Wage Differentials and Economic Growth

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I alone am responsible for any errors or omissions that remain.

Pasquale M. Sgro

GENERAL REMARKS

This book is concerned with one branch of growth theory, namely descriptive growth theory.[1] Our concern is to examine the dynamic (time) paths of macroeconomic variables. Given specific parametric values and functional forms we will look at how the economy will grow. The growth pattern will include the movement of prices, quantities, stocks and flows through time.

It is typically assumed in growth theory that both the factors and goods markets are perfectly competitive. In particular, this implies, among other things, that the reward to each factor is identical in each sector of the economy. In this book the assumption of identical factor rewards is relaxed and the implications of an intersectoral wage differential is analyzed. There is also some discussion on the effects of minimum wages legislation on long-run growth.

This book is not intended to be a comprehensive survey of standard neo-classical growth theory.[2] Our particular concern is looking at some effects in both the short-run and long-run, of wage differentials in two-sector growth models. In particular, the implications of such differentials for the existence and uniqueness of short-run equilibrium as well as the balanced growth path are examined. There is also a discussion (Part Three) on the interaction between international trade and economic growth. A substantial amount of work has appeared in the area of wage differentials and growth in the academic journals, but, until now, and to my knowledge, there has not been any textbook on the subject.

The exposition in the book relies on both mathematics and geometry. Where possible, diagrams have been used extensively to illustrate and clarify some of the results; the purpose being to make the analysis and results accessible to a larger audience. Nevertheless, for a proper study of growth theory, and since we are dealing with the time paths of variable, some elementary differential calculus is essential. In most cases, however, an intuitive explanation of the result is also given.[3] The book can be used as a supplementary text for a growth theory course as well as being suitable as a reference for advanced undergraduate students.

The book is organized into three parts. In Part One the standard perfectly competitive two-sector growth model is developed by commencing first with the simple variety of the two-sector model, namely the fixed coefficient variety; this is presented in Chapter 1, while Chapter 2 of Part One deals with the more general variable coefficients case. These two chapters provide the basic models with which we will work throughout the book. A treatment of the important properties and key relationships is provided.

In Part Two, both the models as presented in Part One are modified to include an intersectoral wage differential. The consequences and implications of the wage differential for both short-run and long-run equilibrium are then fully explored. There is as well, a discussion of the effects of introducing minimum wages into the variable coefficients model discussed in Chapter 4.

The final section of the book, Part Three, deals with the interaction between international trade and growth in the form of capital accumulation. In this part, Chapter 5 presents the standard perfectly competitive trade and growth model while Chapter 6 extends the model, presented in Chapter 5, to include wage differentials.[4]

Finally, a selective list of references is provided at the end of the book for readers who wish to pursue the topic further.

Notes

1. The other branch is usually referred to as normative growth theory which is concerned with how an economy *should* grow. This is sometimes also called 'optimal growth' theory.

2. Books already exist which perform this task admirably. Burmeister and Dobell (15), Dixit (23) and Wan (83) to name three.

3. If readers find some of the mathematical details difficult, they should omit them, at least initially, and confine themselves to the descriptive explanation of the result.

4. A number of results in Chapters 3 and 6 are taken from the author's doctoral dissertation submitted at La Trobe University.

CONTENTS

PART ONE

Two-sector Growth Models

In Part One, the basic growth models that will be used throughout the book are set out. The first chapter is concerned with setting up the simplest type of two-sector model, namely the fixed production coefficient case, while in Chapter 2, the more general variable production coefficient case is presented. The main results are presented in a concise manner to facilitate comparison with Part Two where we introduce intersectoral wage differentials and minimum wages into these growth models.

The aim in Part One is not to present an exhaustive survey of the two-sector growth model but rather to present and focus attention on certain important results which are most affected by the introduction of wage differentials and minimum wages.

CHAPTER 1

A Two-sector Fixed Coefficient Growth Model

1.1 Introduction

In this chapter, the simplest version of the two-sector growth model will be constructed. There are two productive sectors which produce consumption and investment goods respectively, using two factors of production, capital and labour. The proportions in which capital and labour are used in the production process in each sector are fixed. For the demand side of the model, two alternative savings assumptions will be used. Although the fixed coefficient model is essentially a technologically determined model, the use of different savings assumptions highlights the important role that demand plays in this model. The first savings function used is what is called the 'proportional savings' function; that is, a given constant average propensity to save (out of national income). The second savings function is known as the 'classical savings' function. With this second function, the assumption is made that the savings role is carried out by two classes of individuals, namely wage earners and profit earners. It is assumed that each class of individuals has some given constant average propensity to save (out of income).

