Lecture Notes in Applied and Computational Mechanics 65

Czesław I. Bajer
Bartłomiej Dyniewicz

Numerical Analysis of Vibrations of Structures under Moving Inertial Load

Springer

Numerical Analysis of Vibrations of Structures under Moving Inertial Load

Czesław I. Bajer and Bartłomiej Dyniewicz

Springer

Authors
Czesław I. Bajer
Polish Academy of Sciences
and Warsaw University of Technology
Warsaw
Poland

Bartłomiej Dyniewicz
Polish Academy of Sciences
Warsaw
Poland

ISSN 1613-7736
e-ISSN 1860-0816
ISBN 978-3-642-29547-8
e-ISBN 978-3-642-29548-5
DOI 10.1007/ 978-3-642-29548-5
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2012935520

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Lecture Notes in Applied and Computational Mechanics 65

Series Editors

Prof. Dr.-Ing. Friedrich Pfeiffer
Lehrstuhl B für Mechanik
Technische Universität München
Boltzmannstraße 15
85748 Garching
Germany
E-mail: pfeiffer@amm.mw.tu-muenchen.de

Prof. Dr. Peter Wriggers
FB Bauingenieur- und Vermessungswesen
Inst. Baumechanik und Numer. Mechanik
Universität Hannover
Appelstr. 9 A
30167 Hannover
Germany
E-mail: wriggers@ikm.uni-hannover.de

For further volumes:
http://www.springer.com/series/4623

Preface

Computer methods and simulations allow engineers to study complex problems in detail. They can take into account various factors that influence the investigated phenomenon. Moreover, they can examine problems throughout a wide range of parameters. Such cases occur when the existing or designed structures are to carry heavier loads and should optimally resist external forces that involve static displacements or vibrations. Generally, vibrational or wave problems in structural dynamics require a detailed study of numerous cases.

Moving inertial loads are applied to structures in civil engineering, robotics, and mechanical engineering. Some fundamental books exist, as well as thousands of research papers. Well known is the book by L. Frýba, *Vibrations of Solids and Structures Under Moving Loads*, which describes almost all problems concerning non-inertial loads. Unfortunately, this wide literature is rarely reflected in computer codes. Well known commercial packages enable the analysis of complex mechanical problems, with material and geometrical non-linearities, but they fail in the case of moving loads.

This book presents broad description of numerical tools successfully applied to structural dynamic analysis. Unfortunately none of the classical methods can be directly applied to non-classical problems. Moving mass problems are an example of such a group of problems. It can be generally considered as problems with distributed parameters. Physically we deal with non-conservative systems. Mathematically they are described by linear partial differential equations with variable coefficients. We will focus our discussion on the moving inertial particle rather than on the structure carrying the massless load. The discrete approach formulated with the use of the classical finite element method (FEM) results in elemental matrices, which can be directly added to global structure matrices. The classical approach is considered in the simplest case in our book as the finite element method applied to space with another method applied to integration of the time derivatives. A more general approach is carried out with the space-time finite element method. It can be considered as an extension, to the time domain, of the well known finite element method: the spatial finite element gains an additional time dimension. In such a case, a trajectory of the moving concentrated parameter in space and time can be simply defined. What is more, elemental characteristic matrices can easily be derived and the formulation is relatively clear. The crucial point, however, is the uniform treatment of the space and time dependent terms in the differential equations. Discussion and the experience gained then allow of a better understanding of a formulation in the case of the Newmark method, central difference method, and other time integration methods commonly used in structural dynamics.

We consider structures described by pure hyperbolic differential equations such as strings and structures described by hyperbolic–parabolic differential equations such as beams and plates. More complex structures such as frames, grids, shells, and three-dimensional objects, can be treated with the use of the solutions given in this book.

The problems treated in the monograph can be related to problems of mathematical physics. The resulting matrices that describe the influence of the moving inertial particle can be directly implemented in computer codes.

This monograph would not have been possible without the support of the project Lider/26/40/L-2/10/NCBiR/2011 and project of Foudation for Polish Science – *START*.

