



# *Excursions* IN MODERN MATHEMATICS

Sixth Edition

PETER  
TANNENBAUM

# Excursions in Modern Mathematics

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**Sixth Edition**

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# Excursions in Modern Mathematics

***To the members of the board of Last Tango***



# Preface

To most outsiders, modern mathematics is unknown territory. Its borders are protected by dense thickets of technical terms; its landscapes are a mass of indecipherable equations and incomprehensible concepts. Few realize that the world of modern mathematics is rich with vivid images and provocative ideas.

Ivars Peterson,  
*The Mathematical Tourist*

Everything should be made as simple as possible, but not simpler.

Albert Einstein

*Excursions in Modern Mathematics* is, as I hope the title might suggest, a collection of “trips” into that vast and alien frontier that many people perceive mathematics to be. While the formal purpose of this book is quite conventional—it is intended to serve as a textbook for a college-level liberal arts mathematics course—its philosophy and contents are not. My goal is to show the open-minded reader that mathematics is a lively, interesting, useful, and surprisingly rich human activity.

The “excursions” in this book represent a collection of topics chosen to meet the following simple criteria.

- **Applicability.** No need to ask here that great existential question of college mathematics: “What is this stuff good for?” The connection between the mathematics presented in these excursions and down-to-earth, concrete real-life problems is transparent and immediate.
- **Accessibility.** As a general rule, the excursions in this book do not presume a background beyond standard high school mathematics—by and large, Intermediate Algebra and a little Euclidean Geometry are appropriate and sufficient prerequisites. (In the few instances in which more advanced concepts are unavoidable, an effort has been made to provide enough background to make the material self-contained.) A word of caution—this does not mean that the excursions in this book are easy! In mathematics, as in many other walks of life, simple and straightforward is not synonymous with easy and superficial.
- **Modernity.** Unlike much of traditional mathematics, which is often hundreds of years old, most of the mathematics in this book has been discovered within the last 100 years, and in some cases within the last couple of decades. Modern mathematical discoveries do not have to be the exclusive province of professional mathematicians.
- **Aesthetics.** The notion that there is such a thing as beauty in mathematics is surprising to most casual observers. There is an important aesthetic component in mathematics and, just as in art and music (which mathematics very much resembles), it often surfaces in the simplest ideas. A fundamental objective of this book is to develop an appreciation for the aesthetic elements of mathematics.

## Outline

The excursions in the book are divided into four independent parts. Each of these parts consists of four chapters plus a “Mini-Excursion.” (Mini-Excursions are a new feature added to this edition. More details on these Mini-Excursions are given in the “What’s New in the Sixth Edition” section on page xvii.)

- **Part 1 The Mathematics of Social Choice.** This part deals with mathematical applications in social science. How are elections decided? (Chapter 1); How can power be measured? (Chapter 2); How can competing claims on property be resolved in a fair and equitable way? (Chapter 3); How are seats apportioned

in a legislative body? (Chapter 4); and, What is the current method for apportioning seats in the United States House of Representatives? (Mini-Excursion 1).

- **Part 2 Management Science.** This part deals with methods for solving problems involving the organization and management of complex activities—that is, activities involving either a large number of steps and/or a large number of variables. Typically, these problems revolve around questions of efficiency: some limited or precious resource (time, money, raw materials) must be managed in such a way that waste is minimized. Among the many applications discussed in this part are: What is a good delivery route for a mail carrier? (Chapter 5); How can a salesman organize a multi-city sales trip and save money? (Chapter 6); What is the best way to design a communication network? (Chapter 7); How do we create a schedule for constructing a home in the least amount of time? (Chapter 8); and, How do we color a map using the fewest number of colors? (Mini-Excursion 2).
- **Part 3 Growth and Symmetry.** This part deals with nontraditional notions of growth and shape. What are the mechanisms that govern spiral growth in natural shapes? (Chapter 9); How can mathematics be used to predict the rise and fall of populations? (Chapter 10); How can we analyze the symmetries of an object or a pattern? (Chapter 11); What type of geometric shape is a head of cauliflower, our circulatory system, or a mountain range? (Chapter 12); and How does time affect the value of money? (Mini-Excursion 3).
- **Part 4 Statistics.** In one way or another, statistics affects all of our lives. Government policy, insurance rates, our health, our diet, and public opinion are all governed by statistical data. This part deals with some of the most basic aspects of statistics. How should data be collected? (Chapter 13); How can large amounts of data be summarized so that the data makes some sense? (Chapter 14); How can we measure the uncertainty produced by random phenomena? (Chapter 15); How can we use statistical laws to predict long-term patterns in random events? (Chapter 16); and How do we separate foolish risks from risks that are worth taking? (Mini-Excursion 4).