Results on the existence and uniqueness of equilibrium are derived under each alternative savings function assumption. Finally, the growth process is examined and conditions for the existence and uniqueness of a globally stable balanced growth capital/labour ratio are derived.

1.2 The Formal Model

The production functions for commodities X_1 and X_2 can be written as follows:

$$X_1 = F_1(K_1,L_1) = \mathrm{Min}(K_1,L_1) \tag{1.2.1}$$

$$X_2 = F_2(K_2,L_2) = \mathrm{Min}(K_2,L_2) \tag{1.2.2}$$

where K_1, K_2, L_1, L_2 are respectively the amounts of capital and labour used in sectors one and two.[1] We assume in general that the two production functions are linearly homogeneous; as they are in this special case. This assumption enables us to express equations (1.2.1) and (1.2.2) in the following unit isoquant form

$$\text{Min}\left\{\frac{K_i}{X_i},\frac{L_i}{X_i}\right\} = F_i(K_i/X_i, L_i/X_i) = F_i(a_{Ki}, a_{Li}) = 1 \quad i = 1,2 \quad (1.2.3)$$

where a_{Ki} and a_{Li} represent the inputs of factors K and L necessary to produce one unit of output i. Since the model we are concerned with in this chapter is the case where production coefficients are fixed, a_{Ki} and a_{Li} are assumed fixed. The full employment of both factors' assumption can be written as:

$$a_{K1}X_1 + a_{K2}X_2 \leq \bar{K} \qquad (1.2.4)$$

$$a_{L1}X_1 + a_{L2}X_2 \leq \bar{L} \qquad (1.2.5)$$

The inequalities in (1.2.4) and (1.2.5) indicate that the two factors are not always fully employed. The bars over the variables indicate that both factors are in inelastic supply at any moment in time. Assuming perfect competition in the product markets and that both goods are produced, unit costs reflect product prices.[2] Thus we have

$$a_{L1}w + a_{K1}r = P_1 \qquad (1.2.6)$$

$$a_{L2}w + a_{K2}r = P_2 \qquad (1.2.7)$$

where r and w stand for the reward to capital and labour in each industry respectively and P_i for the price of the ith good. The production side of the model is thus described by equations (1.2.3)–(1.2.7).

To close the model, the demand side will now be considered. As mentioned in the introduction to this chapter, two alternative functional forms for the demand (or savings) relationship will be used.

First, if the preferences of all individuals are identical, convex and homothetic-to-the-origin, then the relative demand curve (X_2/X_1) is well behaved and in particular, is independent of the level of national and personal income.[3] The relative demand curve $X_2/X_1 = D(P_2/P_1)$ can therefore be written where $D'(\cdot) < 0$.[4] An important example of this type of demand curve is that used in neoclassical savings models in which commodity two is the investment good, commodity one is the consumption good and preferences are of the Cobb-Douglas type. This implies the relative demand curve $X_2/X_1 = \left(\frac{s}{1-s}\right)/P$ where s is the fixed rate of savings ($\equiv$ investment) and $P = P_2/P_1$. Thus, for the 'proportional savings function' form we can write the demand function as

$$\frac{X_2}{X_1} = \left(\frac{s}{1-s}\right)/P = e/(P) = D(P) \qquad (1.2.8)$$

where $e = \left(\frac{s}{1-s}\right)$ and is a constant.[5]

Second, we assume that there are two distinct classes, wage earners and profit earners, but that all members of a given class have identical demand patterns which can be described by relative demand functions. In this case the relative demand function depends on the distribution of income and an excess demand 'correspondence' is now possible. By 'correspondence' we mean situations in which, say at a given set of prices, the excess demand for a good must be represented by a set rather than a number. Let D_w and D_r denote monotonic decreasing relative demand functions for wage earners and profit earners respectively. These arise from identical people in each class, with homothetic convex-to-the-origin indifference maps.

