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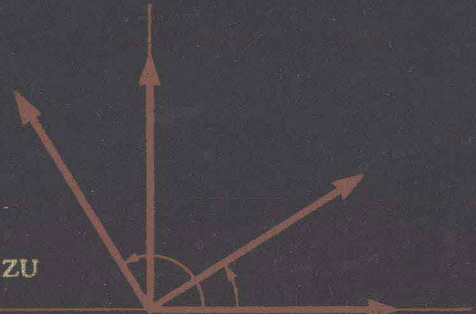

Fundamentals of Linear Algebra

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FUNDAMENTALS OF LINEAR ALGEBRA

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Preface

The importance of linear algebra in the undergraduate mathematics curriculum is now so well recognized that it is hardly necessary to make any comment on why linear algebra should be taught. For the practical problem of how to teach it, one has to find an appropriate answer—depending on the purpose of a given course and the level of the students in it.

The present book is primarily intended for use in a two-semester course on linear algebra combined with the elements of modern algebra and analytic geometry of n dimensions. It is designed, however, to be usable for a one-semester course on linear algebra alone or a one-semester course on linear algebra with analytic geometry. The presentation can be adjusted to an elementary level following a calculus course or to a more advanced level with a rigorous algebraic approach. Some specific recommendations are given in the Suggestions for Class Use.

The following is a brief description of the main content. After an introductory chapter (Chapter 1) explaining the motivations of the subject from various points of view, we develop the basic concepts and results on vector spaces, linear mappings, matrices, systems of linear equations, and bilinear functions in Chapters 2, 3, and 4. In Chapter 5 we introduce some basic algebraic concepts (fields, polynomials and their factorizations, rings, extensions of fields, modules) in order to make it possible to develop linear algebra in a general algebraic setting. In Chapter 6 the theory of determinants is treated for matrices over a commutative ring with identity by way of alternating n-linear functions. In Chapter 7 we discuss minimal polynomials and characteristic polynomials (including the Cayley-Hamilton theorem) and their applications (in particular, the Jordan forms); Schur's lemma and complex structures are also treated. In Chapter 8 we deal with inner product spaces and prove the spectral decomposition theorems for normal transformations, in particular, hermitian, unitary, symmetric, and orthogonal transformations. As explained in the Suggestions for Class Use, we indicate various proofs for these theorems in the exercises.

Chapters 9 and 10 provide a linear algebra approach to analytic geometry of n dimensions, which is the most efficient way of introducing rigorously geometric concepts in affine and euclidean spaces. The introductory material in Sections 1.3 and 1.4 serves as a preview for the full geometric

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development of linear algebra in these last two chapters. The knowledge of n-dimensional analytic geometry is basic for the study of topology, algebraic geometry, and differential geometry; nevertheless, the author has often noticed remarkable lack of that basic knowledge among many students of mathematics.

We have given many examples in order to illustrate important points of discussion and computational techniques or to introduce the standard models for the concepts at hand. Exercises at the end of each section are of the following three kinds:

1. To test the understanding of basic concepts and techniques given in the text
2. To offer more challenging problems of genuine interest based on the text material
3. To provide supplementary results and alternative proofs in order to amplify the understanding of the text material

Problems of the second and third kinds are starred.

After learning the material in this book, a reader will certainly be ready to proceed to a more advanced study of linear algebra in its most prolific sense; for example, through the theory of modules to homological algebra, through the theory of matrix groups to Lie groups and Lie algebras, through the theory of exterior algebras to differential and integral calculus on differentiable manifolds, through the theory of tensor algebras, projective and other geometries to differential geometry, through the theory of Banach and Hilbert spaces to functional analysis, and so on. We should have liked to include at least an elementary introduction to some of these subjects, but they had to be left out entirely. It is hoped that the present book will give the reader a balanced background in linear algebra before he specializes in various directions.

In concluding the preface I should like to acknowledge the invaluable help I have received from Mr. Carl Pomerance, a student at Brown University, who has critically read the manuscript and suggested numerous improvements in the presentation. My thanks go also to Mrs. Marina Smyth for her expert help in proofreading.