Warsaw, February 2012

Czesław I. Bajer
Bartłomiej Dyniewicz

Contents

Chapter 1
Introduction

Computer methods are commonly used now in engineering design, manufacturing, and applications. They replace experimental methods of verification, especially if experiments are expensive, time consuming, or difficult to perform. Static analysis, plastic deformations, optimization, and free vibrations are fields sufficiently well explored, and now possess efficient numerical procedures implemented in commercial software. But the case of moving loads is not represented in such computer codes. Design engineers use simplifications and approximations known from analytical solutions. These are often adequate if the load does not change the dynamical properties of the structure, i.e. is massless. In the case of an inertial load we do not have adequate tools.

In this book we will present numerical methods which enable us to solve problems of the vibrations of structures subjected to inertial moving loads. Only simple and particular cases of problems with moving inertial load can be solved analytically. Such problems usually require numerical computations at the final stage or, if we use discrete methods, during the whole analysis. Analytical and semi-analytical solutions are indispensable when we verify our numerical results. Therefore we will present semi-analytical solutions as a base for a better understanding of both the differential equations that govern the motion of these structures, and features and properties of solutions. Engineers, researchers and students will find here matrices and algorithms ready for use, material which will enable them understanding mechanical problems, and an elaboration of the software procedures for basic or more complex structural elements.

Inertial loads moving on strings, beams, and plates at sub- or super-critical speed are of special interest. Theoretical solutions are applied to many practical problems: train–track interaction, vehicle–bridge interaction, pantograph collectors in railways, magnetic rails, guideways in robotic solutions, etc. Such problems have been widely treated in the literature. Attempts at solving such problems began in the middle of the 19th century. However, up to now we have not had a complete and closed analytical solution. The term describing the concentrated mass motion is the reason for the difficulties. Systems of differential equations of variable coefficients, which, except in a few cases, do not have analytical solution, are serious roadblocks

C.I. Bajer and B. Dyniewicz: Numerical Analysis of Vibrations of Structures, LNACM 65, pp. 1–20.
springerlink.com

to obtaining closed-form solutions. These types of equations are finally solved by numerical means.

Structures subjected to moving loads are often encountered in engineering practice. Such are the bridges and viaducts loaded with vehicles [147], flyovers for traditional or magnetically lifted trains, road or airfield plates, sliding robot manipulators, machine tools, weapon firing barrels, ropes of transporting systems, and current collectors for power supply systems for rail vehicles (Figure 1.1). They are exposed to much larger displacements than when under static loads or slowly sliding loads. This becomes obvious if we look at the undeformed structure at rest, which is suddenly subjected to a force. Such a structure starts to vibrate around its

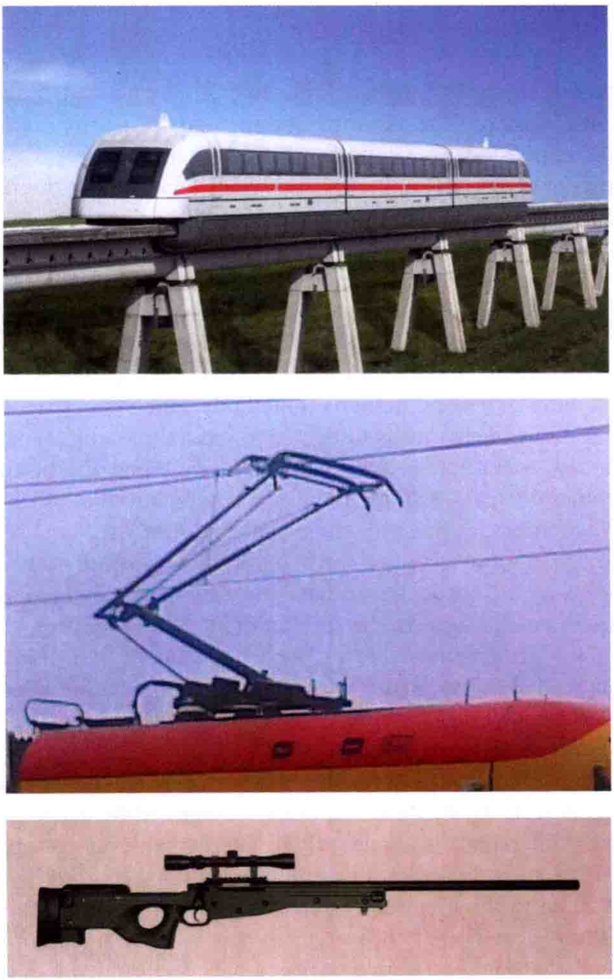

Fig. 1.1 Examples of problems with moving mass.

