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Tenth International Conference on Numerical Methods in Fluid Dynamics

Proceedings, Beijing 1986

Edited by F. G. Zhuang and Y. L. Zhu

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Proceedings of the Conference
Held at the Beijing Science Hall, Beijing, China
June 23–27, 1986

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PREFACE

This issue of Lecture Notes in Physics contains the Proceedings of the Tenth International Conference on Numerical Methods in Fluid Dynamics, held at the Beijing Science Hall in China, June 23-27, 1986. The Proceedings include all the papers presented at the Conference, namely, the inaugural lecture by K. Feng, the invited lectures by V.P. Dymnikov, M.Y. Hussaini, P. Kutler, M. Napolitano, N. Satofuka, F.G. Zhuang, and H.X. Zhang, as well as 108 contributed papers arranged in alphabetical order of the first author's name. The contributed papers were selected from abstracts submitted from all over the world by four Committees on Paper Selection based in China, Europe, the U.S.A., and the U.S.S.R. and headed by the editors (China), Temam (Europe), Holt (U.S.A.), Chernyi and Rusanov (U.S.S.R.).

The Conference was attended by over 200 scientists. In addition to the strong representation from China, a large number of scientists from the U.S.A., Japan, France, Italy, West Germany, the U.S.S.R., the Netherlands, Ireland, Canada, the United Kingdom, Belgium, Sweden, Australia, Brazil, and Norway participated at the Conference. A list of the participants is given at the end of the Proceedings.

We served as general conference cochairmen and are indebted to the many colleagues who helped with the details of the meeting. In particular, our thanks go to all the members of the International Organizing Committee and the Local Committee for the Conference, who were in charge of all academic activities, as well as to Mr. C.S. He of the Chinese Aerodynamics Research Society and Mr. Y. Cao of the China International Conference Center for Science and Technology, who supervised all of the local arrangements.

Financial support for the Conference was provided by the China Aerodynamics Research and Development Center. Peking University, the Computer Center of Academia Sinica and the Institute of Computer Technology of Academia Sinica helped the Conference in many ways. We greatly appreciate their supports.

We are also indebted to Prof. W. Beiglböck and Ms. C. Pendl for valuable assistance in preparing these Proceedings.

August 1986

F.G. Zhuang and Y.L. Zhu
(Editors)

编 者 前 言

第十届国际流体力学数值方法会议於一九八六年六月廿三～廿七日在北京科学会堂召开．本书是该会议的论文集．其中我们收集了会议中的全部文章．它们是冯康教授的开幕学术演讲，Dymnikov 博士，Hussaini 博士，Kutler 博士，Napolitano 教授，Satofuka 教授，庄逢甘和张涵信教授的特邀报告，以及１０８篇“入选”文章．在此文集中，每一类文章是以第一作者的姓名按字母顺序编排的．“入选”文章是由四个选文委员会根据提交来的文章选定的．这四个选文委员会分别设在中国，欧洲，美国和苏联， 主席是本卷的编者〔中国〕，Temam 教授〔欧洲〕，Holt 教授〔美国〕，Chernyi 和 Rusanov 教授〔苏联〕．

参加此届会议，除了大量的中国科学家以外，还有来自美国，日本，法国，意大利，联邦德国，苏联，荷兰，爱尔兰，加拿大，英国，比利时，瑞典，澳大利亚，巴西，挪威的许多科学家，共计二百多位．本会议录的末尾给出了出席者的名单．

作为此届大会主席，我们在此衷心感谢在会议的组织工作中给了我们各种帮助的所有同事们，特别是在学术活动方面做了许多工作的国际和国内组织委员会的委员们，和负责会务工作的中国空气动力学研究会的贺长胜等同事和中国科协国际会议中心的曹跃等同事．

