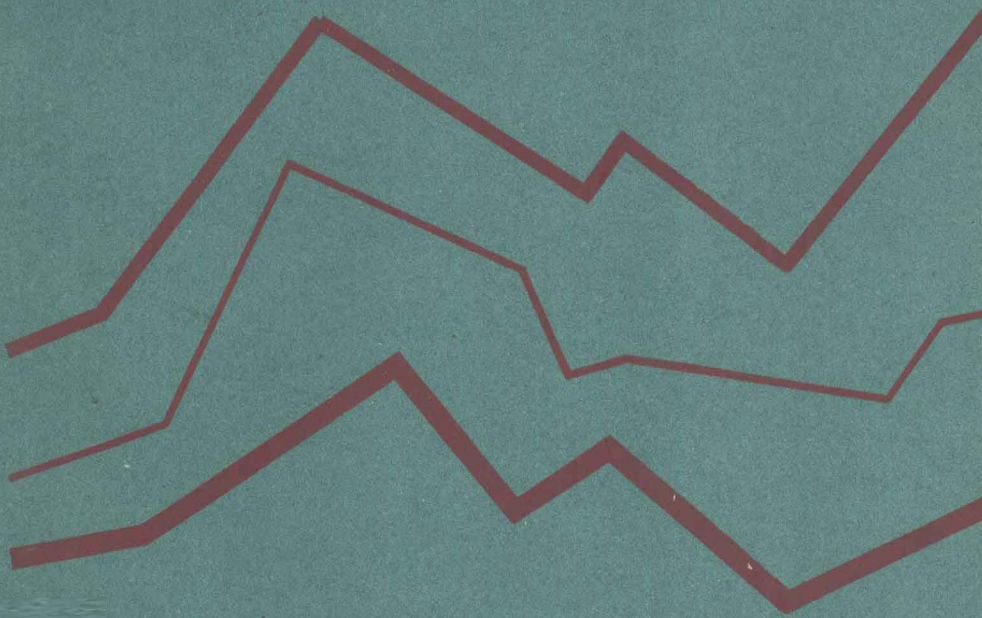
AN INTRODUCTION TO MATHEMATICAL METHODS IN ECONOMICS



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PREFACE

This book has been specifically written to assist students who are taking an introductory course in mathematical methods and their application to economics. In particular, the book has been deliberately designed so as to meet the needs of the substantial number of students who not only have a very elementary knowledge of mathematics, but who also have only a very elementary ability in mathematical manipulation. Since I feel strongly that such students need special assistance, Chapter 1 has been utilized to provide a fuller explanation of how both the method of presentation and the level of difficulty of this book have been carefully chosen so as to supply this special assistance.

It is a pleasure to express my gratitude to the many people who have helped and encouraged me in the course of writing this book. First, the economics students, past and present, that I have taught introductory mathematical methods in economics, are to be thanked for convincing me that there is a need for this type of book. Also, these students must be thanked for their questions and comments, which have had an important influence on both the method of presentation and the level of difficulty of this book. Second, Professor Norman Gibson, Professor John Spencer, and the rest of my colleagues in the Economics Department of the New University of Ulster, are to be thanked for their advice and encouragement. Third, Professor John Spencer (New University of Ulster), David Dinour (New University of Ulster), Gerry Steele (University of Lancaster), and

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J. Colin Glass

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CHAPTER

ONE

INTRODUCTION

1-1 THE DREAD OF MATHEMATICS IN ECONOMICS

Many students, who come to a University or a Polytechnic to study economics, are drawn to the study of economics not only by the wide range of employment opportunities for economics graduates, but also by the wide range of interesting issues (such as wages, inflation, balance of payments, resource allocation, economic growth, pollution, etc.) that are studied under the general heading of economics. This motivation to study economics generally arises from the fact that, before coming to a University or a Polytechnic, many students have enjoyed both the study of economics at school and the discussion of economic problems on television and in the newspapers. However, upon commencing the study of economics at a University or a Polytechnic, many students find to their dismay that the study of economics also entails learning some mathematics. In particular, for the many economics students who have a poor mathematics background, this discovery, that some mathematics is necessary for the study of economics, not only creates anxiety but also results in many students beginning to wonder whether or not they have chosen the correct course of study. Typically, the student in such a situation has two burning questions:

1. How much mathematics is necessary for a reasonable understanding of economics?
2. Will I be able to master this amount of mathematics?

