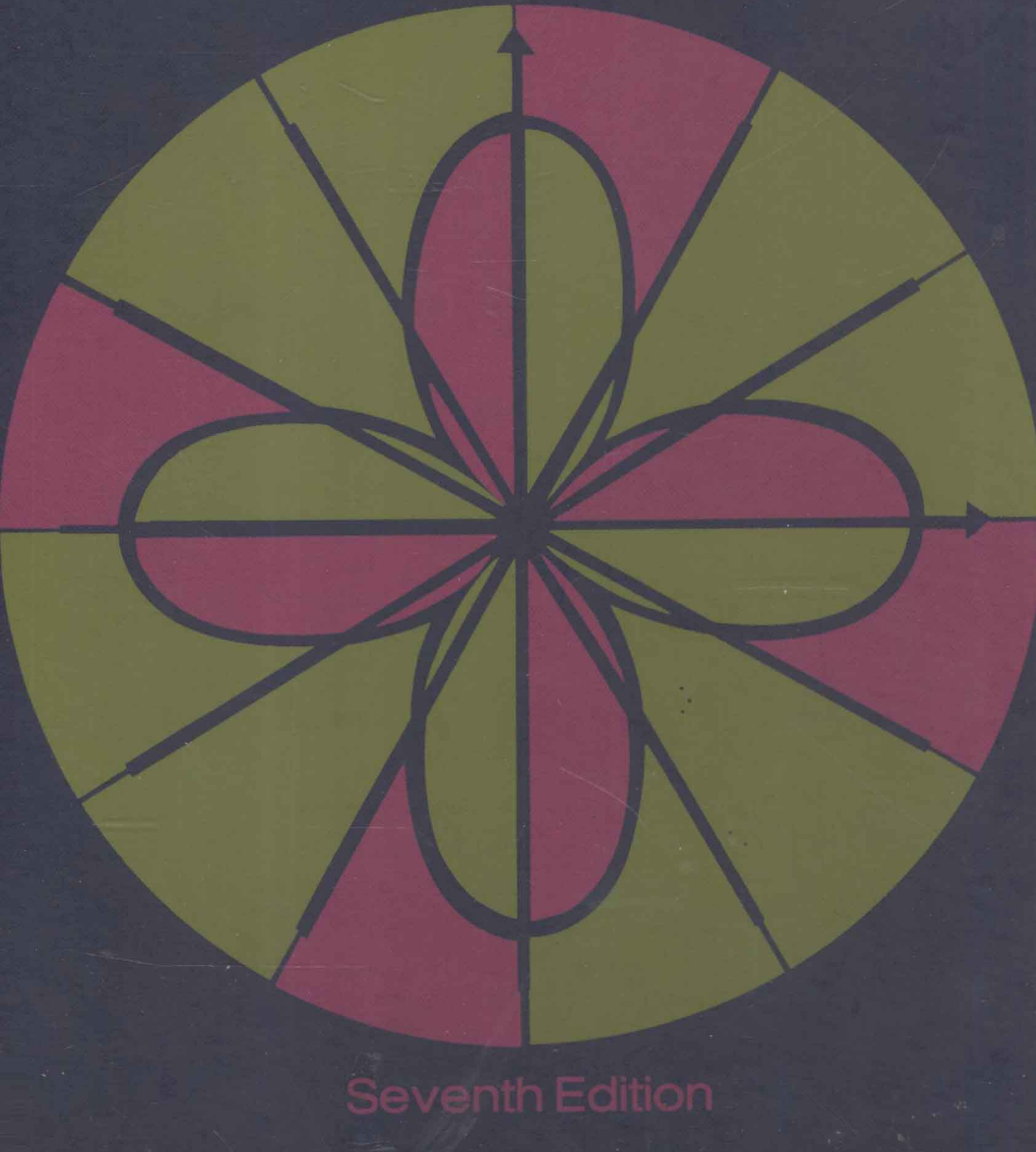
PLANE TRIGONOMETRY
WITH TABLES



Seventh Edition

Fred W. Sparks / Paul K. Rees / Charles Sparks Rees

PLANE TRIGONOMETRY

SEVENTH EDITION

Fred W. Sparks
Texas Tech University

Paul K. Rees
Louisiana State University

Charles Sparks Rees
University of New Orleans

Prentice-Hall, Inc., Englewood Cliffs, New Jersey 07632

Library of Congress Cataloging in Publication Data

Sparks, Fred Winchell, (date)

Plane trigonometry.