KATSUMI NOMIZU

Suggestions for Class Use

1. *For a short course* the following material may be used: Sections 1.1, 1.2 (or 1.3); Chapter 2; Chapter 3; Chapter 6; Section 7.1; Sections 8.1, 8.2, followed by one or two theorems selected out of Theorems 8.21, 8.22, 8.25, 8.30 (and their corollaries). For this selection observe the following:

a. Throughout the whole treatment, treat vector spaces and matrices over the real number field $\mathcal{R}$ or the complex number field $\mathcal{C}$. Thus the notation $\mathcal{F}$ will always stand for $\mathcal{R}$ or $\mathcal{C}$. In Chapter 6, a commutative ring with identity is to be replaced by $\mathcal{R}$ or $\mathcal{C}$. In Section 7.1, the characteristic polynomial has to be defined less formally.

b. Instead of (*a*), one may insert Section 5.1 between Sections 2.2 and 2.3 so that one can treat vector spaces and matrices over an arbitrary field $\mathcal{F}$.

c. For the proofs of Theorems 8.21, 8.22, 8.25, 8.30 follow the suggestions in suggestion 5.

2. *For a short course with emphasis on geometry* the following material may be used: Sections 1.3, 1.4; Chapter 2; Sections 3.1 to 3.3; Sections 6.1 to 6.4; Sections 8.1, 8.2 followed by Theorem 8.30; Chapter 9; Chapter 10.

a. One may treat only vector spaces and matrices over $\mathcal{R}$.

b. For the proof of Theorem 8.30, follow the suggestions in 5.

3. *A more satisfactory treatment of linear algebra with the elements of modern algebra* can be given by Chapters 1 to 8.

a. Sections 1.3, 1.4 may be omitted, although it is always recommended to illustrate various concepts on vector spaces and linear transformations by using geometric interpretations.

b. For less emphasis on algebra, one may introduce only the material in Chapter 5 that is absolutely necessary for the development of linear algebra as the need arises.

4. There are various ways of arriving at the *spectral theorems for normal transformations (matrices)*, in particular hermitian and unitary transformations (matrices) in the complex case and symmetric and orthogonal transformations (matrices) in the real case. They are developed along the following lines in the main text.

a. Theorem 8.19 (complex normal) and Theorem 8.20 (real normal) are based on Theorem 7.8 (on the minimal polynomial). Theorem 8.21 (hermitian) and Theorem 8.22 (unitary) follow immediately by using the results in Section 8.4.

b. Theorem 8.25 (symmetric) follows from Theorem 8.20 as soon as Theorem 8.24 (that a symmetric transformation has the real characteristic roots) is proved, and this is proved in two ways.

c. Theorem 8.30 (orthogonal) is proved first for the two-dimensional case and then by using the argument on the minimal polynomial (Theorem 7.5).

5. For alternative proofs of the spectral theorems we suggest the following:

a. Theorem 8.21 (hermitian) can be proved as in Exercise 8.4, number 15 (thus before introducing normal transformations and without reference to the minimal polynomial).

b. Theorem 8.22 (unitary) can be proved as in Exercise 8.4, number 16, in the same way as (*a*).

c. Theorem 8.19 (complex normal) can be proved as in Exercise 8.5, number 8, and Theorem 8.20 (real normal) as in Exercise 8.5, number 9.

d. Theorem 8.25 (symmetric) can be proved as in Exercise 8.16, number 18, once the existence of an eigenvalue (a real characteristic root) is established (as in Exercise 8.6, number 16 or 17, without reference to hermitian transformations).

e. Theorem 8.30 (orthogonal) can be proved as in Exercise 8.7, number 8 (by using Proposition 8.16) or as in Exercise 8.7, number 9 (by using Theorem 8.25).

f. There are other variations; see Exercises 8.5, numbers 7 and 15, Exercise 8.6, number 19, and Exercise 8.7, numbers 12 and 13.