equilibrium position in the unladen state, through the state of static equilibrium under load, to obtain an amplitude equal to twice the static deflection under load. Therefore, the rapid entry of a loading force has a similar effect. The passage of the load, if takes place cyclically with a certain frequency, will increase the deflection. If its frequency is associated with the passage speed in such a way that at the exit of one load the next will enter, then we obtain the dependence of the maximum deflection of the structure under load on the speed of the travel of the load $w_{max}(\mathrm{v})$.

The maximum deflection occurs at the point located generally around 0.5–0.7 of the span, depending on the speed of travel. Proceeding further, we can examine the speed at which the maximum deflection of the journey will be greatest. This speed is called the critical speed. In the case of a string, the critical velocity corresponds to the wave propagation velocity c. The critical speed is the important feature from a practical point of view. It determines the most unfavorable value of the deflection, to which the structure must be made resistant. For this reason, the study of a structure under a moving load is an important engineering problem. Unfortunately, the existing commercial packages do not perform computational simulations of such tasks. As we will see, the problem is difficult and this can be ascribed to the lack of appropriate computerized procedures.

Here, attention should be given to the classification of the loads. The simplest case is shown in Figure 1.2*a*. It is irrelevant that the force applied directly to the structure is replaced with an oscillator, which will have non-zero mass. Although the mass effect will be visible in the results, it will not be the result of a task with the inertial load [112]. Moreover, although the impact of the mass of the oscillator will increase with increasing spring stiffness, then the solution does not tend uniformly to the solution for the case of an inertial load. Additional degree of freedom oscillations introduce additional artificial effects in the form of resonance: the increase or decrease in amplitude at certain speeds. A mass load is shown in Figure 1.2*b*. The mass motion affects the outcome at non-zero displacements $w(x,t)$ when $0 \leq t < l/\mathrm{v}$. Otherwise, the participation of factors causing displacement is required, such as the pulse force (Figure 1.2).

Let us take a railway wheel with mass 500 kg. Together with the axle and the axle box its inertia exceeds a ton. The rail has a linear mass density of 60 kg/m. The influence of this concentrated mass significantly changes the dynamic properties of a structure (Figure 1.3). We claim that a significant part of the rail wheel should be

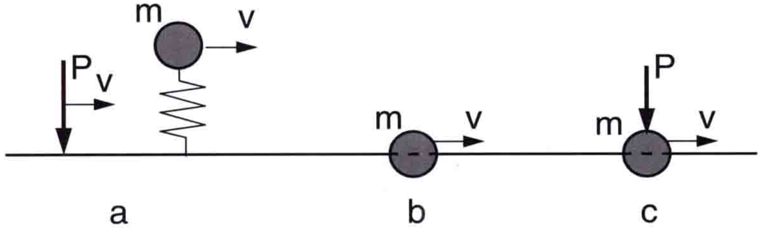

Fig. 1.2 Loads: a) massless, b) inertial, c) inertial and gravity.

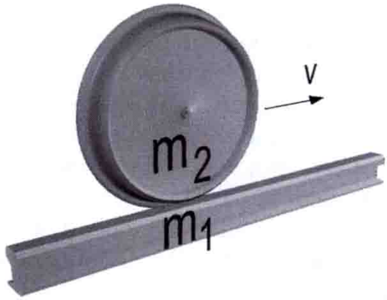

Fig. 1.3 The wheel inertia influencing the rail motion

unsprung. In engineering practice we wish to take into account a real structure, with all the atypical elements for an analytical model, i.e. ballast as an elastic foundation, sleepers as periodic supports, elastic pads, influence of several wheelsets, coupling and interacting with a boogie, etc. Accurate results are fundamental for decisions at the design stage. An accurate estimation of the dynamic influence is essential for proper modelling. Accurate results are important not only for increasing the durability and reliability of systems: predicting the level of the dynamic response of structures under a moving load facilitates the protection of the environment, especially populated urban centres or historic places.

