SIMULATION OF WAITING-LINE SYSTEMS

SUSAN L. SOLOMON

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PREFACE

Simulation is simultaneously one of the easiest to understand and one of the most misunderstood of the management science techniques. Mathematicians, computer scientists, and engineers sometimes denigrate simulation because it is purportedly not based on elegant, theoretical, general models as is, for example, linear programming. Managers and business people sometimes have been told that simulation is a panacea that always solves any problem; these individuals may then be dismayed to find that simulation may be more expensive, more time consuming, and less accurate than they were lead to believe.

The purpose of this book is to present a balanced, realistic picture of the entire process of simulation at a level consistent with the backgrounds and interests of most managers and students of business. Real-world ''glitches'' and often-forgotten caveats are discussed and dealt with. While waiting-line systems are the main vehicle for discussion, the principles apply to many types of simulation. Anyone conversant with computer usage and introductory applied statistics should be adequately prepared for the technical material. Nonparametric statistics are used almost exclusively because they are intuitively easy to understand and avoid many assumptions. Large-sample applications of nonparametric statistics call for χ^2 or normal distributions with which most business students are familiar. The book is intended as a text in a three-semester hour or four-quarter hour survey course in simulation or as a self-study practical introduction to simulation. Numerical answers to selected end-of-chapter problems are given. A complete statistical support package (SIMSTAT) written in conversational FORTRAN is provided along with an extensive bibliography, not just of individual articles but of professional simulation organizations and their regular publications. Chapter appendices explain how the examples given in the chapter were obtained by using the programs in the SIMSTAT package. A few simple illustrations are offered and are carried through the narrative as well as the problem sections of the book to provide clarity, continuity, and perspective on the overall modeling process.

Nearly two decades of teaching and research in simulation have convinced the author that it is possible and desirable to provide beginners with a theoretical and practical foundation in the principles of model building, the distinction between simulation and other modeling methods, statistical analysis of model inputs and outputs, and simulation programming in sufficient detail to enable the student to formulate and implement a complete model of a simple, real-world waiting-line system within a semester or quarter. This author prefers to acquaint the student with all aspects of simulation modeling in the context of a simple situation rather than immerse him or her in one aspect, such as simulation programming, in great detail. In my classes, each student or pair of students must submit such a project contributing about 50 percent of the total course grade. The projects must be completed in segments synchronized with chapters in the book to ensure that students do not fall irretrievably behind. Class members brainstorm unforeseen difficulties encountered by their colleagues. It takes discipline and perseverance to accomplish these objectives within a limited time span, but it can be done. The SIMSTAT statistical support package is expressly designed to enable students to tackle end-of-chapter and project problems without spending scarce time on review of statistical methods, table lookups, data transcription, formula interpretation, and repetitious arithmetic. Each student is provided with statement of computer costs incurred in the simulation effort at the end of the course.

Chapter 1 is an introduction to modeling and model types. Chapter 2 discusses goodness-of-fit tests. Chapter 3 introduces analytical waiting-line models along with a methodology for evaluating the cost components of service systems that can be extended to simulation models. Chapter 4 examines random variate generation and criteria for their suitability. Chapters 5, 6, and 7 give more depth of coverage of GPSS than do most other simulation survey books but without the detail of compiler operation and options characteristic of books dedicated to the GPSS language. Chapter 8 encompasses tests for transients and serial correlation in simulation output data and methods for coping with them. Chapter 9 enables the novice modeler finally to use ''sanitized'' simulation output data to make estimates of system performance or to test hypotheses about differences in performance of alternative configurations. Chapter 10 offers a selection of simulation studies which were chosen according to the following criteria: readability, relevance of subject area, recentness, practicality, quality and completeness of modeling effort, and apparent benefits or cost savings. This chapter includes some waiting-line simulations as well as some other types of simulation, some simulations in the private sector, and some in the public sector. Chapter 11 investigates the management of the modeling process as well as the future of simulation. Techniques employed throughout the book were chosen for their simplicity of comprehension and execution rather than for the efficiency or state-of-the-art aspects. It is assumed that a reader who decides to become a simulation professional will undertake further continuing study.

I would like to thank Geoffrey Gordon of IBM for his reviews of the entire manuscript, and Eastern Washington University for providing me with professional leave and computer support without which this work would have been impossible.

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chapter one

THE NATURE AND PURPOSE OF MODELS

THE RATIONALE FOR MODELING

Our world is complex and is growing ever more so. The complexity results from both a rapidly increasing store of facts and the intricate interrelationships among these facts. A decision maker, no matter how bright and how well trained, finds it difficult to sift the relevant facts from those which are irrelevant and to fathom the complexities of the system that the decision will affect. Whether the problem is to modify a currently existing system or to design a brand-new system, the development of a model of the system on which to experiment is often desirable. First, the decision maker can try out alternatives (and probably make some mistakes) using the model rather than the real world as the laboratory. This is not only less embarrassing but is usually less costly and difficult than experimentation with the real system. Also, when the real system is not yet in existence, waiting for it to be implemented before experimentation can impose a long delay and incur the possibility that the implemented system will be unsatisfactory.