6. Finally, a word about the *terminology used in the text.* We assume familiarity with the notation concerning sets, mappings, and equivalence relations. We also assume that a reader is acquainted with the principle of mathematical induction. Since these ideas are now introduced at an early stage in many courses in calculus, we shall give only a very concise explanation of the terminology in the Appendix.

Contents

1 Introduction

The purpose of this chapter is to motivate the study of linear algebra. We shall show how the concepts of vector spaces, linear mappings, and matrices arise naturally from various types of problems in mathematics. Discussions are rather informal and are not to be considered part of a systematic development (which starts in Chap. 2); Definition 1.3, on matrix multiplication, and Definition 1.5, on identity matrices, will be referred to later.

1.1 SYSTEMS OF LINEAR EQUATIONS

To start with an easy example, we recall how we can solve a system of two linear equations in two unknowns:

$$\begin{aligned} ax + by &= u, \\ cx + dy &= v, \end{aligned} \tag{1.1}$$

where a, b, c, d and u, v are given (real) numbers and x, y are unknowns. We assume that a, b are not both 0 and c, d are not both 0. Multiplying the first equation by d and subtracting from it b times the second equation, we obtain

$$(ad - bc)x = du - bv.$$

Similarly, by eliminating x, we obtain

$$(ad - bc)y = av - cu.$$

If $ad - bc \neq 0$, then we find the solution

$$x = \frac{du - bv}{ad - bc}, \qquad y = \frac{av - cu}{ad - bc}.$$

In the case where $ad - bc = 0$, we proceed as follows. If $c \neq 0$, let $k = a/c$, so that $a = ck$. Substituting this in $ad = bc$, we have $cdk = cb$. Since $c \neq 0$, we get $b = dk$. If $c = 0$, then $d \neq 0$ and we let $k = b/d$ and still obtain $a = ck$. If $k = 0$, we have $a = b = 0$, contrary to the assumption. Thus we see that there is $k \neq 0$ such that

$$a = ck \qquad \text{and} \qquad b = dk.$$

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Multiplying the second equation of (1.1) by k, we have

$$ckx + dky = kv,$$

that is,

$$ax + by = kv.$$

If $u = kv$, then this is the same as the first equation of (1.1). In this case the system (1.1) has infinitely many solutions (x,y), which are determined from the first equation by giving arbitrary values to x (or y). On the other hand, if $u \neq kv$, then (x,y) satisfying the second equation does not satisfy the first equation; that is, the system (1.1) has no solution at all.

Summing up, we have:

1. If there is no number $k \neq 0$ such that $a = ck$, $b = dk$, then (1.1) has a unique solution.

2. If there is a number $k \neq 0$ such that $a = ck$, $b = dk$, and $u = vk$, then (1.1) has infinitely many solutions.

3. If there is a number $k \neq 0$ such that $a = ck$, $b = dk$ but $u \neq vk$, then (1.1) has no solution at all.

A geometric interpretation is the following. Each of the equations in (1.1) represents a straight line on the plane with the usual coordinate system. In case 1, two straight lines are distinct and meet at one point. In case 2, two lines coincide. In case 3, two lines are distinct but parallel to each other.

We shall now consider a more general system of linear equations —a system of m linear equations in n unknowns $x_1, \ldots, x_n$—

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= u_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= u_2, \\ \cdots\cdots\cdots\cdots\cdots\cdots &\cdots, \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= u_m, \end{aligned} \tag{1.2}$$

where the essential data of the system are the mn coefficients a_{ij}, $1 \leq i \leq m$, $1 \leq j \leq n$, and the numbers u_i, $1 \leq i \leq m$. We may express them by writing

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_m \end{bmatrix}$$

On the other hand, the set of unknowns x_i, $1 \leq i \leq n$, can be written as

$$\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}.$$

Definition 1.1

A display of mn numbers a_{ij} in the form above is called an $m \times n$ *matrix;* we denote this matrix, for example, by

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

or, more briefly, by

$$\mathbf{A} = [a_{ij}], \qquad 1 \leq i \leq m,\ 1 \leq j \leq n.$$

Similarly, we set

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_m \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}.$$

Although $\mathbf{u}$ is an $m \times 1$ matrix and $\mathbf{x}$ an $n \times 1$ matrix, it is more common to say that $\mathbf{u}$ is an m-dimensional *vector* and $\mathbf{x}$ an n-dimensional vector.