Includes index.

1. Trigonometry, Plane. I. Rees, Paul Klein, (date) joint author. II. Rees, Charles Sparks, joint author. III. Title.

QA533.S82 1977 516'.24 76-26508

ISBN 0-13-679209-X

10 9 8 7 6 5 4 3 2

Printed in the United States of America

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA PTY. LIMITED, *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL OF INDIA PRIVATE LIMITED, *New Delhi*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*
PRENTICE-HALL OF SOUTHEAST ASIA PTE. LTD., *Singapore*
WHITEHALL BOOKS LIMITED, *Wellington, New Zealand*

IDENTITIES AND FORMULAS

1.1 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

1.2 π radians $= 180°$

1.3 $s = r\theta$

1.4 $v = r\omega$

2.1 $x^2 + y^2 = 1$

4.1 $\sin\theta \csc\theta = 1$

4.2 $\cos\theta \sec\theta = 1$

4.3 $\tan\theta \cot\theta = 1$

4.4 $\tan\theta = \sin\theta/\cos\theta$

4.5 $\cot\theta = \cos\theta/\sin\theta$

4.6 $\sin^2\theta + \cos^2\theta = 1$

4.7 $1 + \tan^2\theta = \sec^2\theta$

4.8 $1 + \cot^2\theta = \csc^2\theta$

5.1 $\cos(A - B) = \cos A \cos B + \sin A \sin B$

5.2 $\cos(\pi/2 - B) = \sin B$

5.3 $\sin(\pi/2 - B) = \cos B$

5.4 $\cos(-B) = \cos B$

5.5 $\sin(-B) = -\sin B$

5.6 $\cos(A + B) = \cos A \cos B - \sin A \sin B$

5.7 $\cos 2A = \cos^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

$= 2\cos^2 A - 1$

5.8 $\cos\frac{1}{2}\theta = \pm\sqrt{\dfrac{1 + \cos\theta}{2}}$

5.9 $\sin(A + B) = \sin A \cos B + \cos A \sin B$

5.10 $\sin 2A = 2\sin A \cos A$

5.11 $\sin\frac{1}{2}\theta = \pm\sqrt{\dfrac{1 - \cos\theta}{2}}$

5.12 $\sin(A - C) = \sin A \cos C - \cos A \sin C$

5.13 $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

5.14 $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

5.15 $\tan\frac{1}{2}\theta = \dfrac{\sin\theta}{1 + \cos\theta}$

$= \dfrac{1 - \cos\theta}{\sin\theta}$

5.16 $\tan(A - B) = \dfrac{\tan A - \tan B}{1 - \tan A \tan B}$

5.17 $2\sin A \cos B = \sin(A + B) + \sin(A - B)$

5.18 $2\cos A\sin B = \sin(A + B) - \sin(A - B)$

5.19 $2\cos A\cos B = \cos(A + B) + \cos(A - B)$

5.20 $2\sin A\sin B = -\cos(A + B) + \cos(A - B)$

5.21 $\sin C + \sin D = 2\sin\frac{1}{2}(C + D)\cos\frac{1}{2}(C - D)$

5.22 $\sin C - \sin D = 2\cos\frac{1}{2}(C + D)\sin\frac{1}{2}(C - D)$

5.23 $\cos C + \cos D = 2\cos\frac{1}{2}(C + D)\cos\frac{1}{2}(C - D)$

5.24 $\cos C - \cos D = -2\sin\frac{1}{2}(C + D)\sin\frac{1}{2}(C - D)$

6.1 $a/\sin A = b/\sin B = c/\sin C$

6.2 $K = \frac{1}{2}bh = \frac{1}{2}ab\sin C$

6.3 $K = b^2\sin A\sin C/\sin B$

6.4 $c^2 = a^2 + b^2 - 2ab\cos C$

6.5 $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

6.6 $\dfrac{a - b}{a + b} = \dfrac{\tan\frac{1}{2}(A - B)}{\tan\frac{1}{2}(A + B)}$