WHAT IS A SYSTEM?

A system may be defined as a collection of inputs which pass through certain processing phases to produce outputs. A manufacturing system may use crude oil as an input and a cracking plant for processing and produce various types of oil and gasoline as outputs.

The performance of the system might be evaluated by criteria such as how many gallons of output result from each gallon of input, how long it takes to transform the inputs into outputs, how expensive it is to transform the inputs into outputs, and how much space the processing phases occupy.

In many systems there is a mechanism for evaluating the performance of the system and altering the nature of the process depending on the degree to which performance criteria have been satisfactorily met. If the oil refinery's ratio of inputs used to ouputs produced is considered too low by some predefined criterion, this may be reported to a manager or an operator who can alter the mix of inputs, change the nature of processing, or vary the composition of outputs in an effort to improve the performance of the system. When the performance of the system is monitored and evaluated, with the evaluation affecting the future state of the system, the system is said to incorporate feedback.

A service system, likewise, has inputs, processing phases, and outputs, as well as measures of system performance. The inputs are customers and any items the customer wishes to obtain; the processing components are servers, equipment, and space; and the outputs are satisfied customers. Performance measures for a service-oriented system might include the amount of time a customer spends waiting for service or waiting for processing to be completed, the number of customers waiting in line, or the proportion of time the servers are idle. By changing the number or quality of servers or equipment, the manager can bring the performance of the system into better alignment with predefined criteria.

Not all variables which are relevant to system performance are under the manager's control. A downturn in the economy or the emergence of a strong new competitor can influence system performance in a negative manner, but they cannot be wished away by the manager. However, the manager may evaluate their probable effect on the system and manipulate those variables which he or she can alter in an attempt to compensate. Thus, the manager might elect to improve the quality of merchandise or service or to cut costs in an endeavor to offset the competitor's challenge. Variables which affect a system but cannot be manipulated directly by the manager are called *exogenous variables*. Variables which can be controlled by the manager are called *endogenous variables*. Variables which are exogenous to one system may be endogenous to another. For example, the new competitor is exogenous to the service system of the manager just described, but they are both endogenous to the system which is the economic structure of the United States.

STANDARDIZED VERSUS CUSTOM MODELS

Management science, decision science, and operations research are all names for a set of quantitative tools which have assisted decision makers with their task. Many of these tools specify which variables, or sets of facts, are to be examined, what the nature of their interrelationships is presumed to be, and what results or performance measures may be observed as output of the model. To the extent that these tools are computer-based, the decision maker reaps the additional advantage of speed and accuracy of computation.

For many well-known decision situations, these restrictive, "packaged" models yield good results. Production and operations managers are familiar with models of inventory systems in which the relevant variables—volume of sales per unit of time, cost of ordering new merchandise, cost of carrying unsold inventory, purchase price per unit incorporating possible discounts, and cost of running out of stock—are related in an intuitively plausible mathematical expression to provide the manager with answers to inevitable questions—how many units to order at a time, how often to place an order, how often stockouts will occur, and how much this inventory management policy will cost per unit time.

Other tools are more general in their applicability, but by convention, they have tended to be used more frequently in some decision situations than in others. An example is linear programming, a technique which enables the decision maker to optimize some objective such as finding the combination of possible products or services that minimizes cost or maximizes profits. Unlimited optimization is generally precluded by some restrictions or constraints, such as meeting specified levels of demand, confining labor-intensive activities to a 40-hour week, limiting usage of materials to the quantities on hand, or ensuring that the proportion of a certain input or output is not more than, not less than, or exactly equal to some fraction of the total input or output.

A bit of imagination or exposure to more advanced treatment of the subject may lead the decision maker to employ linear programming in a decision situation other than the traditional product mix determination. For example, linear programming may be used to route deliveries from warehouses to retail stores, to assign people to jobs, to budget capital and operating expenditures, or to schedule and monitor the progress of large-scale projects. Many more potential but less obvious applications exist. Sensitivity analysis methods permit the decision maker to ask and answer "what if" questions about the effects of variations of the stated objective function and the constraints.

Nevertheless, any general-purpose packaged model has limitations which may not be ignored on pain of inaccurate results and false conclusions. The standard formulation of the inventory model assumes that the cost of holding unsold inventory is proportional to the average level of that inventory. This assumption is reasonable for the segment of holding costs, which comprises costs of spoilage, pilferage, obsolescence, and insurance and which may be assumed to increase and decrease with the level of inventory over the order cycle. However, one of the largest components of holding cost is the cost of warehousing; a firm usually needs and pays for sufficient warehouse space to hold the maximum as opposed to the average amount of inventory during the order cycle, since that maximum is the quantity initially delivered with each new order.