## Exercises and Projects

An important goal for this book is that it be flexible enough to appeal to a wide range of readers in a variety of settings. The exercises, in particular, have been designed to convey the depth of the subject matter by addressing a broad spectrum of levels of difficulty—from the routine drill to the ultimate challenge. For convenience (but with some trepidation) the exercises are classified into three levels of difficulty:

- **Walking.** These exercises are meant to test a basic understanding of the main concepts, and they are intended to be within the capabilities of students at all levels.
- **Jogging.** These are exercises that can no longer be considered as routine—either because they use basic concepts at a higher level of complexity, or they require slightly higher-order critical thinking skills, or both.
- **Running.** This is an umbrella category for problems that range from slightly unusual or slightly above average in difficulty to problems that can be a real challenge to even the most talented of students.

A **Projects and Papers** section following the exercise sets at the end of each chapter offers some potential topics and ideas for further exploration and enrichment. In most cases, the projects are well suited for group work, be it a handful of students or an entire small class.

## What's New in the Sixth Edition

While much has changed in this book since the first edition, I have made a conscious effort to resist changing its overall structure. Four parts consisting of four chapters each has a nice symmetry to it, and users have made it clear that they like the flexibility that this structure offers. At the same time, many users have expressed a desire to have additional topics covered. In an effort to balance these competing interests, a new feature makes its debut in this edition—the “Mini-Excursion.”

If one thinks of a chapter in this book as a snorkeling trip along a colorful coral reef, then a Mini-Excursion is a free dive into somewhat deeper waters along a particularly interesting section of the reef. In this edition I have introduced four Mini-Excursions, one for each of the four parts:

- Mini-Excursion 1 (Apportionment Today: *The Huntington-Hill Method*) discusses the current method used to apportion the United States House of Representatives, and enriches (both mathematically and politically) the material introduced in Chapter 4.
- Mini-Excursion 2 (A Touch of Color: *Graph Coloring*) discusses the dual concepts of graph coloring and chromatic number, and some of the applications of these concepts. This is an enrichment topic tangentially related to some of the ideas introduced in Chapters 5 and 6.
- Mini-Excursion 3 (The Time Value of Money: *Annuities*) builds on the exponential growth model and its applications to financial mathematics (introduced in Chapter 10) to explore the concepts of fixed and deferred annuities.
- Mini-Excursion 4 (The Cost of Risk-Taking: *Expected Value*) explores the idea that risks can be quantified and managed. This Mini-Excursion is an enrichment topic for the material introduced in Chapter 15.

The Mini-Excursions are available as printable PDF files at the book's Web site [www.prenhall.com/tannenbaum](http://www.prenhall.com/tannenbaum), as well as in hard copy form in the Student Resource Guide.

As with previous editions, numerous changes were made throughout the text to improve the flow and readability and make the material more user-friendly. Some of the more noticeable and significant changes are:

- The introduction of marginal notes for suggestions, references, and comments.
- The end of each chapter opener now includes a brief narrative of the chapter's organization.
- Approximately 250 new exercises.
- Many old examples have been updated or expanded. Many new examples have been added.
- Chapter 7 has undergone a significant but not extreme makeover. The chapter now uses the concept of a *network* as the underlying paradigm for the study of connectivity and trees.