Existing finite element (FEM) modelling software allows us to perform computations in several stages grouped into a batch procedure. In such a case, one can split the dynamic problem into a sequence of static problems with structures subjected to gravitational and inertial forces. Such a solution corresponds to the simple lumping of a moving mass into nodal points, as depicted in Figure 1.4. Such an approach is correct only at extremely low speeds of the mass, practically quasistatic. Problems containing beams as a supporting structure in the case of a moderate speed of a mass

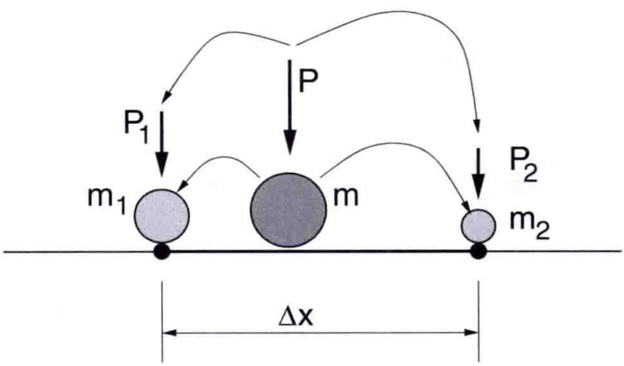

Fig. 1.4 Ad-hoc mass lumping to nodes.

usually give limited solutions. Unfortunately they are neither convergent nor stable. Even a small variation in the parameters produces significant alterations of the results. The pure hyperbolic differential equation describing the string motion results in divergent solutions.

Travelling loads are generally unlikely to be solved by commercial codes. Most of the existing systems for dynamic simulations usually do not allow us to solve even simple problems comprising travelling massless point forces, travelling distributed non-inertial loads, or even travelling and elastically joined moving substructures. Inertial moving loads are completely unimplemented by computer systems. The intuitive approach to discrete analysis with ad-hoc lumping of forces and masses to neighbouring nodes always fails. Sometimes, especially in the case of beams, numerical solutions are limited, but significantly inaccurate. We emphasize here that the travelling mass problem is not trivial, even if at first sight it seems to be.

1.1 Literature Review

In the literature, numerous historical reviews concerning the moving load problem exist (for example Panovko [106], Yakushev [146], Dmitrijev [43]). In most cases the moving massless constant force was considered as a moving load. This type of problem results in closed solutions. Unfortunately, the problem of inertial loads is still open. Saller in [123] considered a moving mass for the first time. He proved, in spite of essential simplifications, the significant influence of the moving mass on the beam dynamics. In the 1930s, two contributions appeared, important for researchers working in the field of moving loads. Inglis [70] applied simplifications and the solution was expressed by only the first term of a trigonometric series. The time function fulfilled a second order differential equation with variable coefficients. This equation was derived by considering the acceleration under the moving mass, expressed by the so-called Renaudot formula. In fact it is the derivative computed with the chain rule. The final solution of the differential equation with variable coefficients was proposed as an infinite series. It results in an approached solution.

Schallenkamp [124] proposed another approach to the problem of a moving mass. However, his attempt only allows us to describe the motion under the moving mass. The method of separation of variables by the expansion of the unknown function into a sine Fourier series was applied. Boundary conditions in the beam were taken into account in a natural way. The ordinary differential equation, which describes the motion under the moving mass, was expressed in generalized coordinates by using the second Lagrange equations. The generalized force was derived from the virtual work principle. Schallenkamp's consideration is relatively complex and converges slowly since the final solution is expressed in terms of a triple infinite series.

The works of Inglis and Schallenkamp can be considered as the basis for the analysis of the problem of a moving mass in the succeeding works of Bolotin [36, 35], Morgajewskij [101] and others. An excellent and important monograph in this field was written by Szcześniak [133]. One can find there hundreds of references concerning moving loads on beams and strings. In [138] the authors consider a simply supported beam modelled by the Bernoulli–Euler theory. The equation of motion is written in an integral-differential form with Green's function terms. In order to solve this equation, a dual numerical scheme was used. A backward difference technique was applied to treat the time parameter and numerical integration was used for the spatial parameter. This method of solution, though applied to higher velocities, still requires complex mathematical operations. Each solution enables us to determine only the displacements under the moving load and does not give solutions for a wide range of parameters x and t. Only one closed analytical solution can be found in the literature. Smith [127] proposed a purely analytical solution for the inertial moving load, however, only in the case of a massless string. The basic motion equation, without the term which describes the inertia of the string, was transformed to the hyper-geometrical equation. It has an analytical solution in terms of infinite series. Frýba [56] applied the same approach and found a closed analytical solution for the particular case. However, the formula given in [56] has mistakes.