Fortunately, a well-trained, experienced, clever, and aware decision maker can modify the standard inventory model to accommodate some variations. Similarly, an informed user of linear programming who encounters a crucial nonlinearity in the situation to be modeled may select a nonlinear optimization technique as an alternative. Eventually, however, the multitude of idiosyncrasies in a particular situation may render all known packaged models ineffective. At that point, the decision maker must assess whether the deviations of the particular situation from the general formulation of a packaged model are sufficiently important to warrant the cost of constructing a complex, customized model. Sometimes the decision maker will tolerate modest discrepancies in order to use a packaged model which is convenient, accessible, and inexpensive. The construction of a fairly simple customized model may be no more expensive and considerably more accurate than the use of a package. This possibility should be explored before choosing the vehicle for modeling.

ATTRIBUTES OF MODELS

A model may or may not be a faithful replication of reality or proposed reality. For reasons of complexity and cost previously mentioned, most decision makers prefer models which extract the salient characteristics of reality while omitting the "noise" or irrelevancies. In some cases all characteristics are relevant, and identical or scale models are constructed, often at great expense. The aerospace industry exemplifies instances in which faithful replication is an essential prerequisite to a successful model in which the decision maker can have confidence. While the cost of model construction and testing is high, the decision maker can rest assured that most design defects will be exposed, discovered, and rectified as a result of experimentation with the model rather than after a real-system catastrophe.

Models may include characteristics which are vastly different from the real system as long as those characteristics have no impact on the performance outputs which are relevant to the decision maker. No one would take the trouble to include irrelevancies in a customized model, but they may be present in a packaged model. A human fashion model may display attributes of attractiveness, proportion, style, and grace which are desirable for the apparel marketing system, yet the same model may possess or lack intelligence, knowledge, lucidity, and other characteristics which are generally construed as valuable but which are not germane to the task at hand.

In general, simplicity is a virtue in modeling. Simplicity clarifies the system in the eyes of the decision maker, illuminating alternatives which might be profitable. A simple model is also less expensive to construct and manipulate. However, simplicity should not be pursued at the expense of relevant elements.

The decision maker must assess whether the model is to describe an optimum condition or reality, which is generally suboptimal. The fashion model may represent an ideal but may be very different from reality as characterized by an average customer for the garments which the model displays. In designing a new system, the search for optimality may weigh more heavily than in modifying an existing system in which few variables are left to the decision maker to manipulate. The manager may wish that the number of customers desiring service while the employees are having lunch would be few, or zero, but may have no control over this variable and should plan the service system to accommodate rather than ignore this unpleasant reality.

Some models, particularly packages such as inventory control and linear programming, lend themselves well to the goal of optimization as well as experimentation with selected alternatives. However, many models are not amenable to mathematical optimization. In these instances, the decision maker must be content with the facility to test a set of possible model configurations and select the best of the sample tested, realizing that it is unlikely to be a universal optimum but the cost of infinite search is prohibitive.

The question of optimization requires the decision maker to focus on objectives—what is to be maximized or minimized and how it is to be measured. In a service system, the manager may wish to minimize the cost of staffing subject to the constraint that the average waiting time per customer should not exceed 5 minutes. This statement facilitates development and evaluation of models incorporating various numbers and qualities of servers. However, it makes an implicit assumption about the value of customer waiting time and the probability that a disgruntled customer will elect to do business elsewhere after enduring a long wait.

TYPES OF MODELS

According to one dimension of classification, the most common types of models are physical, schematic, mathematical, and heuristic. A physical model may be an identical replication of the real system, such as an experimental aircraft or a fashion model, or it may be scaled down, such as the wind tunnel version of the same aircraft or a doll analogous to the fashion model. A schematic model is a pictorial representation of the system, such as a blueprint or a graph. A mathematical model consists of expressions containing variables, constants, and operators which describe the process of interest. An heuristic model is a collection of descriptors and decision rules, usually computer-based, which is not limited by the physical, diagrammatic, or mathematical bounds of the other types of models. While mathematical models may be implemented on a computer, they are restricted to purely mathematical operations such as arithmetic, algebra, and calculus. Heuristic models may be programmed to search data sets and perform logical comparisons as well as mathematics.

Inventory and linear programming models are analytic in nature. That is, they offer a general formulation of a common class of problems for which the user needs only to supply specific parameters. Synthetic models, on the other hand, allow the modeler to extrapolate from a particular situation to the more general case which has not been previously described and evaluated.

Another distinction in model types is static versus dynamic. A static model is a snapshot of a system at a particular point in time; a dynamic model conveys the essence of changes in the system over time. The contrast is akin to the accountant's balance sheet, which reflects the stock of certain variables of the firm at the end of an accounting period, as compared with the profit and loss or cash flow statements, which itemize movements among categories during the accounting period. A