## Ancillary Materials Available with This Edition

- **Companion Web Site** ([www.prenhall.com/tannenbaum](http://www.prenhall.com/tannenbaum)) Features a syllabus manager, online quizzes, Internet projects, graphing calculator help, spreadsheet basics and Excel projects, PowerPoint Presentation downloads, Java applets, the Mini-Excursions, and dozens of additional resource links.
- **Student Resource Guide** (0-13-187382-2) In addition to the worked out solutions to odd-numbered exercises from the text, this guide contains “selected hints” that point the reader in one of many directions leading to a solution. Mini-Excursions in full color and keys to student success, including lists of skills that will help prepare for chapter exams, are also included.
- **Instructor’s Resource Manual** (0-13-187381-4) Contains solutions to all the exercises in the text. Also contains an Instructor’s Guide that includes Learning Outcomes, Skill Objectives, Ideas for the Classroom, and Worksheets.
- **Test Item File** (0-13-187380-6) PDF version of TestGen. Available for download from the Instructor’s Resource Center.
- **TestGen** (0-13-187379-2) Test-generating software that creates randomized tests and offers an onscreen LAN-based testing environment, complete with Instructor Gradebook.
- **Course Compass Access Card** (0-13-242929-2)
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## Acknowledgments

In developing this new edition, I consulted extensively with Dale Buske of St. Cloud State University. Dale read the manuscript several times and made numerous comments and suggestions. In addition, Dale contributed greatly to the development of the new mini-excursions, as well as to the expanded exercise sets. Dale’s contributions to this edition went above and beyond the ordinary and I owe Dale a special debt of gratitude.

I was also fortunate to have an excellent group of reviewers who reviewed the 5th edition and made many excellent suggestions as to how it might be improved:

Lowell Abrams, *George Washington University*  
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Many people contributed to previous editions of this book. Special thanks go to Robert Arnold, my original co-author, to Dale Buske (again), and to the following reviewers: Teri Anderson, Carmen Artino, Donald Beaton, Terry L. Cleveland, Leslie Cobar, Crista Lynn Coles, Ronald Czocho, Nancy Eaton, Lily Eidswick, Kathryn E. Fink, Stephen I. Gendler, Marc Goldstein, Josephine Guglielmi, William S. Hamilton, Harold Jacobs, Tom Kiley, Jean Krichbaum, Kim L. Luna, Mike Martin, Thomas O'Bryan, Daniel E. Otero, Philip J. Owens, Matthew Pickard, Lana Rhoads, David E. Rush, Kathleen C. Salter, Theresa M. Sandifer, Paul Schembari, Marguerite V. Smith, William W. Smith, David Stacy, Zoran Sunik, John Watson, Sarah N. Ziesler.

Many tasks go into overseeing the production of a book from beginning to end, and it is the responsibility of the production editor to see that everything falls into place. Barbara Mack, my production editor, succeeded admirably in smoothly bringing this project to a timely completion. Barbara is, by far, the best production editor I have ever worked with.

Heather Scott was responsible for the design of this edition. Thanks to her talent we managed a rare feat—a book that is better looking, has more content, and is 100 pages shorter than its predecessor! And then, there is Sally Yagan, editor extraordinaire. There is an editor behind every book, but few that can match her vision, “can-do” attitude, and leadership.

## A Final Word

This book grew out of the conviction that a liberal arts mathematics course should teach students more than just a collection of facts and procedures. The ultimate purpose of this book is to instill in the reader an overall appreciation of mathematics as a discipline and an exposure to the subtlety and variety of its many facets: problems, ideas, methods, and solutions. Last, but not least, I have tried to show that mathematics can be interesting and fun.

