that we draw upon (in some cases quite heavily):

1. J.-P. Doignon and M. Regenwetter. "An approval-voting polytope for linear orders." *Journal of Mathematical Psychology*, **41**:171–188, 1997.
2. M. Regenwetter, J. Adams, and B. Grofman. "On the (sample) Condorcet efficiency of majority rule: An alternative view of majority cycles and social homogeneity." *Theory and Decision*, **53**:153–186, 2002.
3. M. Regenwetter and B. Grofman. "Approval voting, Borda winners and Condorcet winners: Evidence from seven elections." *Management Science*, **44**:520–533, 1998a.
4. M. Regenwetter and B. Grofman. "Choosing subsets: A size-independent probabilistic model and the quest for a social welfare ordering." *Social Choice and Welfare*, **15**:423–443, 1998b.
5. M. Regenwetter, B. Grofman, and A. A. J. Marley. "On the model dependence of majority preferences reconstructed from ballot or survey data." *Mathematical Social Sciences (special issue on random utility theory and probabilistic measurement theory)*, **43**:453–468, 2002.
6. M. Regenwetter, A. A. J. Marley, and B. Grofman. "A general concept of majority rule." *Mathematical Social Sciences (special issue on random utility theory and probabilistic measurement theory)*, **43**:407–430, 2002.
7. I. Tsetlin and M. Regenwetter. "On the probabilities of correct or incorrect majority preference relations." *Social Choice and Welfare*, **20**:283–306, 2003.
8. I. Tsetlin, M. Regenwetter and B. Grofman. "The impartial culture maximizes the probability of majority cycles." *Social Choice and Welfare*, **21**:387–398, 2003.

9. M. Regenwetter, A. A. J. Marley, and B. Grofman. "General concepts of value restriction and preference majority." *Social Choice and Welfare*, **21**:149–173, 2003.

We are indebted to all of these journals for granting us reprint permission as follows:

Materials from Doignon and Regenwetter (1997) are reprinted from *Journal of Mathematical Psychology*, volume 41, pages 171–188, Copyright (1997), with kind permission from Elsevier.

Materials from Regenwetter, Adams, and Grofman (2002a) are reprinted from *Theory and Decision*, volume 53, pages 153–186, Copyright (2002), with kind permission of Kluwer Academic Publishers.

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Materials from Regenwetter and Grofman (1998b), Tsetlin and Regenwetter (2003), Tsetlin, Regenwetter and Grofman (2003), and Regenwetter, Marley and Grofman (2003) are reprinted from Social Choice and Welfare, Choosing subsets: A size-independent probabilistic model and the quest for a social welfare ordering, volume 15, pages 423–443, Copyright (1998); On the probabilities of correct or incorrect majority preference relations, volume 20, pages 283–306, Copyright (2003); The impartial culture maximizes the probability of majority cycles, volume 21, pages 387–398, Copyright (2003); General concepts of value restriction and preference majority, volume 21, pages 149–173, Copyright (2003); with kind permission by Springer-Verlag.

Materials from Regenwetter, Grofman, and Marley (2002b) and from Regenwetter, Marley, and Grofman (2002c) are reprinted from Mathematical Social Sciences, volume 43, pages 453–468 and pages 407–430, Copyright (2002), with kind permission from Elsevier.

Chapter 1 draws heavily on Tsetlin et al. (2003) and on Regenwetter and Grofman (1998b). Chapter 2 draws heavily on Regenwetter et al. (2002c) and Regenwetter et al. (2003) and also makes use of a small portion of Regenwetter et al. (2002a). Chapter 3 integrates large parts of Regenwetter et al. (2002b) and Regenwetter et al. (2003). Chapter 4 draws on portions of Doignon and Regenwetter (1997), and more substantially on Regenwetter and Grofman (1998a) and on Regenwetter et al.

(2002b). Chapter 5 draws on Regenwetter et al. (2002a), and heavily on Tsetlin and Regenwetter (2003). Chapter 6 draws in part on Regenwetter and Grofman (1998a) and Regenwetter and Grofman (1998b). We are particularly grateful to James Adams for permission to use some material in Regenwetter et al. (2002a); and to Jean-Paul Doignon for permission to report some of the theorems in Doignon and Regenwetter (1997).

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Introduction and Summary

INTRODUCTION

Behavioral Social Choice Research

This book develops conceptual, mathematical, methodological, and empirical foundations of *behavioral social choice research*. Behavioral social choice research (or, more briefly, *behavioral social choice*) encompasses two major interconnected paradigms: the development of *behavioral social choice theory* and the evaluation of that theory with empirical data on social choice behavior.

The fundamental purpose of a behavioral theory of social choice processes is the development of descriptive models for real actors' social choice behavior and the statistical evaluation of such models against empirical data. Our notion of behavioral social choice research builds on and, at the same time, complements much of classical social choice theory in the tradition of leading figures such as the Marquis de Condorcet, Duncan Black, Kenneth Arrow, and Amartya Sen. Most classic approaches follow an axiomatic, normative line of reasoning. They formulate desirable properties of "rational" social choice and provide numerous "possibility" or "impossibility" theorems that classify groups of such axioms into whether or not they lead to 'feasible' aggregation procedures, given various theoretical assumptions about the nature, domain, and distribution of individual preferences (McLean and Urken, 1995). A principal task of behavioral social choice research is to evaluate such normative benchmarks of rational social choice against empirical evidence on real world social choice behavior. Consistently throughout this book we attempt to evaluate

our models against a wide range of empirical evidence drawn from large-scale real-world data sets from three different countries. To the extent that classical/normative theories fail to be descriptive of observed social choice behavior, they motivate and inspire the development of (alternative) behavioral theories that complement classical approaches by descriptively capturing the social choice behavior of real actors.

We see our work as building on the pioneering literature that integrates formal models with the analysis of real world social choice data (e.g., Chamberlin et al., 1984; Felsenthal et al., 1986, 1993; Felsenthal and Machover, 1995; Laver and Schofield, 1990; Niemi, 1970; Riker, 1958). We provide a general probabilistic modeling and statistical sampling and inference framework for the descriptive theoretical and empirical investigation of social choice behavior of real-world decision makers, but we place a major emphasis on majority rule decision making (Condorcet, 1785). Our general framework is formulated in terms of an extremely broad domain of permissible preference representations and it is applicable to an extremely broad range of empirical rating, ranking, and choice paradigms.

Six Major Contributions

While we conceptualize behavioral social choice theory as encompassing a very broad spectrum of research paradigms,[1] we focus here exclusively on the foundations for such a theory. Our main contributions are sixfold:

1. We argue for the limited theoretical relevance and demonstrate the lack of empirical evidence for cycles in mass electorates by replacing "value restriction"[2] and similar classic domain restriction conditions, as well as the "impartial culture" assumption, with more realistic assumptions about preference distributions.
2. We expand the classical domains of permissible preference states by allowing for more general binary preference relations than linear or weak orders and by considering probabilistic representations of preference and utility.
3. We develop methodologies to (re)construct preference distributions from incomplete data, that is, data which do not provide either complete rankings or complete sets of pairwise comparisons.

[1] For example, in addition to the study of committee voting and mass election processes, we see behavioral social choice theory as encompassing the empirical study of coalitions, of information pooling (such as occurs in juries), and of a wide variety of other collective choice processes.

[2] A definition of this (and related) terms is provided later in the text.