Define s_r and s_w as the constant savings rate of profit earners and wage earners respectively. Given homothetic preferences for both classes, demand X_{ij} for commodity i ($i = 1,2$) by class j ($j = w,r$) can be expressed as follows:

$$X_{1w} = (1-s_w)\ell(P)Y(P)/P_1$$

$$X_{1r} = (1-s_r)(1-\ell(P))Y(P)/P_1$$

$$X_{2w} = s_w\ell(P)Y(P)/P_2$$

$$X_{2r} = s_r(1-\ell(P))Y(P)/P_2$$

where $Y(P)$ is national income at relative prices P and ℓ and $(1-\ell)$ are respectively the wage earners' and profit earners' share of national income. Hence the aggregate relative demand $(X_{2w} + X_{2r})/(X_{1w} + X_{1r})$ is given by the function

$$X_2/X_1 = \frac{s_w \ell(P) + s_r(1-\ell(P))}{(1-s_w)\ell(P) + (1-s_r)(1-\ell(P))} / P \qquad (1.2.9)$$

where P is defined as in (1.2.8). The variable ℓ is also a function of P. Thus for the 'classical savings function' form, the relative demand function can be written as in (1.2.9). This completes the specification of demand side of the model.

The production and demand sides are now combined to discuss the attainment of short-run equilibrium. The different savings assumptions will be considered in turn.

1.3 Short-Run Equilibrium

Suppose that industry two is capital intensive, that is $a_{K2}/a_{L2} > a_{K1}/a_{L1}$. Furthermore, to simplify the algebra we will assume for the moment that the supply of labour is fixed. Given constant returns to scale, and denoting the aggregate capital/labour endowment ratio by k ($\equiv K/L$), the following relationships hold. If $k \leq a_{K1}/a_{L1}$, regardless of demand, labour is redundant while if $k \geq a_{K2}/a_{L2}$, regardless of demand, capital is redundant, if $a_{K1}/a_{L1} < k < a_{K2}/a_{L2}$ both factors are fully employed. Thus:

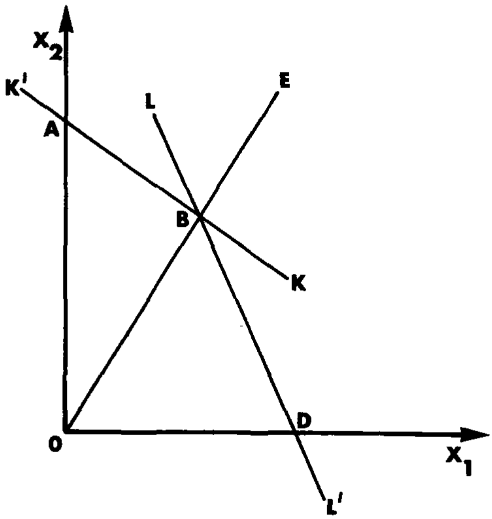
$$k \quad \begin{matrix} \notin \\ \in \end{matrix} \quad \left(\frac{a_{K1}}{a_{L1}}, \frac{a_{K2}}{a_{L2}}\right)$$

as full employment of both factors does or does not prevail.

In terms of Figure 1.3.1, the capital and labour constraints defined by (1.2.4) and (1.2.5) respectively are represented by KK' and LL'. These constraints define the production possibility set $OABD$. The diagram is drawn on the assumption that industry two is capital intensive. Along AB, labour is unemployed, along BD capital is unemployed. At point B, both factors are fully employed. In the case shown in Figure 1.3.1, the full employment (of both factors) output ratio is represented by the slope of the ray OE.

Given that the total factor supply is fixed, the relative supply curve, which can be written as $X_2/X_1 = S(P_2/P_1)$, depends on the stock of capital. For $k \leq a_{K1}/a_{L1}$ and using (1.2.6) and (1.2.7),

Figure 1.3.1



$$\frac{X_2}{X_1} = \begin{cases} 0 \\ \in(0,\infty) \text{ as } \\ \infty \end{cases} \frac{P_2}{P_1} \begin{matrix} < \\ = \\ > \end{matrix} \frac{a_{K2}}{a_{K1}} \tag{1.3.1}$$

Thus, if both commodities are to be produced when labour is redundant, the price ratio must be (the negative of) the slope of the capital constraint.

