

College Algebra

A Graphing Approach

SECOND EDITION

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College Algebra

Preface

College Algebra: A Graphing Approach, Second Edition, is the premier text for a reform-oriented course. Designed to build a strong foundation in algebra, the text encourages students to develop a firm grasp of the underlying mathematical concepts while using algebra as a tool for solving real-life problems. The comprehensive text presentation invites discovery and exploration, while the integrated technology and consistent problem-solving strategies help the student develop strong college algebra skills.

College Algebra Reform

The college algebra course has changed over the past few years in response to the growing discussion of reform in mathematics education. Generally speaking, these changes have focused on the following areas: technology, real-life applications, problem-solving, and communicating about mathematics. The Second Edition embodies the spirit of these reform ideals without compromising the mathematical integrity of the course presentation. All text elements from the previous edition were considered for revision and many new examples, exercises, and applications were added.

Technology Graphing technology is consistently incorporated throughout the Second Edition. The visualization and exploration capabilities of technology encourage the student to participate actively in the learning process, to develop their intuitive understanding of mathematical concepts, and to solve problems using actual data. Thus, students learn how algebra functions as a modeling language for real-life problems. Technology is used as a tool, drawn into the discussion whenever it offers a useful perspective on the topic at hand. For example, the power of graphing technology may be used to guide the students through thought-provoking explorations or to show alternative problem-solving techniques. Where appropriate, situations in which the results obtained through the use of technology may be misleading are also noted.

The Second Edition assumes that the student will use a graphing calculator on a daily basis in the course. Integrated throughout the text at point of use are many opportunities for investigation using technology (e.g., see page 318) and exercises that require the use of a graphing utility (e.g., see page 188). The text also carefully shows how to use graphing technology to best advantage (e.g., see page 185).

Whenever possible, references to graphing technology are generic. In a few cases, however, the text includes programs that will enable the student to investigate particular mathematical concepts (e.g., see page 134). Comparable programs for a wide variety of Texas Instruments, Casio, Sharp, and Hewlett-

Packard graphing calculators—including the most current models—are given in the appendix.

To accommodate a variety of teaching and learning styles, *College Algebra: A Graphing Approach*, Second Edition, is also available in a multimedia, CD-ROM format. *Interactive College Algebra: A Graphing Approach* offers students a variety of additional tutorial assistance, including examples and exercises with detailed solutions; pre-, post-, and self-tests with answers; and *TI-82* and *TI-83* graphing calculator emulators. (See pages xviii–xx for more detailed information.)

Real-Life Applications To emphasize for students the connection between mathematical concepts and real-world situations, up-to-date, real-life applications are integrated throughout the text. These applications appear as chapter introductions with related exercises (e.g., see page 75 and 115), examples (e.g., see page 360), exercises (e.g., see page 87), Group Activities (e.g., see page 97), and Chapter Projects (e.g., see page 455).

Students have many opportunities to collect and interpret data, to make conjectures, and to construct mathematical models in the examples, exercises, Group Activities, and Chapter Projects. Students work on modeling problems with experimental and theoretical probabilities (e.g., see page 591), use mathematical models to make predictions or draw conclusions from real data (e.g., see page 354), compare models (e.g., see page 307), and apply curve-fitting techniques to create their own models from data (e.g., see page 230). In the process, the Second Edition gives students many more opportunities to use charts, tables, scatter plots, and graphs to summarize, analyze, and interpret data.

Problem Solving The primary goal of any mathematics textbook is to encourage students to become competent and confident problem solvers. Many aspects of this revision focused on this goal—including the addition of new features such as Chapter Projects, Explorations, and Group Activities, as well as extensive and careful revision of the examples and exercise sets. Students are asked to use numerical, graphical, and algebraic techniques, and the use of graphing technology as a problem-solving tool is encouraged as appropriate (e.g., see page 215). Throughout, students are encouraged to follow a consistent approach to solving applied problems: Construct a verbal model, label terms, construct an algebraic model, solve the problem using the model, and check the answer in the original statement of the problem.