*Peter Tannenbaum*

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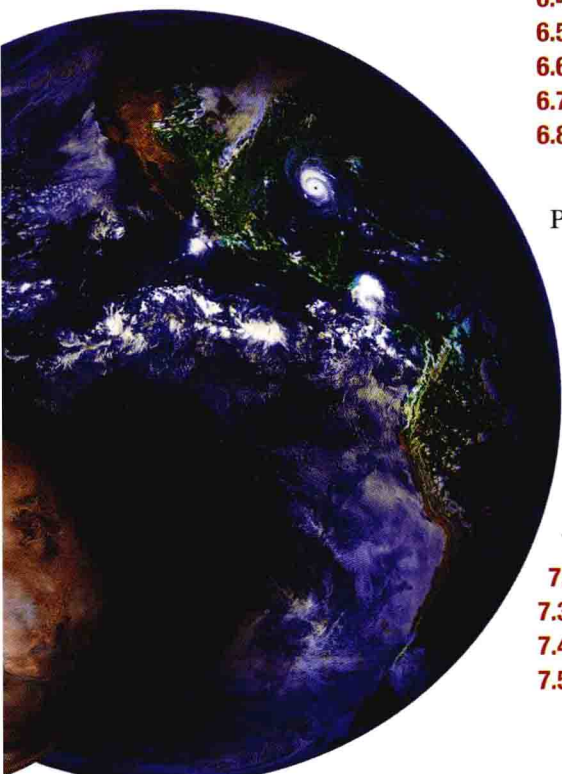
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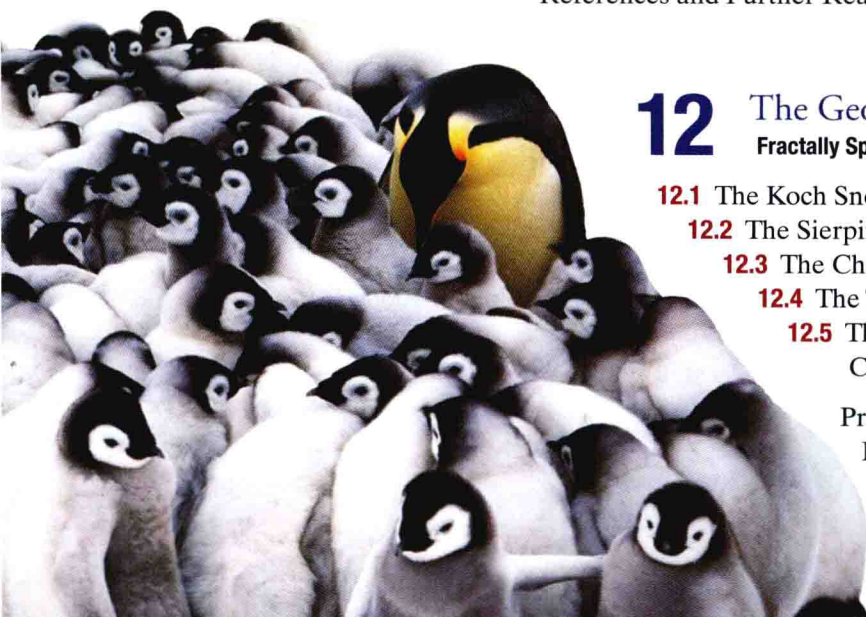
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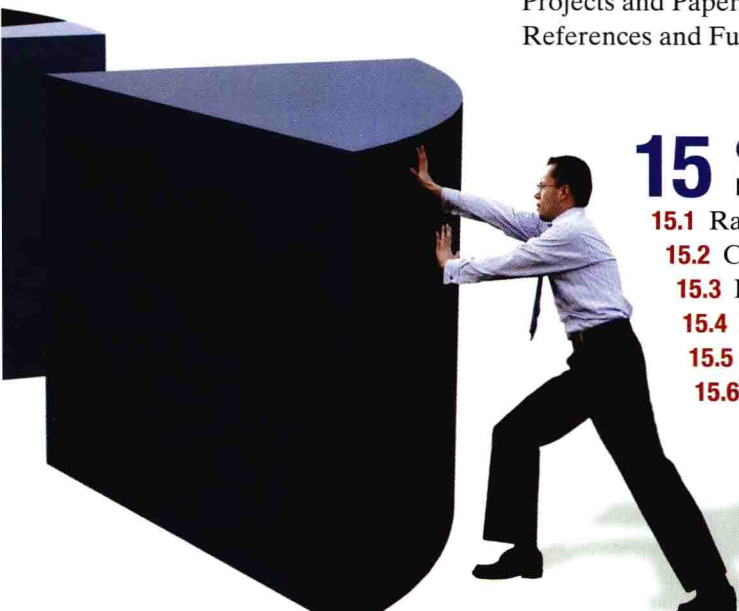
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