Fortunately, for the majority of undergraduate degree courses in economics, the answer to (1) is that only a certain basic level of mathematics is

necessary for a reasonable understanding of economics. Also, fortunately, the answer to (2) is yes. Experience has demonstrated that the majority of students with little or no mathematics background are capable of attaining a reasonable working knowledge of mathematics. In fact, this book has been deliberately written not only to demonstrate the validity of the answers to (1) and (2), but also to remove the dread of mathematics in economics.

The following section indicates the way in which both the method of presentation and the level of difficulty of this book have been carefully chosen so as to meet the needs of economics students with little or no mathematics background.

1-2 THE METHOD OF PRESENTATION AND THE LEVEL OF DIFFICULTY

Economics students at universities and polytechnics are increasingly required to undertake at least an introductory course in mathematical methods and their application to economics. *An Introduction to Mathematical Methods in Economics* is specifically written for students taking such courses. In particular, the method of presentation has been designed to allow for the following facts:

1. Most students possess not only a very elementary knowledge of mathematics, but also a very elementary ability in mathematical manipulation.
2. Most students are studying mathematical methods and economics concurrently.

The implication of (1) and (2) is that students are often in the uncomfortable position of applying a shaky grasp of newly-learnt mathematical methods to barely-grasped, newly-learnt, economic analysis. It is for this very reason that the basic approach of *An Introduction to Mathematical Methods in Economics* is to explain both the mathematical methods and the economic applications in a way which truly recognizes most students' elementary knowledge of mathematics and economics. To achieve this, the method of presentation consists of:

(*a*) An exposition of mathematical methods which not only commences at a level which assumes little or no prior knowledge of mathematics, but which also proceeds at a realistic pace for students with such a limited mathematics background.
(*b*) A brief explanation of why each mathematical method is important for handling aspects of economic analysis.

(*c*) A non-rigorous, yet adequate, exposition of each mathematical method, amply illustrated by fully-worked examples. To enhance understanding, the exposition is presented in a step-by-step manner and illustrated, where possible, by diagrams. In addition, further problems (denoted as Practice Problems) are provided not only to give students practice at solving mathematical problems, but also to develop mathematical manipulation skill. To achieve this aim, the Practice Problems have been carefully designed not only to test the student's understanding, but also to bolster his or her confidence. Consequently, the Practice Problems have been carefully constructed so as to avoid both excessively difficult problems and problems involving excessive algebraic manipulation. Moreover, not only are all the (non-graphical) answers to the Practice Problems given at the back of the book, but in many cases fully-worked answers are also given.

(*d*) The application of each mathematical method, to various aspects of economic analysis, in a manner which explicitly assumes little prior knowledge of economics. This means that the book is reasonably self-contained in the sense that the economic application of each mathematical method is made in conjunction with both a verbal and (where possible) a diagrammatic explanation of the economic analysis involved. Also, to enhance the integration of this book with the student's concurrent economics courses, the majority of the economic applications relate to topics included in most first-year, undergraduate, courses in microeconomics and macroeconomics. Moreover, as in the exposition of each mathematical method, the exposition of each economic application is not only amply illustrated by fully-worked examples, but is also followed by Practice Problems (with, in many cases, fully-worked answers at the back of the book). Once more the Practice Problems have been carefully designed so as to both test and encourage, rather than frustrate and discourage, the student. Finally, further unworked problems (denoted as Additional Problems) are included at the end of each chapter containing economic applications. The latter problems are suitable for class work.

(*e*) A restriction of the mathematical methods and the economic applications to those areas which are generally the subject-matter of introductory 'mathematics for economists' courses. In addition, special emphasis has been given to the discussion of optimization problems, since these are by far the predominant problems that are analysed in undergraduate economics courses.