Definition 1.2

Two $m \times n$ matrices

$$\mathbf{A} = [a_{ij}] \qquad \text{and} \qquad \mathbf{B} = [b_{ij}]$$

are said to be *equal* (written $\mathbf{A} = \mathbf{B}$) if $a_{ij} = b_{ij}$ for every pair (i,j), where $1 \leq i \leq m$ and $1 \leq j \leq n$.

Similarly, two m-dimensional vectors

$$\mathbf{y} = [y_i] \qquad \text{and} \qquad \mathbf{z} = [z_i]$$

are equal ($\mathbf{y} = \mathbf{z}$) if $y_i = z_i$ for every i, $1 \leq i \leq m$.

System (1.2) is thus described by an $m \times n$ matrix $\mathbf{A}$ and an m-dimensional vector $\mathbf{u}$; the set of unknowns is expressed by an n-dimensional vector $\mathbf{x}$. We shall even write system (1.2) in the form

(1.3) $$\mathbf{Ax} = \mathbf{u}.$$

This matrix notation for system (1.2) will be justified when $\mathbf{Ax}$ on the left-hand side acquires the meaning of a "product" of the matrix $\mathbf{A}$ and the vector $\mathbf{x}$. For this purpose we shall define matrix multiplication in the following way.

Definition 1.3

Given an $m \times n$ matrix $\mathbf{A} = [a_{ij}]$, $1 \le i \le m$, $1 \le j \le n$, and an $n \times p$ matrix $\mathbf{B} = [b_{jk}]$, $1 \le j \le n$, $1 \le k \le p$, we form an $m \times p$ matrix $\mathbf{C} = [c_{ik}]$, $1 \le i \le m$, $1 \le k \le p$, where

$$(1.4) \qquad c_{ik} = \sum_{j=1}^{n} a_{ij}b_{jk}, \qquad 1 \le i \le m,\ 1 \le k \le p.$$

The matrix **C** is called the *product* of **A** and **B** and is denoted by **AB**.

In the special case where **B** is an $n \times 1$ matrix, namely, an n-dimensional vector $[b_j]$, $1 \le j \le n$, we have an $m \times 1$ matrix, namely, an m-dimensional vector, as the product $\mathbf{AB} = [c_i]$, $1 \le i \le m$, where

$$(1.4') \qquad c_i = \sum_{j=1}^{n} a_{ij}b_j, \qquad 1 \le i \le m.$$

Example 1.1

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 2(-1) + 1(0) & 2(4) + 1(-2) \\ 1(-1) + 3(0) & 1(4) + 3(-2) \end{bmatrix} = \begin{bmatrix} -2 & 6 \\ -1 & -2 \end{bmatrix}.$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} 5 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 2(5) + (-1)(-1) + 1(4) \\ 3(5) + (-2)(-1) + 4(4) \end{bmatrix} = \begin{bmatrix} 15 \\ 33 \end{bmatrix}.$$

Now (1.3) acquires the following meaning: The product **Ax** is equal to the vector **u**; in fact,

$$(1.3') \qquad \sum_{j=1}^{n} a_{ij}x_j = u_i, \qquad 1 \le i \le m,$$

which is exactly the same as system (1.2).