Recent papers have contributed analyses of complex problems of structures subjected to moving inertial loads [144] or oscillators [29, 97, 112]. Variable speeds were analysed in [3, 58, 99]. The equivalent mass influence is analysed in [57]. An infinitely long string subjected to a uniformly accelerated point mass was also treated [121] and analytical solution of the problem concerning the motion of an infinite string on a Winkler foundation subjected to an inertial load moving at a constant speed was given [74].

In one of our papers [48], we considered small vibrations of the massless and massive string subjected to a moving inertial load. We proposed an analytical–numerical solution of the problem. The final equation has the form of a matrix differential equation of the second order. Numerical integration results in a solution over a wide range of the velocity: under-critical and over-critical. It exhibits a discontinuity of the mass trajectory at the end support point. This new feature had not yet been reported in the literature. A closed-form solution in the case of a massless string was analysed and its discontinuity was proved mathematically. Fully numerical results obtained for the inertial string had a similar property. Since small vibrations are analysed, the discontinuity effect discussed in the paper was of purely mathematical interest.

The results are compared with the approximate numerical solutions obtained by the finite element method (FEM). The string is subjected to a moving oscillator. In the case of a rigid spring, we approach the analytical solution. However, in the case of higher speeds (greater than 20% of the critical wave speed), the accuracy of the FEM solution is poor.

A review of the literature devoted to numerical methods applied to moving mass problems will be given in Chapter 4.

1.2 Solution Methods

In the early period of the rapid development of computer methods, falling in the eighties of the last century, researchers analysed and described the basic properties of discrete methods of calculation. These included:

- the impact of the finite element mesh density on the results, the estimation of the error of approximation,
- reducing the size of the task by using techniques of static and dynamic condensation, the division into subsystems, etc.
- creation and study of the properties of new, more accurate finite element models, mainly bending elements, the analysis of the locking of degrees of freedom, over-stiffening, the inclusion of complex constitutive relations,
- the development of methods for the integration of the differential equations of motion, characterized by unconditional stability, low computational cost, and appropriately matched characteristics of spurious damping.

The capabilities of the known techniques were combined (finite difference and finite element method) and new methods were formulated (the boundary element method, moving elements, meshless methods). The limited computational ability of computers still forced work on improving the performance of the computing algorithms. With the increasing power of processors and reductions in the costs of memory, the effort of software developers has shifted to improve the utilisation of existing computational programs: improved data input methods and attractive forms of visualisation of the results. Computer programs were widely used in engineering practice.

Today, computer modelling generally involves the phenomenon of change over time. Both knowledge and computer tools allow you to take into account many factors influencing the processes in structures with complex shape. At the same time, less and less importance is attached to the evaluation of the correctness of the results, and attention is focused on a faithful reproduction of the geometry. Geometric modelling, an appropriate choice of the type of finite elements, and then imaging the stress fields, are the activities which usually limit the operation engineer, i.e., the user of the computing package. Less time is devoted to understandng the numerical and mechanical properties of the models created. Hence, in many cases, the results obtained are difficult to interpret. Effects arising from properties of the numerical model emerge. Sometimes they may be mistakenly regarded as the characteristic features of the phenomenon studied. Differences in the results obtained with two different commercial packages are no longer a cause for concern. Knowledge and experience is slowly being replaced by knowledge about the flaws in package design and how to overcome the technical difficulties encountered. Packages designed for crashworthiness analysis can be a good example. The explicit time integration method of differential equations used in the computations turns out to be unstable in the case of lightweight discrete elements, or those with small dimensions, or which are relatively rigid. In this case, there is a way to prevent instability by artificially increasing the weight of selected points. A stable solution is obtained, and then the