It should be noted that the pedagogical approach, to the exposition of mathematical methods in economics, as outlined above, represents an attempt to avoid the disadvantages of what might be termed the 'purist' and the 'cookbook' approaches to the exposition of mathematical methods in

economics. To explain what is meant by this statement, let us firstly note that the purist approach is characterized by, first, a desire to lay a very strong foundation which will be sufficient to support the build-up of mathematical analysis to the highest possible level, and, second, a rigorous, extensive, exposition of each mathematical method, backed up by mathematical proofs of each step and followed by challenging, unworked (and often unanswered), problems that are designed to really make the student think. As such, while the purist approach rightly stresses the vital importance of a good foundation, it has the unfortunate disadvantage of being both too rigorous and too extensive for the student with a poor mathematics background. In contrast, the cookbook approach is characterized by, first, a desire to proceed as quickly as possible to the use of each mathematical method, and, second, a very sketchy explanation of each mathematical method, followed by a recipe of steps of formulas for solving problems. As such, while the cookbook approach has the laudable goal of 'mathematical economics without tears', it has the very serious disadvantage of not only laying an inadequate foundation for the subsequent building up of mathematical ability, but also of giving an unwarranted degree of confidence initially. Unlike both the purist and the cookbook approaches, the approach of this book has the goal of laying a foundation which is neither excessively rigorous nor excessively extensive, but which still represents an adequate foundation for the mathematical methods discussed.

The next section discusses certain points which are important to remember when learning mathematical methods.

1-3 POINTS TO REMEMBER WHEN LEARNING MATHEMATICAL METHODS

The student who is commencing an introductory course of mathematical methods in economics should, first of all, try to take a realistic mental attitude towards such a course. In particular, the student should take comfort in the fact that it is by no means impossible for students with little or no prior knowledge of mathematics to successfully complete such courses—the numerous economics students, who successfully complete such courses year by year, are a sufficient witness to that fact. Just as the student will encounter problems in learning economics and, with perseverance, will eventually overcome these problems, so the student will also encounter and eventually overcome problems in learning mathematical methods in economics. For example, the author can still recall (with satisfaction) how, the first time he lectured on an introductory course of mathematical methods in economics, the lectures commenced with a pretty female student weeping unhappily (because she was utterly convinced that she would never

be able to master mathematical methods) and finished with the same young lady passing the course examination with flying colours!

The correct mental attitude to learning mathematical methods is more important than perhaps many students recognize. For example, when a student, studying a chapter in an economics textbook, is faced with a seemingly incomprehensible paragraph, he or she does not despair or immediately begin to think that he or she will never learn economics. Instead, the student reads the paragraph over again a few times and usually the problem disappears. If the problem still remains, the student will not stop reading, but instead will continue to read on through the chapter in the hope that things will eventually fall into place (as they usually do) as he or she proceeds. It is important that the student should have exactly the same mental attitude whenever he or she is faced with a problem in the study of mathematical methods in economics.

It is also important to remember that many things often look complex initially, but in practice turn out to be much less complex. For example, a book on how to drive a car may seem very complicated to the learner-driver, yet the actual operation of driving a car is not nearly so complicated as it appears. Also, the apparent complexity soon diminishes with practice. This is exactly the situation with mathematical methods. In other words, it is essential for the student to have plenty of practice on the actual operation of each mathematical method. Consequently, the student should not only read through each series of mathematical steps, but should also stop and work through the same series of steps on a sheet of paper. This simple procedure will: first, enhance the student's understanding, especially when it is realized that the actual mathematical operation is not as difficult as it looks at first sight; second, let the student see that one's own mathematical work looks much less complicated than someone else's mathematical work; and, third, give the student practice in mathematical manipulation.

The previous point is also important in the sense that it reminds us that, while certain things are not so hard to do in practice, they may be quite difficult to explain. For example, to change the illustration, it is by no means easy to write a simple explanation of how to tie a shoe-lace. No doubt, the person who has never seen a shoe, or a shoe-lace, would not only find the explanation (and the diagrammatic illustrations) extremely hard to follow, but would also find it very difficult to see the usefulness of such an operation. Yet, with practice in the actual operation of tying shoe-laces, most people not only quickly learn how to tie shoe-laces, but also quickly come to appreciate the usefulness of such an operation. The student will do well to remember this point when faced with a seemingly difficult and pointless mathematical method. In such a case, the student who practises writing out the steps of the particular mathematical method will, on proceeding through the chapter, not only find the mathematical method much less difficult, but will also eventually come to appreciate the usefulness

of the particular mathematical method. In other words, it is sometimes necessary to accept the 'how' of a mathematical method and proceed some way before understanding the 'why' of that method.