Like the previous edition, the Second Edition has an abundance of exercises that are designed to develop skills. The text also includes many other types of exercises that offer students the opportunity to refine their problemsolving skills, such as exercises that require interpretations (e.g., see page 329), those having many correct answers (e.g., see page 288), and multipart exercises designed to lead the student through problem-solving strategies (e.g., see page 226).

Communicating about Mathematics Each section in the Second Edition ends with a Group Activity. Designed to be completed in class or as homework assignments, the Group Activities give students the opportunity to work cooperatively as they think, talk, and write about mathematics. Students' understanding is reinforced through interpretation of mathematical concepts and results (e.g., see page 250), problem posing and error analysis (e.g., see page 428), and constructing mathematical models, tables and graphs (e.g., see page 418).

Making connections between algebra and real-world situations also helps students understand the underlying theory. Other connections are emphasized in this text as well, including those to probability (e.g., see Chapter 7), geometry (e.g., see page 397), and statistics (see the Sections P.6 and 2.6).

Improved Coverage

Chapter P, Prerequisites, is streamlined in the Second Edition. All or part of this review material may be covered or it may be omitted, offering greater flexibility in designing the course syllabus.

Several topics are now covered earlier in the Second Edition. Linear modeling and scatter plots are covered much earlier, in Chapter 2. Complex numbers also are now in Chapter 2. Rational functions are covered with polynomials in Chapter 3. The discussion of exponential and logarithmic functions appears a chapter earlier, in Chapter 4. Coverage of systems of equations and inequalities was moved to Chapter 5 and now includes partial fractions and linear programming.

New sections on exploring data have been added to Chapters P, 2, and 4. The coverage of matrices in Chapter 6 was expanded to include determinants of matrices and applications. Conics and translations of conics have been moved to a new chapter, Chapter 8, along with new coverage of parametric equations.



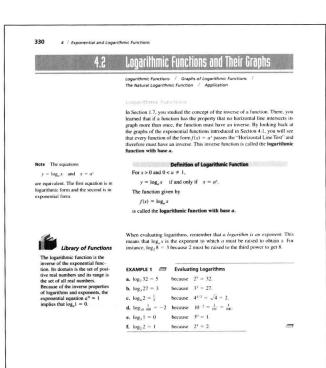
Features of the Second Edition

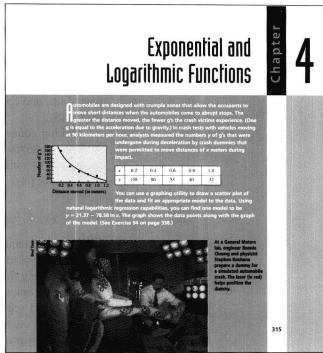
Chapter Opener Each chapter opens with a look at a real-life application. Real data is presented using graphical, numerical, and algebraic techniques.

Notes Notes anticipate students' needs by offering additional insights, pointing out common errors, and describing generalizations.

Theorems, Definitions, and Guidelines

All of the important rules, formulas, theorems, guidelines, properties, definitions, and summaries are highlighted for emphasis. Each is also titled for easy reference.

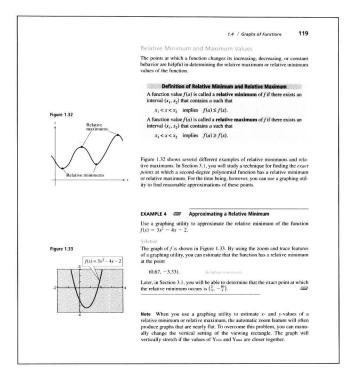


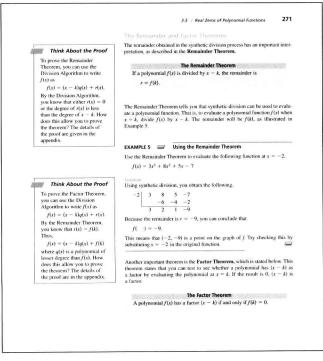


Section Outline Each section begins with a list of the major topics covered in the section. These topics are also the subsection titles and can be used for easy reference and review by students. In addition, an exercise application that uses a skill or illustrates a concept covered in the section is highlighted to emphasize the connection between mathematical concepts and real-life situations.