中国空气动力研究和发展中心给了这次会议以财政上的支持．北京大学，中国科学院计算中心和计算所也对会议给予了很多支持．我们在此一并表示感谢．

最后，我们还要对 Beiglbock 教授和 Pendl 女士在准备此会议录方面所给予的热情帮助表示诚挚的谢意．

庄逢甘　　　朱幼兰

一九八六年八月

ACKNOWLEDGEMENTS

At the end of the 10th International Conference on Numerical Methods in Fluid Dynamics, Professor Henri Cabannes, of Mécanique Théorique, Université Pierre et Marie Curie, Paris, stepped down as Secretary of the Organizing Committee, to be replaced by Dr. Soubbaramayer. Professor Cabannes has served in this capacity since the beginning of the 3rd International Conference on Numerical Methods in Fluid Dynamics, which he organized in Paris in 1972, and has worked without respite to ensure the success of the conference series. He established a permanent office for the Organizing Committee in Paris, attended to all correspondence connected with reports of past conferences and preparation of forthcoming conferences, gave invaluable guidance to the committee on such matters as the choice of sites for the conference, selection of speakers and financial support. Because of the unusual international character of the conference, Professor Cabannes has had to exercise considerable diplomatic skill and use a great deal of his valuable time in resolving organizational and personnel problems which arose during the 14 years of his tenure. Past participants in the conference will surely wish to join the Organizing Committee in giving warm thanks to Professor Cabannes for his long and consistent service to the conference. We hope that his advice will continue to be available to the committee far into the future.

October 1986 The Organizing Committee

INTERNATIONAL CONFERENCE ON
NUMERICAL METHODS IN FLUID DYNAMICS

First Conference:	Novosibirsk, USSR, 1969
Second Conference:	Berkeley, California, USA, 1970
Third Conference:	Paris, France, 1972
Fourth Conference:	Boulder, Colorado, USA, 1974
Fifth Conference:	Enschede, the Netherlands, 1976
Sixth Conference:	Tbilisi, USSR, 1978
Seventh Conference:	Stanford University and NASA/Ames, USA, 1980
Eighth Conference:	Aachen, West Germany, 1982
Ninth Conference:	Saclay, France, 1984
Tenth Conference:	Beijing, China, 1986

SYMPLECTIC GEOMETRY AND NUMERICAL METHODS IN FLUID DYNAMICS

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1. INTRODUCTION

It is an honor and a pleasure for me to present the inaugural talk at the Tenth International Conference on Numerical Methods in Fluid Dynamics in Beijing. I want to thank the Organizing Committee, its Secratory, Prof. H. Cabannes, the Conference Chairman, Prof. F.G.Zhuang, and the Co-chairman, Prof. Y.L.Zhu for the kind invitation.

We present a brief survey of considerations and results of a study [1,2,3,4,6], undertaken by the author and his group, on the links between the Hamiltonian formalism and the numerical methods for solving dynamical problems expressed in the form of the canonical system of differential equations

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}, \quad \frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \quad i = 1,\cdots, n \tag{1.1}$$

with given Hamiltonian function $H(p_1,\cdots, p_n, q_1,\cdots, q_n)$.

The canonical system (1.1) with remarkable elegance and symmetry was introduced by Hamilton as a general mathematical scheme, first for problems of geometrical optics in 1824, then for conservative dynamical problems in 1834. The approach was followed and developed further by Jacobi into a well-established mathematical formalism for analytical dynamics, which is an alternative of, and equivalent to, the Newtonian and Lagrangian formalisms. The geometrization of the Hamiltonian formalism was undertaken by Poincare in 1890's and by Cartan, Birkhoff, Weyl, Siegel, etc., in the 20th century; this gave rise a new dicipline, called symplectic geometry, which serves as the mathematical foundation of the Hamiltonian formalism.

It is known that, Hamiltonian formalism, apart from its classical links with analytical mechanics, geometrical optics, calculus of variations and non-linear PDE of first order, has inherent connections also with unitary representations of Lie groups, geometric quantization, pseudo-differential and Fourier integral operators, classification of singularities, integrability of non-linear evolution equations, optimal control theory, etc.. It is also under extension to infinite dimensions for various field theories, including fluid dynamics, elasticity, electrodynamics, plasma physics, relativity, etc.. Now it is almost certain that all real physical processes with negligible dissipation can be described, in some way or other, by Hamiltonian formalism, so the latter is becoming one of the most useful tools in the mathematical arsenal of physical and engineering sciences. In this way, a systematic study of numerical methods of Hamiltonian systems is motivated and would eventually lead to more general applicability and more direct accessibility of the Hamiltonian formalism. We try to conceive, design, analyse and evaluate difference schemes and algorithms specifically within the framework of symplectic geometry. The approach proves to be quite successful as one might expect, we actually derive in this way numerous "unconventional" difference schemes. Due to historical reasons, classical symplectic geometry, however, lacks the "computational" component in the modern sense. Our present study might be considered as an attempt to fill the blank.

