LECTURE NOTES IN PHYSICS

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(Eds.)

Analytical and Numerical Approaches to Mathematical Relativity

With a Foreword by Roger Penrose

Springer

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Foreword

The general theory of relativity, as formulated by Albert Einstein in 1915, provided an astoundingly original perspective on the physical nature of gravitation, showing that it could be understood as a feature of a curvature in the four-dimensional continuum of space-time. Now, some 90 years later, this extraordinary theory stands in superb agreement with observation, providing a profound accord between the theory and the actual physical behavior of astronomical bodies, which sometimes attains a phenomenal precision (in one case to about one part in one hundred million million, where several different non-Newtonian effects, including the emission of gravitational waves, are convincingly confirmed). Einstein's tentative introduction, in 1917, of an additional term in his equations, specified by a "cosmological constant", appears now to be observationally demanded, and with this term included, there is no discrepancy known between Einstein's theory and classical dynamical behavior, from meteors to matter distributions at the largest cosmological scales. One of Einstein's famous theoretical predictions that light is bent in a gravitational field (which had been only roughly confirmed by Eddington's solar eclipse measurements at the Island of Principe in 1919, but which is now very well established) has become an important tool in observational cosmology, where gravitational lensing now provides a unique and direct means of measuring the mass of very distant objects.

But long before general relativity and cosmology had acquired this impressive observational status, these areas had provided a prolific source of mathematical inspiration, particularly in differential geometry and the theory of partial differential equations (where sometimes this had been applied to situations in which the number of space-time dimensions differs from the four of direct application to our observed space-time continuum). As we see from several of the articles in this book, there is still much activity in all these mathematical areas, in addition to other areas which have acquired importance more recently. Most particularly, the interest in black holes, with their horizons, their singularities, and their various other remarkable properties, both theoretical and in relation to observed highly dramatic astronomical phenomena, has also stimulated much important research. Some have interesting mathematical implications, involving particular types of mathematical argumentations, such as the involvement of differential topology and the study of families of geodesics, and some having relevance to deep foundational issues relating to quantum theory and thermodynamics. We find a good representation of these discussions here. Some distinct progress in the study of asymptotically flat space-times is also reported here, which greatly clarifies the issue of what can and cannot be achieved using the method of conformal compactification.

In addition to (and sometimes in conjunction with) such purely mathematical investigation, there is a large and important body of technique that has grown up, which has been made possible by the astonishing development of electronic computer technology. Enormous strides in the computer simulation of astrophysical processes have been made in recent years, and this has now become an indispensable tool in the study of gravitational dynamics, in accordance with Einstein's general relativity (such as with the study of black-hole collision that will form an essential part of the analysis of the signals that are hoped to be detected, before too long, by the new generations of gravitational wave detectors). Significant issues of numerical analysis inevitably arise in conjunction with the actual computational procedures, and issues of this nature are also well represented in the accounts presented here.

It will be seen from these articles that research into general relativity is a thoroughly thriving activity, and it is evident that this will continue to be the case for a good many years to come.

July, 2005 *Roger Penrose*

Preface

Recent years have witnessed a tremendous improvement in the experimental verification of general relativity. Current experimental activities substantially outrange those of the past in terms of technology, manpower and, last but not least, money. They include earthbound satellite tests of weak-gravity effects, like gravitomagnetism in the Gravity-Probe-B experiment, as well as strong-gravity observations on galactic binary systems, including pulsars. Moreover, currently four large international collaborations set out to directly detect gravitational waves, and recent satellite observations of the microwave background put the science of cosmology onto a new level of precision.

All this is truly impressive. General relativity is no longer a field solely for pure theorists living in an ivory tower, as it used to be. Rather, it now ranges amongst the most accurately tested fundamental theories in all of physics. Although this success naturally fuels the motivation for a fuller understanding of the computational aspects of the theory, it also bears a certain danger to overhear those voices that try to point out certain, sometimes subtle, deficiencies in our mathematical and conceptual understanding. The point being expressed here is that, strictly speaking, a theory-based prediction should be regarded as no better than one's own structural understanding of the underlying theory. To us there seems to be no more sincere way to honor Einstein's "annus mirabilis" (1905) than to stress precisely this – his – point!

Accordingly, the purpose of the 319th WE-Heraeus Seminar "Mathematical Relativity: New Ideas and Developments", which took place at the Physikzentrum in Bad Honnef (Germany) from March 1 to 5, 2004, was to provide a platform to experts in Mathematical Relativity for the discussion of new ideas and current research, and also to give a concise account of its present state. Issues touching upon quantum gravity were deliberately not included, as this was the topic of the 271st WE-Heraeus Seminar in 2002 (published as Vol. 631 in the LNP series). We broadly categorized the topics according to their mathematical habitat: (i) differential geometry and differential topology, (ii) analytical methods and differential equations, and (iii) numerical methods. The seminar comprised invited one-hour talks and contributed half-hour talks. We are glad that most of the authors of the one-hour talks followed our invitation to present written versions for this volume.

We believe that the account given here is representative and of a size that is not too discouraging for students and non-experts.

Last but not least we sincerely thank the Wilhelm-and-Else-Hereaeus-Foundation for its generous support, without which the seminar on Mathematical Relativity would not have been possible and this volume would not have come into existence.

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July, 2005

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