6.7 $s = \frac{1}{2}(a + b + c)$

6.8 $\tan\frac{1}{2}A = r/(s - a)$

$\tan\frac{1}{2}B = r/(s - b)$

$\tan\frac{1}{2}C = r/(s - c)$

6.9 $K = rs$

6.10 $r = \sqrt{\dfrac{(s - a)(s - b)(s - c)}{s}}$

7.1 $f(\theta + p) = f(\theta)$

8.1 Arcsin or $\text{Sin}^{-1} = \{(x, y) \mid x = \sin y, -\pi/2 \le y \le \pi/2\}$

8.2 Arccosine or $\text{Cos}^{-1} = \{(x, y) \mid y = \text{Cos}^{-1} x, 0 \le y \le \pi\}$

8.3 Arctangent or $\text{Tan}^{-1} = \{(x, y) \mid y = \text{Tan}^{-1} x, -\pi/2 < y < \pi/2\}$

8.4 Arccotangent or $\text{Cot}^{-1} = \{(x, y) \mid y = \text{Cot}^{-1}, 0 < y < \pi\}$

8.5 Arcsecant or $\text{Sec}^{-1} = \{(x, y) \mid y = \text{Sec}^{-1} x, 0 \le y \le \pi, y \ne \pi/2\}$

8.6 Arccosecant or $\text{Csc}^{-1} = \{(x, y) \mid y = \text{Csc}^{-1}, -\pi/2 \le y \le \pi/2, y \ne 0\}$

9.1 $r = \sqrt{a^2 + b^2}$

9.2 $\theta = \arctan b/a$

9.3 $a + ib = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

9.4 $CC' = rr'[\cos(\theta + \theta') + i\sin(\theta + \theta')]$

9.5 $\dfrac{r_1\,\text{cis}\,\theta_1}{r_2\,\text{cis}\,\theta_2} = -\dfrac{r_1}{r_2}\,\text{cis}\,(\theta_1 - \theta_2)$

9.6 $(r\,\text{cis}\,\theta)^n = r^n\,\text{cis}\,n\theta$

10.1 $x = r\cos\theta$ and $y = r\sin\theta$

10.2 $r = \sqrt{x^2 + y^2}$

A1.1 $\log_b N = L$ if and only if $b^L = N$

A1.2 $\log_b MN = \log_b M + \log_b = N$

A1.3 $\log_b M/N = \log_b M - \log_b N$

A1.4 $\log_b N^p = p\log_b N$

A2.1 $\dfrac{a - b}{c} = \dfrac{\sin\dfrac{A - B}{2}}{\cos\dfrac{C}{2}}$

A2.2 $\dfrac{a + b}{c} = \dfrac{\cos\dfrac{A - B}{2}}{\sin\dfrac{C}{2}}$

PLANE TRIGONOMETRY

PREFACE

In writing the manuscript for the seventh edition of *Plane Trigonometry*, we have made those changes that we think will make the book even more readable and teachable than its predecessors. We think that the student of average ability and industry will be able to read the book with understanding and we think that the below average student will be able to profit by use of the book. The above average student will do well regardless of the book or instructor.

The entire manuscript has been touched up but the major rewriting and rearrangement is in the following areas: The first two chapters, Introductory Concepts and The Trigonometric Functions, and the chapters on identities and equations. Each type of equation is taken up as soon as the required identities are available. The introductory section and those on Standard Position, The Trigonometric Ratios, The Trigonometric Functions, Domain and Range, The Reference Angle, and Introduction to Trigonometric Equations are entirely rewritten. The sections on Significant Digits, Computation, and Calculations Involving Approximations are put in Chapter One so as to be available for use in connection with the work on central angle, arc length, angular speed, and linear speed.