For some capital/labour stocks, there exists a critical output ratio Λ, defined as the full employment of both factors, output ratio, such that $\Lambda'(k) > 0$, $k \in [a_{K1}/a_{L1}, a_{K2}/a_{L2}]$; where $\Lambda(k) = 0$, $k \leq a_{K1}/a_{L1}$; while for $\Lambda(k) = \infty$, $k \geq a_{K2}/a_{L2}$. Thus, for $X_2/X_1 < \Lambda(k)$,[6]

$$\frac{P_2}{P_1} = \frac{a_{L2}}{a_{L1}} \tag{1.3.2}$$

from which

$$\frac{X_2}{X_1} = \in (0, \Lambda(k)). \tag{1.3.3}$$

For $X_2/X_1 > \Lambda(k)$,

$$\frac{P_2}{P_1} \begin{matrix}>\\=\end{matrix} \frac{a_{K2}}{a_{K1}} \text{ as } \frac{X_2}{X_1} = \begin{cases}\infty \\ \in(\Lambda(k), \infty)\end{cases} \tag{1.3.4}$$

For $X_2X_1 = \Lambda(k)$, the price ratio is between the limits $(\frac{a_{L2}}{a_{L1}}, \frac{a_{K2}}{a_{K1}})$ defined by (1.3.2) and (1.3.4). Thus the relative supply curve for the commodity two, capital intensive case is shown as *ADEZ* in Figure 1.3.2.[7]

Assume that, for the demand side, the proportional savings function applies. From equation (1.2.8), we have a monotonic decreasing relative demand curve labelled $D(P_2/P_1)$ in Figure 1.3.2.[8] Given the shape of the relative supply curve in Figure 1.3.2, then corresponding to any downward sloping demand curve, there will be one and only one intersection of the relative demand and supply curves. Equilibrium exists and is unique in this case. For the case where commodity one is capital intensive, the relative supply curve, *ADEZ*, is shown in Figure 1.3.3. Again given the proportional savings function, equilibrium exists and is unique.

Figure 1.3.2

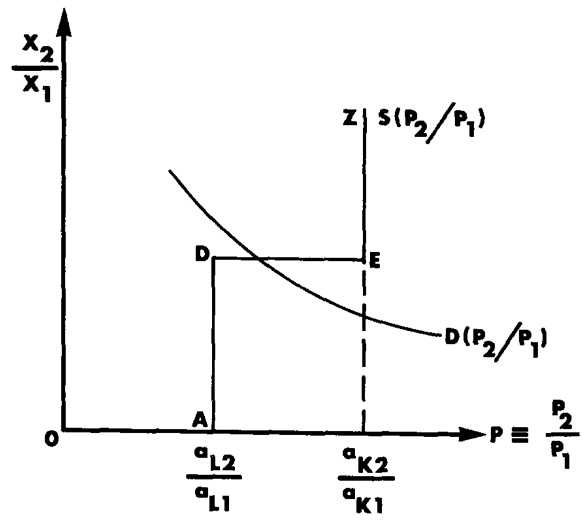
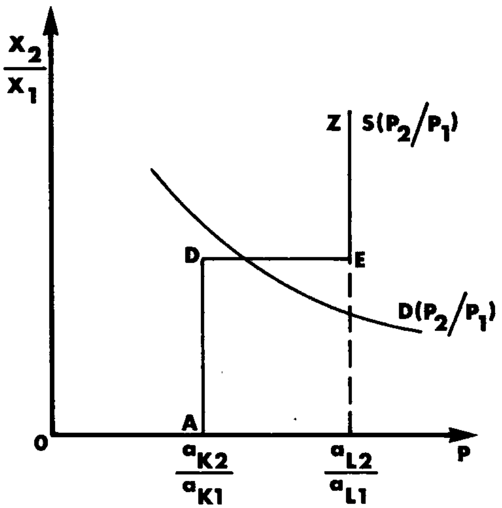


Figure 1.3.3



Proposition 1.1.
If we assume a proportional savings function, equilibrium exists and is unique.

Consider the second case of a classical savings function. The problem of determining the uniqueness and existence of equilibrium is more complicated. Conditions are needed for the relative demand function to be monotonic.

Consider equation (1.2.9), if a point is chosen on that part of the production possibility frontier where capital is redundant, $\ell = 1$ so that (1.2.9) reduces to:

$$D_w = X_2/X_1 = \{s_w/(1-s_w)\}/P \tag{1.3.5}$$

while along that position of the frontier where labour is redundant, $\ell = 0$ so that

$$D_r = X_2/X_1 = \{s_r/(1-s_r)\}/P \tag{1.3.6}$$

The relative demand curve is constructed for given stocks of factors. Given a price ratio, some point on the production possibility frontier is determined, from considerations of profit maximization.