In the following, vectors are always represented by column matricės, matrix transpose is denoted by prime '. Let $z=(z_1,\cdots,z_n, z_{n+1},\cdots,z_{2n})'=(p_1,\cdots,p_n,q_1,\cdots, q_n)'$,

$$H_z = [\frac{\partial H}{\partial p_1},\cdots,\frac{\partial H}{\partial p_n},\frac{\partial H}{\partial q_1}\cdots,\frac{\partial H}{\partial q_n}]',$$

$$J_{2n}= J =\begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}, \qquad J' = J^{-1} = -J.$$

(1.1) can be written as

$$\frac{dz}{dt} = J^{-1} H_z , \tag{1.2}$$

defined in phase space R^{2n} with a standard <u>symplectic structure</u> given by the non-singular anti-symmetric closed differential 2-form

$$\omega = \Sigma\, dz_i \wedge dz_{n+i} = \Sigma\, dp_i \wedge dq_i .$$

According to <u>Darboux Theorem</u>, the symplectic structure given by any non-singular anti-symmetric closed differential 2-form can be brought to the above standard form, at least locally, by suitable change of co-ordinates.

The <u>Fundamental Theorem on Hamiltonian Formalism</u> says that the solution $z(t)$ of the canonical system (1.2) can be generated by a <u>one-parameter group</u> $G(t)$, depending on the given Hamiltonian H, of <u>canonical transformations</u> of R^{2n} (locally in t and z) such that

$$z(t) = G(t)\, z(0).$$

A transformation $z \to \hat{z}$ of R^{2n} is called <u>canonical</u>, or <u>symplectic</u>, if it is a local diffeomorphism whose Jacobian $\frac{\partial \hat{z}}{\partial z} = M$ is everywhere symplectic, i.e.

$$M'JM = J, \quad \text{i.e.} \quad M \in Sp(2n).$$

The canonicity of $G(t)$ implies the preservation of 2-form ω, 4-form $\omega\wedge\omega$, $\cdots$, 2n-form $\omega\wedge\omega\wedge\cdots\wedge\omega$. They constitute the class of <u>conservation laws of phase area</u> of even dimensions for the Hamiltonian system (1.2).

Moreover, the Hamiltonian system possesses another class of conservation laws related to the <u>energy</u> $H(z)$. A function $\varphi(z)$ is said to be an <u>invariant integral</u> of (1.2) if it is invariant under (1.2)

$$\varphi(z(t)) \equiv \varphi(z(0))$$

which is equivalant to

$$\{\ \varphi,\ H\ \} = 0,$$

where the Poisson Bracket for two functions $\varphi(z)$, $\psi(z)$ are defined as

$$\{\ \varphi,\ \psi\ \} = \varphi_z' J^{-1}\ \psi_z\ .$$

H itself is always an invariant integral, see, e.g., [5].

The above digressions on Hamiltonian systems suggest the following guidelines for the numerical study of dynamical problems: The problem should be expressed in some suitable Hamiltonian formalism. The numerical schemes should preserve as much as possible the characteristic properties and inner symmetries of the original system. The transition from the k-th time step z^k to the next (k+1)-th time step z^{k+1} should be canonical for all k and, moreover, the invariant integrals of the original system should remain invariant under these transitions.

2. CANONICAL DIFFERENCE SCHEMES FOR LINEAR CANONICAL SYSTEMS

Consider the case for which the Hamiltonian is a quadratic form

$$H(z) = \frac{1}{2}\ z'Sz, \qquad S' = S, \qquad H_z = Sz, \tag{2.1}$$

then the canonical system

$$\frac{dz}{dt} = Lz, \qquad L = J^{-1}S \tag{2.2}$$

is linear, where L is infinitesimally symplectic, i.e. L satisfies $L'J + JL = 0$. The solution of (2.2) is

$$z(t) = G(t)\ z(0),$$

where $G(t) = \exp tL$, as the exponential transform of infinitesimally symplectic tL, is symplectic.