Since most students are familiar with logarithms and their use, the chapter on that subject has been put in an appendix. There are, however, exercises given for the use of those students who may need drill in the use of logarithms. We show how to solve triangles by use of a calculator. We differentiate between a relation, a function, and a function value. The solution of right triangles is now in Chapter Three instead of Chapter Two as in the sixth edition. We still begin the study of the composite angle with the cosine of the difference of two angles and follow that by the function values of the complement of an angle. The function values of a composite angle then follow readily.

The chapter on a composite angle is followed by Graphs of the Functions, Logarithmic Solution of Right Triangles, and Inverse Functions in order to keep a proper balance between analytical and numerical work. The emphasis is on the analytical aspects of the subject.

We still define and illustrate each concept before using it. The examples illustrate the text material immediately preceding them and serve as models for the problems that follow.

We continue giving problem lists a normal lesson apart and in groups of four similar ones. These features make it easy even for the inexperienced teacher to assign the right amount of material and to give a good coverage of the concepts that are involved. This edition contains some 1600 problems in 40 exercises, including seven review exercises besides the four exercises and 200 problems on logarithms. There is a review exercise at the end of each chapter that contains more than two exercises. The review exercises contain some problems of a rather challenging nature. Answers are given to three fourths of the problems in the back of the book and the others are available in a pamphlet.

There is enough material in the book to afford the instructor a considerable amount of choice in the selection of material to be used in his course. There are enough problems so that only about one fourth of them will be needed in a semester for an average class.

We are grateful to those persons mentioned in the preface to the original 1937 edition and to those who have used the first six editions and have made suggestions for changes. The seventh edition will be a better book than it would have been without the help of those people.

The original authors want to take this opportunity to welcome Charles Sparks Rees as a coauthor. He is the son of one of us and the namesake of the other.

FRED W. SPARKS
PAUL K. REES
CHARLES SPARKS REES

REFERENCE MATERIAL

THE GREEK ALPHABET

Letters		Names	Letters		Names	Letters		Names	Letters		Names
A	α	Alpha	H	η	Eta	N	ν	Nu	T	τ	Tau
B	β	Beta	Θ	θ	Theta	Ξ	ξ	Xi	Υ	υ	Upsilon
Γ	γ	Gamma	I	ι	Iota	O	o	Omicron	Φ	ϕ	Phi
Δ	δ	Delta	K	κ	Kappa	Π	π	Pi	X	χ	Chi
E	ϵ	Epsilon	Λ	λ	Lambda	P	ρ	Rho	Ψ	ψ	Psi
Z	ζ	Zeta	M	μ	Mu	Σ	$\sigma\ \varsigma$	Sigma	Ω	ω	Omega

DEFINITIONS AND THEOREMS FROM PLANE GEOMETRY

1. In a right triangle, the two sides which inclose the right angle are called the *sides* or the *legs* of the triangle. The side opposite the right angle is the *hypotenuse*.
2. *Pythagorean Theorem*. The sum of the squares of the sides of a right triangle is equal to the square of the hypotenuse.
3. The sum of the two acute angles of a right triangle is 90°.
4. The sum of the three angles of a triangle is 180°.
5. If the acute angles of a right triangle are 30° and 60°, the hypotenuse is twice the shorter side.
6. If two sides of a triangle are equal, it is called an isosceles triangle. If all three sides are equal, it is an equilateral triangle.
7. In an isosceles triangle, the angles opposite the equal sides are equal. In an equilateral triangle, each angle is equal to 60°.
8. If two angles of a triangle are equal, the sides opposite them are equal and the triangle is isosceles.
9. If the three angles of a triangle are respectively equal to the three angles of another, the triangles are similar.
10. In two similar triangles, the ratios of the pairs of sides opposite the equal angles are equal.
11. The radius of a circle is perpendicular to the tangent at the point of tangency.
12. Tangents from an external point to a circle are equal and make equal angles with the straight line joining the point to the center of the circle.