Library of Functions The concept of the function is introduced in Chapter 1. In the material that follows, the icon appears each time a new type of function is described in detail.

Intuitive Foundation for Calculus Special emphasis is given to the algebraic skills that are needed in calculus. Many examples in the Second Edition discuss algebraic techniques or graphically show concepts that are used in calculus, providing an intuitive foundation for future work.

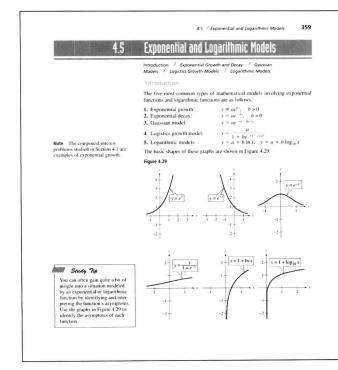




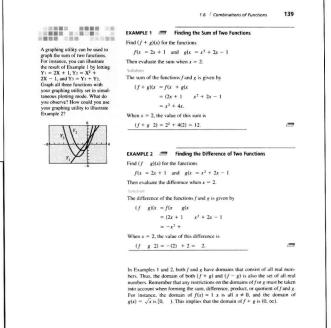
Think About the Proof Located in the margin adjacent to the corresponding theorem, each Think About the Proof feature offers strategies for proving the theorem. Detailed proofs for all theorems are given in Appendix B.

Technology Technology is integrated throughout the text at point of use as a tool for visualization, investigation, and verification. Instructions for using graphing utilities are given as necessary.

Study Tips Study Tips appear in the margin at point of use and offer students specific suggestions for studying algebra.



Exploration Throughout the text, the Exploration features encourage active participation by students, strengthening their intuition and critical thinking skills by exploring mathematical concepts and discovering mathematical relationships. Using a variety of approaches—including visualization, verification, use of graphing utilities, pattern recognition, and modeling—students are encouraged to develop a conceptual understanding of theoretical topics.





Many of the properties of real numbers are valid for complex numbers as well. Here are some examples.

2.3 Complex Numbers

Associative Property of Addition and Multiplication Commutative Property of Addition and Multiplication Distributive Property of Multiplication Over Addition

Notice how these properties are used when two complex numbers are multiplied.

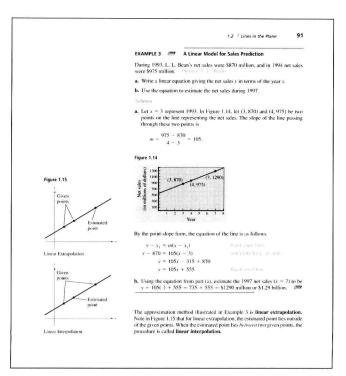
 $\begin{aligned} (a+bi)(c+di) &= a(c+di) + bi(c+di) \\ &= ac + (adji + (bc)i + (bd)i^2 \\ &= ac + (adji + (bc)i + (bd)i - 1) \\ &= ac - bd + (adji + (bc)i \end{aligned} \qquad \begin{array}{ll} \text{The standard of the sta$

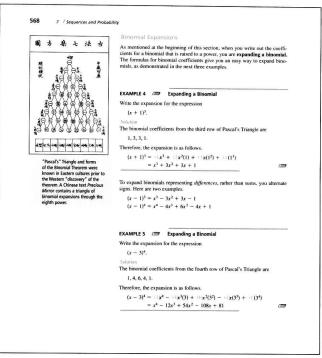
Rather than trying to memorize this multiplication rule, we suggest that you simply remember how the distributive property is used to multiply two complex numbers. The procedure is similar to multiplying two polynomials and combining like terms (as in the FOIL Method).