It is easily seen that the weighted Euler scheme

$$\frac{1}{\tau}\ (z^{k+1} - z^k) = L(\alpha\ z^{k+1} + (1 - \alpha)\ z^k)$$

for the linear system (2.2) is symplectic if and only if $\alpha = \frac{1}{2}$, i.e. it is the case of time-centered Euler Scheme with the transition matrix F_τ,

$$z^{k+1} = F_\tau z^k, \qquad F_\tau = \varphi(\tau L), \qquad \varphi(\lambda) = \frac{1 + \frac{\lambda}{2}}{1 - \frac{\lambda}{2}}\ , \tag{2.3}$$

F_τ, as the Cayley transform of infinitesimally symplectic τL, is symplectic. The 2nd order canonical Euler scheme (2.3) can be generalized to canonical schemes of arbitrary high order [2,3]. For example, by taking the matrix transform function $\varphi(\lambda)$ in (2.3) to be the diagonal Padé approximants $P_m(\lambda)/P_m(-\lambda)$ to the exponential function $\exp\lambda$, where

$$P_0(\lambda)=1,\ P_1(\lambda)=2+\lambda,\ P_2(\lambda)=12+6\lambda+\lambda^2,\cdots,\ P_m(\lambda)=2(2m-1)P_{m-1}(\lambda)+\lambda^2 P_{m-2}(\lambda),$$

we can prove that the difference schemes

$$z^{k+1} = \frac{P_m(\tau L)}{P_m(-\tau L)}\ z^k \qquad m = 1,2,\cdots \tag{2.4}$$

for the system (2.2) are symplectic, A-stable, of 2m-th order of accuracy, and having the same set of guadratic invariant integrals including $H(z)$ as that of system (2.2). The case $m=1$ is the time-centered Euler scheme (2.3).

For the general non-linear canonical system (1.2), the time-centered Euler scheme

$$\frac{1}{\tau}\,(z^{k+1} - z^k) = J^{-1}\,H_z\left(\frac{1}{2}(z^{k+1} + z^k)\right) \tag{2.5}$$

is canonical. However, unlike the linear case, the invariant integrals $\varphi(z)$ of system (1.2), including $H(z)$, are conserved only approximately

$$\varphi(z^{k+1}) - \varphi(z^k) = O(\tau^3).$$

The time-centered Euler schemes (2.3), (2.5) and their canonical generalizations (2.4) are all implicit. For the case of separable Hamiltonian

$$H(p, q) = U(p) + V(q),$$

one can construct time-staggered schemes which are canonical, of 2nd order accuracy and practically explicit [1,2], e.g.,

$$\begin{aligned} \frac{1}{\tau}\,(p^{k+1} - p^k) &= -V_q(q^{k+\frac{1}{2}}), \\ \frac{1}{\tau}\,(q^{k+1+\frac{1}{2}} - q^{k+\frac{1}{2}}) &= U_p(p^{k+1}). \end{aligned} \tag{2.6}$$

The p's are set at integer times $t = k\tau$, q's at half-integer times $t=(k + \frac{1}{2})\tau$. We need averaging, e.g., using

$$q^k = \frac{1}{2}\,(q^{k-\frac{1}{2}} + q^{k+\frac{1}{2}})$$

to compute the invariant integrals $\varphi(p, q)$ and get

$$\varphi(p^{k+1}, q^{k+1}) - \varphi(p^k, q^k) = O(\tau^3).$$

For the comparison of stability for the linear system (2.2) and the canonical schemes (2.4), (2.6) and the application of (2.6) to the wave equation, see [1].

3. CONSTRUCTION OF CANONICAL DIFFERENCE SCHEMES VIA GENERATING FUNCTIONS

A major component of the transformation theory in symplectic geometry is the method of generating functions, see, e.g., [5], which also play a central role for the construction of canonical difference schemes. In [2,4] a constructive general theory of generating functions is given, roughly as follows: Let

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \qquad T^{-1} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix},$$

T be a non-singular real matrix of order $4n$ satisfying

$$T' \begin{bmatrix} 0 & I_{2n} \\ -I_{2n} & 0 \end{bmatrix} T = \mu \begin{bmatrix} -J_{2n} & 0 \\ 0 & J_{2n} \end{bmatrix}, \quad \text{for some } \mu \neq 0. \tag{3.1}$$

T defines a linear transformation in product space $R^{2n} \times R^{2n}$ by

$$\begin{aligned} \hat{w} &= A\hat{z} + Bz \\ w &= C\hat{z} + Dz, \end{aligned} \qquad \begin{bmatrix} \hat{z} \\ z \end{bmatrix}, \begin{bmatrix} \hat{w} \\ w \end{bmatrix} \in R^{2n} \times R^{2n}. \tag{3.2}$$

Let $z \to \hat{z} = g(z,t)$ be a time-dependent canonical transformation defined by

$$g(z, t) = M_0\,G(z, -t) \tag{3.3}$$