Let us first consider a *homogeneous system:*

$$(1.5) \qquad \sum_{j=1}^{n} a_{ij}x_j = 0, \qquad 1 \le i \le m,$$

which is a special case of (1.3′) where all u_i's are 0. Writing **0** for the m-dimensional vector (called the *m-dimensional zero vector*)

$$\begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix},$$

we have the matrix notation for (1.5) in the form

$$(1.5') \qquad \mathbf{Ax} = \mathbf{0}.$$

This system has an obvious solution $x_1 = x_2 = \cdots = x_n = 0$, namely,

$$\mathbf{x} = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \qquad n\text{-dimensional zero vector.}$$

This solution is called the trivial solution of (1.5′); it may be the only solution, or there may be other solutions. At any rate, consider the set S of all solutions [namely, n-dimensional vectors satisfying (1.5′)].

If $\mathbf{x} = [x_i]$ and $\mathbf{x}' = [x_i']$ are in S, then the vectors

$$\begin{bmatrix} x_1 + x_1' \\ x_2 + x_2' \\ \cdot \\ \cdot \\ \cdot \\ x_n + x_n' \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} cx_1 \\ cx_2 \\ \cdot \\ \cdot \\ \cdot \\ cx_n \end{bmatrix}, \qquad c \text{ arbitrary,}$$

which we denote by $\mathbf{x} + \mathbf{x}'$ and $c\mathbf{x}$, respectively, are also solutions. Thus the set of solutions S has the property that $\mathbf{x}, \mathbf{x}' \in S$ implies $\mathbf{x} + \mathbf{x}' \in S$ and $c\mathbf{x} \in S$ for any number c.

Definition 1.4

Let $\mathbf{x} = [x_i]$ and $\mathbf{x}' = [x_i']$ be two n-dimensional vectors. The *sum* $\mathbf{x} + \mathbf{x}'$ is the n-dimensional vector

$$\begin{bmatrix} x_1 + x_1' \\ \cdot \\ \cdot \\ \cdot \\ x_n + x_n' \end{bmatrix},$$

and the *scalar multiple* $c\mathbf{x}$, where c is an arbitrary number, is the n-dimensional vector

$$\begin{bmatrix} cx_1 \\ \cdot \\ \cdot \\ \cdot \\ cx_n \end{bmatrix}.$$

In connection with the sums and scalar multiples of vectors, let us observe that matrix multiplication has the following two properties:

$$\mathbf{A}(\mathbf{x} + \mathbf{x}') = \mathbf{A}\mathbf{x} + \mathbf{A}\mathbf{x}', \tag{1.6}$$

$$\mathbf{A}(c\mathbf{x}) = c(\mathbf{A}\mathbf{x}), \tag{1.7}$$

where $\mathbf{A}$ is an $m \times n$ matrix and $\mathbf{x}$, $\mathbf{x}'$ are n-dimensional vectors. In fact, the ith entry of the vector $\mathbf{A}(\mathbf{x} + \mathbf{x}')$ is equal to

$$\sum_{j=1}^{n} a_{ij}(x_j + x_j') = \sum_{j=1}^{n} a_{ij}x_j + \sum_{j=1}^{n} a_{ij}x_j',$$

which is equal to the ith entry of $\mathbf{Ax} + \mathbf{Ax}'$. Property (1.7) can be verified in a similar way.

The assertion on S which we made above can be considered as a consequence of (1.6) and (1.7); in fact, if $\mathbf{Ax} = \mathbf{0}$ and $\mathbf{Ax}' = \mathbf{0}$, then

$$\mathbf{A}(\mathbf{x} + \mathbf{x}') = \mathbf{Ax} + \mathbf{Ax}' = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

and

$$\mathbf{A}(c\mathbf{x}) = c\mathbf{Ax} = c\mathbf{0} = \mathbf{0},$$

showing that $\mathbf{x} + \mathbf{x}'$ and $c\mathbf{x}$ are solutions.