EXAMPLE 2 🥭 Multiplying Complex Num	nbers
$a_i(i)(-3i) = -3i^2$	Advisigning
= -3(-1)	
= 3	
b. $(2-i)(4+3i) = 8+6i-4i-3i^2$	Property Comment
= 8 + 6i - 4i - 3(-1)	
= 8 + 3 + 6i - 4i	
= 11 + 2i	
e. $(3 + 2i)(3 - 2i) = 9 - 6i + 6i - 4i^2$	
= 9 - 4(-1)	
= 9 + 4	
= 13	
d. $(3 + 2i)^2 = 9 + 6i + 6i + 4i^2$	
= 9 + 4(-1) + 12i	
= 9 - 4 + 12i	
= 5 + 12i	Stappling hour

Historical Notes To help students understand that algebra has a past, historical notes featuring mathematicians and their work and mathematical artifacts are included in each chapter.

Graphics Visualization is a critical problem-solving skill. To encourage the development of this ability, the text has nearly 1500 figures in examples, exercises, and answers to exercises. Included are graphs of equations and functions, geometric figures, displays of statistical information, scatter plots, and numerous screen outputs from graphing technology. All graphs of equations and functions are computer- or calculator-generated for accuracy, and they are designed to resemble students' actual screen outputs as closely as possible. Graphics are also used to emphasize graphical interpretation, comparison, and estimation.

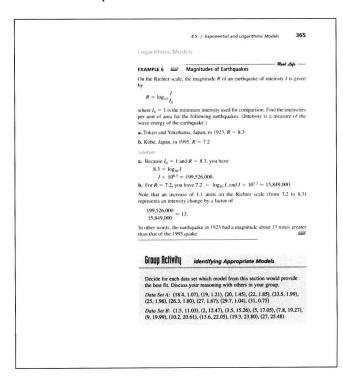


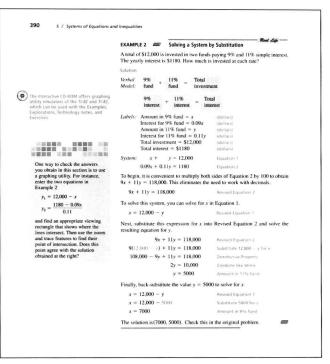


Applications Real-life applications are integrated throughout the text in examples and exercises. These applications offer students constant review of problem-solving skills, and they emphasize the relevance of the mathematics. Many of the applications use recent, real data, and all are titled for easy reference. Photographs with captions in the introduction to the chapter also encourage students to see the link between mathematics and real life.

Examples Each of the more than 400 text examples was carefully chosen to illustrate a particular mathematical concept, problem-solving approach, or computational technique, and to enhance students' understanding. The examples in the text cover a wide variety of problem types, including theoretical problems, real-life applications (many with real data), and problems requiring the use of graphing technology. Each example is titled for easy reference, and real-life applications are labeled. Many examples include side comments in color that clarify the steps of the solution.

Problem Solving The text provides ample opportunity for students to hone their problem-solving skills. In both the exercises and the examples in the Second Edition, students are asked to apply verbal, analytical, graphical, and numerical approaches to problem solving. Students are also encouraged to use a graphing utility as a tool for solving problems. Students are taught the following approach to solving applied problems: (1) construct a verbal model; (2) label variable and constant terms; (3) construct an algebraic model; (4) using the model, solve the problem; and (5) check the answer in the original statement of the problem.

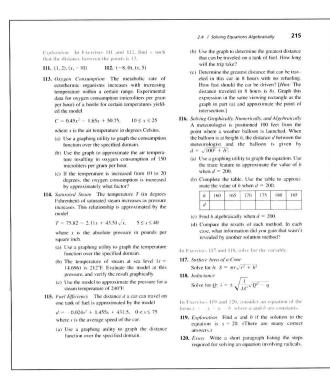


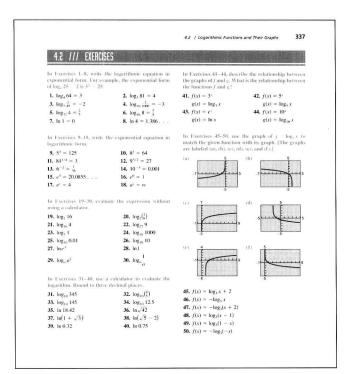


cD-ROM The icon refers to additional features of *Interactive College Algebra: A Graphing Approach* that enhance the text presentation, such as exercises, computer animations, examples, tests, and graphing calculator emulators.