CONTENTS

CHAPTER ONE

INTRODUCTORY CONCEPTS

1.1 The Word Trigonometry

A dictionary states that trigonometry is that branch of mathematics that deals with the relations between the sides and angles of a triangle. This was the original reason for inventing trigonometry but it now deals with six ratios that have tremendous theoretical importance as well as practical value. This book will deal with both of these aspects of the subject.

1.2 Directed Line Segments

A straight line segment is the portion of a straight line between two points on it. We shall consider two concepts that are associated with a straight line segment. They are the length and the direction. The measure of the length of a line segment is usually expressed in terms of a standard unit. For example, in the British system, a length is usually expressed in inches, feet, yards, or miles; and, in the metric system, lengths are usually expressed in millimeters, centimeters, or meters. In this book, we shall often express a length in terms of some convenient segment that is chosen as a unit. For example, in Figure 1.1, if the segment u is chosen as a unit, the length of AB

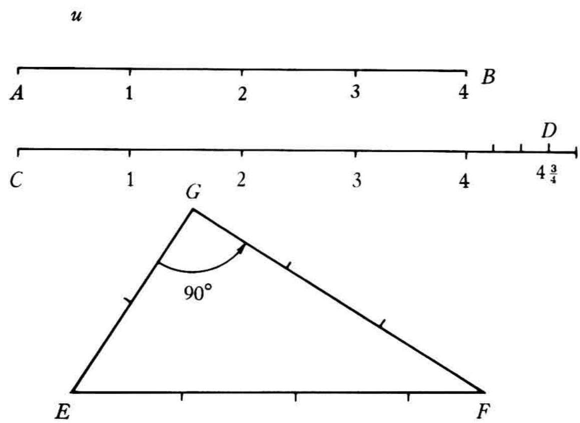


Figure 1.1

is four units, the length of CD is $4\frac{3}{4}$ units, and the length of EF is $\sqrt{2^2 + 3^2} = \sqrt{13} = 3.606\ldots$ units.

It is not sufficient to say merely that the point B in the figure is four units from A. It is really four units to the right of A, and A is four units to the left of B. In mathematics, we use the terms *positive* and *negative* to designate opposite directions. Hence, if we choose to regard the distance from A to B as positive, the distance from B to A is negative.

In this book, we shall usually consider horizontal and vertical lines as directed. On a horizontal line, the direction to the right of a point is positive and that to the left is negative, while on a vertical line the upward direction is positive and the downward is negative.

1.3 The Rectangular Cartesian Coordinate System

It is essential for the work that is to follow that we have some method for locating a point in a plane. The method that we shall use is known as the *rectangular Cartesian coordinate system.* To set up this system, we ordinarily draw a horizontal directed line and a vertical directed line in the plane. We call the horizontal line the *X-axis,* the vertical line the *Y-axis,* and their point of intersection the *origin.* We may locate any point in the plane with reference to these axes by stating its distance to the right or left of the Y-axis and its distance above or below the X-axis. The directed distance from the Y-axis to a point measured parallel to the X-axis is called the *abscissa* of the point and is denoted by x. The directed distance from the X-axis to a point measured parallel to the Y-axis is called the *ordinate* of the point and is denoted by y. The length of the line from the origin to the point is called the *radius vector* of the point. The abscissa is positive or negative according as the point is to the right or to the left of the Y-axis. The ordinate is positive or negative according as the point is above or below the X-axis. The radius vector is always considered positive. We shall use the symbols, x, y, and r to designate the abscissa, the ordinate, and the radius vector, respectively, of a point.

The abscissa and ordinate of a point are known as the *coordinates* of the point and are written as a pair of numbers enclosed in parentheses and separated by a comma, the first designating the abscissa and the second the ordinate.

The coordinate axes divide the plane into four parts called *quadrants*, which are numbered from I to IV in a counterclockwise direction, beginning with the upper right-hand portion. These quadrants will frequently be designated by the symbols Q_1, Q_2, Q_3, and Q_4.