Let us now consider the system (1.3), called an *inhomogeneous system* if $\mathbf{u} \neq \mathbf{0}$; we call (1.5) the homogeneous system associated with (1.3). If $\mathbf{x}$ and $\mathbf{x}'$ are solutions of (1.3), that is,

$$\mathbf{Ax} = \mathbf{u} \qquad \text{and} \qquad \mathbf{Ax}' = \mathbf{u},$$

then, denoting $\mathbf{x}' + (-1)\mathbf{x}$ by $\mathbf{x}' - \mathbf{x}$, we have

$$\begin{aligned} \mathbf{A}(\mathbf{x}' - \mathbf{x}) &= \mathbf{A}(\mathbf{x}' + (-1)\mathbf{x}) = \mathbf{Ax}' + (-1)\mathbf{Ax}' \\ &= \mathbf{Ax}' - \mathbf{Ax} = \mathbf{u} - \mathbf{u} = \mathbf{0}, \end{aligned}$$

by virtue of (1.6) and (1.7). This shows that $\mathbf{x}' - \mathbf{x} = \mathbf{y}$ is a solution of (1.5). Conversely, suppose that $\mathbf{x}$ is a solution of (1.3) and $\mathbf{y}$ a solution of (1.5). Then $\mathbf{x}' = \mathbf{x} + \mathbf{y}$ is a solution of (1.5), because

$$\mathbf{Ax}' = \mathbf{A}(\mathbf{x} + \mathbf{y}) = \mathbf{Ax} + \mathbf{Ay} = \mathbf{u} + \mathbf{0} = \mathbf{u}.$$

We have seen that an arbitrary (or general) solution of system (1.3) is obtained from any particular solution by adding an arbitrary (or general) solution of the associated homogeneous system (1.5).

In order to find a solution of (1.3), one will, of course, try to reduce the system to a system of a simpler form which has the same solutions. One employs a number of elimination steps, the simplest form of which we recalled for system (1.1). One of the processes consists in multiplying one equation by a certain number and adding it to another equation. For example, we replace the jth equation of (1.3) by

$$(ca_{i1} + a_{j1})x_1 + \cdots + (ca_{in} + a_{jn})x_n = cu_i + u_j.$$

For the corresponding matrix $\mathbf{A}$, this process will change the jth row

$$[a_{j1} \quad a_{j2} \quad \cdots \quad a_{jn}]$$

into

$$[ca_{i1} + a_{j1} \quad ca_{i2} + a_{j2} \quad \cdots \quad ca_{in} + a_{jn}]$$

with the accompanying change of **u** into

$$\begin{bmatrix} u_1 \\ \cdot \\ \cdot \\ \cdot \\ u_{j-1} \\ cu_i + u_j \\ u_{j+1} \\ \cdot \\ \cdot \\ \cdot \\ u_m \end{bmatrix}.$$

It is obvious that this sort of operation does not change the solutions. The same thing is true of multiplying one equation by a nonzero number, which is the other kind of process one uses for solving a system of linear equations.

This indicates that the practical method of solving a system of linear equations can be described neatly as a sequence of certain operations performed on the matrix **A** and the vector **u**.

EXERCISE 1.1

1. Compute the following matrix products:

(*a*) $\begin{bmatrix} 2 & -1 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$; (*b*) $\begin{bmatrix} 3 & -1 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$;

(*c*) $\begin{bmatrix} 3 & 1 & 2 \\ 1 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 3 & 2 & 5 \\ 1 & 3 & 4 \end{bmatrix}$; (*d*) $\begin{bmatrix} 1 & -1 & 4 \\ 2 & 3 & 3 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & -1 \\ 1 & 5 \end{bmatrix}$;

(*e*) $[1 \quad -1 \quad 1]\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

2. Are the following products defined?

(*a*) $\begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$; (*b*) $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}[1 \quad -1 \quad 1]$.

3. Solve the following systems of linear equations:

(*a*) $\begin{aligned} x + y + z &= 1, \\ 3x - 2y + 2z &= 3, \\ 2x - y - z &= -1. \end{aligned}$ (*b*) $\begin{aligned} x + y + z &= 1, \\ 3x - 2y + 2z &= 3, \\ 5y + z &= 4. \end{aligned}$

1.2 DIFFERENTIAL EQUATIONS

Let us consider a differential equation

$$\frac{d^2x}{dt^2} + x = 0, \tag{1.8}$$

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