Group Activities The Group Activities that appear at the ends of sections reinforce students' understanding by studying mathematical concepts in a variety of ways, including talking and writing about mathematics, creating and solving problems, analyzing errors, and developing and using mathematical models. Designed to be completed as group projects in class or as homework assignments, the Group Activities give students opportunities to do interactive learning and to think, talk, and write about mathematics.

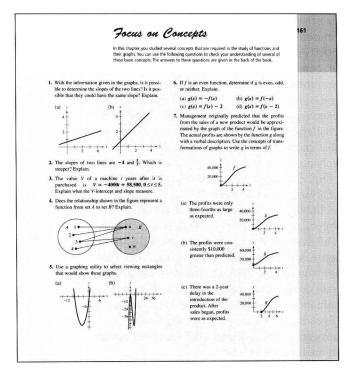
Exercises The exercise sets were completely revised for the Second Edition. More than 5000 exercises with a broad range of conceptual, computational, and applied problems accommodate a variety of teaching and learning styles. Included in the section and review exercise sets are multipart, writing, and more challenging problems with extensive graphics that encourage exploration and discovery, enhance students' skills in mathematical modeling, estimation, and data interpretation and analysis, and encourage the use of graphing technology for conceptual understanding. Applications are labeled for easy reference. The exercise sets are designed to build competence, skill, and understanding; each exercise set is graded in difficulty to allow students to gain confidence as they progress. Detailed solutions to all odd-numbered exercises are given in the Study and Solutions Guide; answers to all odd-numbered exercises appear in the back of the text.

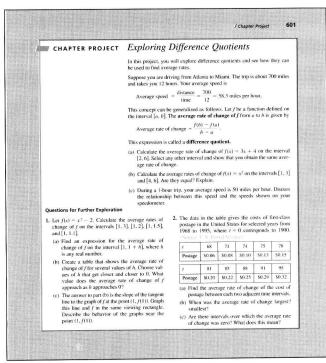




Geometry Geometric formulas and concepts are reviewed throughout the text in examples, Group Activities, and exercises. For reference, common formulas are listed inside the back cover of this text.

Focus on Concepts Each Focus on Concepts feature is a set of exercises that test students' understanding of the basic concepts covered in the chapter. Answers to all questions are given in the back of the text.





Chapter Projects Chapter Projects are extended applications that use real data, graphs, and modeling to enhance students' understanding of mathematical concepts. Designed as individual or group projects, they offer additional opportunities to think, discuss, and write about mathematics. Many projects give students the opportunity to collect, analyze, and interpret data.

1 / Functions and Their Graphs

1 /// REVIEW EXERCISES

In Exercises 1 and 2, complete the table. Ese the resulting solution points to sketch the graph of the equation. Use a graphing utility to verify the graph.

1.
$$y = -\frac{1}{2}x + 2$$

x	-2	0	2	3	4
5					

 $2 \cdot v = v^2 - 3v$

,i	-1	0	1	2	3
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In F vereises 3–12, sketch the graph of the equation by hand. Use a graphing utility to verify the graph.

 $3 \cdot y - 2y - 3 = 0$ $4 \cdot 3y + 2y + 6 = 0$ 5. x - 5 = 07. $y = \sqrt{5 - x}$

6. y = 8 - |x|8. $y = x^2 - 4x$ 10. $y = x^2 - 4x$

9. $y + 2x^2 = 0$ 11. $y = \sqrt{25 - x^2}$

12. $x^2 + y^2 = 10$

In Exercises 13-20, use a graphing utility to graph the

15. $y = \frac{1}{4}x^4 - 2x^2$

13. $y = \frac{1}{4}(x+1)^3$ 14. $y = 4 - (x-4)^2$ 16. $y = \frac{1}{4}x^3 - 3x$ 17. $y = x\sqrt{9 - x^2}$ 18. $y = x\sqrt{x + 3}$

19. y = |x - 4| - 4 **20.** y = |x + 2| + |3 - x|

utility such that the graph of the equation agrees with the given graph.