In Figure 1.2, $X'X$ is the X-axis, $Y'Y$ is the Y-axis, and the point O is

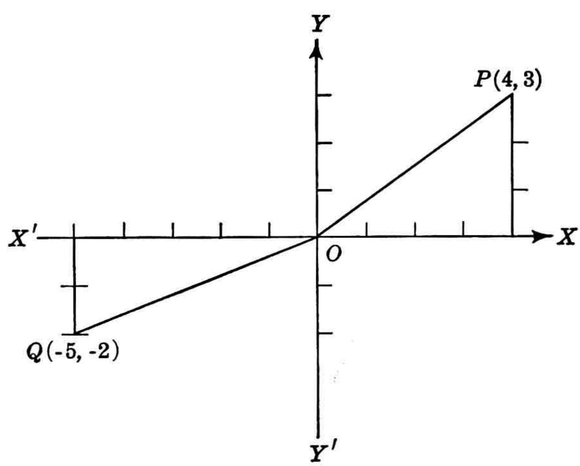


Figure 1.2

the origin. The point P is four units to the right of the Y-axis and three units upward from the X-axis. Hence, its abscissa is 4, its ordinate is 3, and we write its coordinates in the form (4, 3). Likewise, the abscissa of the point Q is -5, its ordinate is -2, and its coordinates are written $(-5, -2)$. The radius vector of the point P is the length of the line OP and the length of OQ is the radius vector of Q.

The process of locating a point by means of its coordinates is called *plotting the point*. Since, in the notation (x, y) for the coordinates of a point, the first number always represents the abscissa and the second number represents the ordinate, (x, y) is called an *ordered pair of numbers*.

1.4 The Distance Formula

In order to obtain a formula for the distance between any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, we shall make use of the Pythagorean theorem and of the distance between two points that are equidistant from a coordinate axis. If the points have the same ordinate as shown in Figure 1.3, then the length of the segment P_1P_2 is $x_2 - x_1$, since x_2 is the distance and direction from the Y-axis to P_2 and x_1 is the distance and direction from the Y-axis to P_1. Similarly, the distance between two points that have the same abscissa is the ordinate of the upper one minus the ordinate of the lower one.

Examples

1. The distance between (2, 3) and (2, 8) is $8 - 3 = 5$.
2. The distance between $(-1, 4)$ and $(5, 4)$ is $5 - (-1) = 6$.

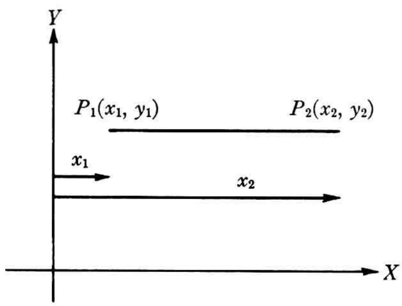


Figure 1.3

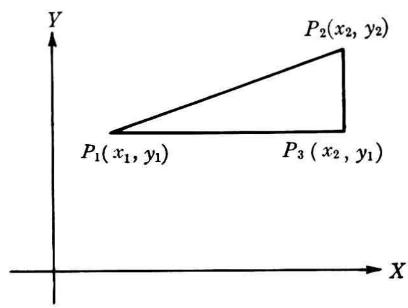


Figure 1.4

We shall now consider any two points as shown in Figure 1.4. In addition to $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, the figure shows, as P_3, the point determined by the intersection of a line parallel to the X-axis from P_1 and a line parallel to the Y-axis from P_2; hence, its coordinates are as shown. Furthermore, P_1P_3 is $x_2 - x_1$, and P_3P_2 is $y_2 - y_1$. Now, applying the Pythagorean theorem, we see that

$$(P_1P_2)^2 = (P_1P_3)^2 + (P_3P_2)^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2;$$

consequently, *the distance* $\boldsymbol{d}$ *between* $\boldsymbol{P_1}$ *and* $\boldsymbol{P_2}$ *is*

$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}. \qquad \textbf{(1.1)}$$