The student with not only a poor mathematics background, but also a poor ability in mathematical manipulation, should recognize that an increased ability in mathematical manipulation will not be achieved overnight. Such an increased ability only comes after a period of regular practice. While this is so, the student should not be discouraged by this fact, especially as both the level of difficulty and the pace of progress of this book have been designed to allow for the development of this ability. Moreover, to aid the development of ability in mathematical manipulation, the student reading this book should:

1. Read each portion of mathematical analysis and then write it out, doing each step for himself or herself.
2. Attempt all the Practice Problems—if you cannot do a particular problem, then re-read the relevant section in the chapter and try the problem again.
3. Check your answers to the Practice Problems against the answers given at the back of the book.
4. Periodically repeat the Practice Problems of chapters already read.
5. The student should also attempt all the (unanswered) Additional Problems.

Finally, the student should take comfort in the fact that even a modest investment in the learning of mathematical methods, such as learning the various mathematical methods included in this book, will produce a surprisingly large return. As the student progresses through *An Introduction to Mathematical Methods in Economics*, he or she will soon discover that the modest range of mathematical methods discussed has a very wide range of important applications in economics. In addition, the student will soon discover that each mathematical method has many applications in economics. The latter fact not only means that, by the repeated application of a particular mathematical method, the student's understanding of that method will be continually reinforced, but also means that his or her mathematical dexterity will be continually improved.

1-4 ALTERNATIVE PATHS OF STUDY THROUGH THE BOOK

As can be seen from the table of contents, the order of presentation in this book essentially consists of the exposition of each mathematical method followed by the application of that method to various aspects of economic

analysis. Also, as can be seen from the table of contents, the book concentrates heavily on elementary calculus methods and their application to economics. In addition, there are two chapters on matrix algebra. It should be noted, however, that the order of chapters for study need not follow that given in the table of contents. For example, the two chapters on matrix algebra may be either studied in their present position or, alternatively, studied as two consecutive chapters at a later stage in the book. This flexibility of order is made possible by the fact that the use of matrix algebra is essentially restricted to chapters four and five—the only other chapter where matrix methods are employed is chapter thirteen (where the particular matrix method involved is merely listed as an alternative method of solving a linear simultaneous-equation system).

The alternative location of chapters four and five is not, of course, the only example of a different, yet feasible, order of presentation. For example, at the author's university, the contents of this book are integrated into the undergraduate economics degree course in the following way:

1. The contents of eleven chapters (in the order: 2, 3, 6–11, 14, 4, and 5) are taught, to first-year students, as an introductory course of mathematical methods in economics. This introductory course (which is regarded as an essential supplement to the first-year microeconomics and macroeconomics courses) accounts, time-wise, for one-sixth of the first-year economics programme.
2. The contents of chapters twelve and thirteen are taught as part of a second-year course in microeconomics. This example indicates that *An Introduction to Mathematical Methods in Economics* may be used both as a textbook for introductory 'mathematics for economists' courses and as a supplement to various courses in microeconomics and macroeconomics.

CHAPTER

TWO

THE MATHEMATICAL REPRESENTATION OF ECONOMIC RELATIONSHIPS

2-1 ECONOMIC MODELS

A brief perusal, of almost any introductory economics textbook, quickly reveals that economics is very much concerned with the important issues of everyday life such as inflation, wages, unemployment, balance of payments, taxation, pollution, and so on. In other words, it is the interesting and challenging task of economics to study such phenomena with a view to understanding them and providing an explanation of them. However, while such a task is both stimulating and worth while, the vast complexity of the modern industrial economy makes this task rather daunting. For example, it is clearly not possible to try to study all economic phenomena at once, since this would involve attempting to handle an immense number of economic interrelationships simultaneously. This means we must decide upon some simpler method of procedure. In this respect, the most obvious method of procedure is to select or isolate some particular economic phenomenon, such as consumer behaviour, for detailed study on its own.

Once we have decided to study, say consumer behaviour, we cannot just go along to the nearest supermarket and 'observe' consumers—the simple reason being that we cannot just 'observe', we need to know *what* to observe. This implies that, in order to proceed, we must make some hypothesis about consumer behaviour that will guide our observations. For example, we may hypothesize that a consumer's demand (per time period) for a good will be influenced by the market price of that good. In particular,