21.
$$y = 0.002x^2 - 0.06x - 1$$

22. $y = 10x^3 - 21x^2$

Figure for 21





Data Analysis. In Exercises 23 and 24, (a) use a graphing utility to plot the data; (b) use a graphing utility Seast squares regression capabilities to find the best-fitting linear model (let t = 0 correspond to 1990a (c) graph the model in the same viewing rectangle with the data, and sketch the model for the data; and (d) use the model to estimate the values of y for the years 1998 and 2000.

23. The total annual expenditures y for the Smithsonian Institution (in millions) each year from 1990 through 1993 are given in the table. (Souther 1)

x	1990	1991	1992	1993
v	302	340	387	395

24. The total annual expenditures y for NASA (in billions) from 1990 through 1993 are given in the table.

x	1990	1991	1992	1993
y	12.4	13.9	14.0	14.3

Cumulative Test for Chapters 3-5

3-5 /// CUMULATIVE TEST

1. $h(x) = -(x^2 + 4x)$ **2.** $f(t) = \frac{1}{4}t(t-2)^2$ **3.** $g(s) = \frac{2s}{s-3}$

- 7. Let x be the amount (in hundreds of dollars) that a company spends on advertising, and let P be the profit (in thousands of dollars), where $P = 230 + 20x \frac{1}{3}x^2$. How much advertising will maximize the profit?
- 8. Find all the zeros of $f(x) = x^{1} + 2x^{2} + 4x + 8$.
- 9. Approximate the real zero of $g(x) = x^3 + 3x^2 6$ to the nearest hundredth.
- 10. Write $2 \ln x \frac{1}{2} \ln(x + 5)$ as a logarithm of a single quantity.
- 11. You deposit \$2500 in an account earning 7.5% interest, compounded continuously. Find the balance after 25 years.

12. $6e^{2x} = 72$

13. $\log_2 x + \log_2 5 = 6$

14. Substitution: $2x - y^2 = 0$

15. Graphical: $y = \log_3 x$

 $\chi - \chi = 4$

- 16. Find the equation of the parabola $y = ax^2 + bx + c$ passing through the points (0, 6), (-2, 2), and $(3, \frac{9}{2})$.
- 17. Derive a set of inequalities to describe the region shown in the figure.
- 11. Let're a set or inequanties to osestine the region shown in the figure.

 S. A merchant plans to sell two models of compart disc players. One model sells for \$275 and yields a profit of \$55, and the other model selfs for \$400 and yields a profit of \$57. The merchant estimates that the total monthly demand with net exceed 300 units. The merchant does not want to invest more than \$100,000 in inventory for these products. Find the number of units of each model that should be stocked in order to maximize profit.



Review Exercises The Review Exercises at the end of each chapter offer students an opportunity for additional practice. Answers to odd-numbered review exercises are given in the back of the text.

3 / Polynomial and Rational Functions

3 /// CHAPTER TEST

lake this nest as concitrould take a first his lass. After you are thine, check your wink of

1. Describe how the graph of g differs from the graph of $f(x) = x^2$.

- 2. Identify the vertex and intercepts of the graph of $y = x^2 + 4x + 3$.

 3. Find an equation of the parabola shown at the right.

 4. The path of a half is given by $y = -\frac{1}{30}x^2 + 3x + 5$, where y is the height in feet and the parabola shown in the content of the parabola shown at the right. x is the horizontal distance in feet. (a) Find the maximum height of the ball.
- (b) Which term determines the height at which the ball was thrown? Does changing this term change the coordinates of the maximum height of the ball? Explain.

(a) $g(x) = 2 - x^2$ (b) $g(x) = (x - \frac{x}{2})^2$

5. Divide by long division: $(3x^3 + 4x - 1) + (x^2 + 1)$. **6.** Divide by synthetic division: $(2x^4 - 5x^2 - 3) \div (x - 2)$.

7. $g(t) = 2t^4 - 3t^3 + 16t - 24$ 8. $h(x) = 3x^5 + 2x^4 - 3x - 2$

9. $f(x) = x^4 - x^3 - 1$

10. $f(x) = 3x^4 + 2x^4 - 12x - 8$

11. 0, 3, 3 + i, 3 - i

12. $1 + \sqrt{3}i$, $1 - \sqrt{3}i$, 2. 2

14. $g(x) = \frac{x^2 + 2}{x - 1}$

15. Find a rational function with vertical asymptotes at $x=\pm 3$ and a horizontal asymptote

Chapter Tests Each chapter that is not followed by a Cumulative Test ends with a Chapter Test, an effective tool for student self-assessment.

Cumulative Tests The Cumulative Tests that follow Chapters 2, 5, and 8 help students judge their mastery of previously covered material as well as reinforce the knowledge they have been accumulating throughout the text—preparing them for other exams and for future courses.

Supplements

College Algebra: A Graphing Approach, Second Edition, by Larson, Hostetler, and Edwards is accompanied by a comprehensive supplements package. Most items are keyed to the text.

Printed Resources

For the student

Study and Solutions Guide by Bruce Edwards, University of Florida, and Dianna L. Zook, Indiana University—Purdue University at Fort Wayne

- · Section summaries of key concepts
- Detailed, step-by-step solutions to all odd-numbered exercises
- · Key solution steps for Chapter Tests and Cumulative Tests
- · Practice tests with solutions
- · Study strategies

Graphing Technology Keystroke Guide: Precalculus

- Keystroke instructions for a wide variety of Texas Instruments, Casio, Sharp, and Hewlett-Packard graphing calculators—including the most current models.
- · Examples with step-by-step solutions
- · Extensive graphics screen output
- · Technology tips

For the instructor

Instructor's Annotated Edition

- Includes the entire student edition of the text, with the student answers section
- Instructor's Answers section: Answers to all even-numbered exercises, and answers to all Explorations, Technology exercises, Group Activities, and Chapter Project exercises
- Annotations at point of use offer specific teaching strategies and suggestions for implementing Group Activities, point out common student errors, and give additional examples, exercises, class activities, and group activities.

Solutions to Even-Numbered Exercises

• Detailed, step-by-step solutions to even-numbered exercises

Test Item File and Instructor's Resource Guide

- Printed test bank with approximately 2000 test items (multiple-choice, openended, and writing) coded by level of difficulty
- Technology-required test items coded for easy reference
- · Bank of chapter test forms with answer keys

- · Two final exam test forms
- Notes to the instructor, including materials for alternative assessment and managing the multicultural and cooperative-learning classrooms

Problem Solving, Modeling, and Data Analysis Labs by Wendy Metzger, Palomar College

- · Multipart, guided discovery activities and applications
- Keystroke instructions for Derive and TI-82
- Keyed to the text by topic
- Funded in part by NSF (National Science Foundation, Instrumentation and Laboratory Improvement) and California Community College Fund for Instructional Improvement

Media Resources

For the student

Interactive College Algebra: A Graphing Approach (See pages xviii–xx for a description, or visit the Houghton Mifflin home page at http://www.hmco.com for a preview.)

- Interactive, multimedia CD-ROM format
- · IBM-PC for Windows

Tutor software

- · Interactive tutorial software keyed to the text by section
- · Diagnostic feedback
- · Chapter self-tests
- Guided exercises with step-by-step solutions
- Glossary

Videotapes by Dana Mosely

- Comprehensive, text-specific coverage keyed to the text by section
- · Real-life application vignettes introduced where appropriate
- Computer-generated animation
- · For media/resource centers
- Additional explanation of concepts, sample problems, and applications
- · Instructional graphing calculator videotape also available

For the instructor

Computerized Testing (IBM, Macintosh, Windows)

- · New on-line testing
- New grade-management capabilities
- Algorithmic test-generating software provides an unlimited number of tests
- Approximately 2000 test items
- Also available as a printed test bank

Transparency Package

• 50 color transparencies color-coded by topic