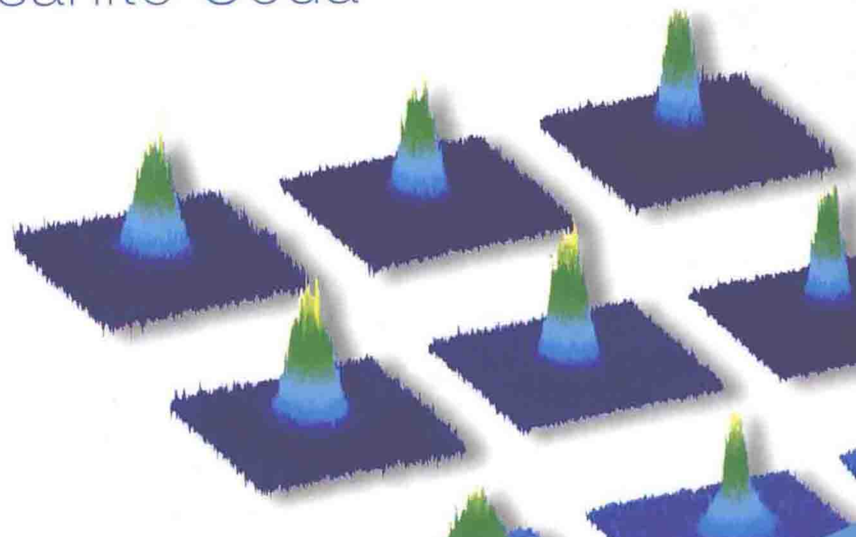
FUNDAMENTALS AND NEW FRONTIERS OF BOSE-EINSTEIN CONDENSATION

Masahito Ueda



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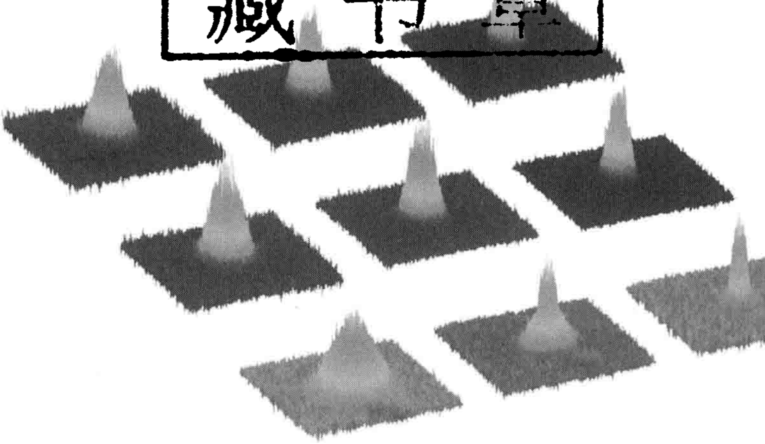

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Preface

Experimental realization of Bose–Einstein condensation (BEC) of dilute atomic gases [Anderson, *et al.* (1995); Davis, *et al.* (1995); Bradley, *et al.* (1995, 1997)] has ignited a virtual explosion of research. The unique feature of the atomic gas BEC is its unprecedented controllability, which makes the previously unthinkable possible. Almost all parameters of the system such as the temperature, number of atoms, and even strength and sign (attractive or repulsive) of interaction can be varied by several orders of magnitude. The interaction between atoms is usually considered to be an immutable, inherent property of individual atomic species. In alkali and some other Bose–Einstein condensates, we can not only control the strength of interaction but also switch the sign of interaction from repulsive to attractive and vice versa [Inouye, *et al.* (1998); Cornish (2000)]. The atomic-gas BEC may thus be regarded as an artificial macroscopic matter wave that act as an ideal testing ground for the investigation of quantum many-body physics. The atomic-gas BEC may also be regarded as an atom laser because the condensate provides a phase-coherent, intense atomic source with potential applications for precision measurement, lithography, and quantum computation. Fermionic species may also undergo BEC by forming molecules or Cooper pairs. Both molecular condensates [Greiner, *et al.* (2003); Zwierlein, *et al.* (2003)] and Bardeen–Cooper–Schrieffer-type resonant superfluids [Regal, *et al.* (2004); Zwierlein, *et al.* (2004)] have been realized using alkali fermions, opening up the new research field of strongly correlated gaseous superfluidity. This book is intended as an introduction to this rapidly developing, interdisciplinary field of research.

Most phase transitions occur due to interactions between constituent particles. For example, superconductivity occurs due to effective interactions between electrons, and ferromagnetism is caused by the exchange interaction between spins. In contrast, BEC is a genuinely quantum-statistical phase transition in that it occurs without the help of interaction (Einstein called it "condensation without interaction" [Einstein (1925)]). The fundamentals of noninteracting BECs are reviewed in Chapter 1.

In a real BEC system, interactions between atoms play a crucial role in determining the basic properties of the system. Neutral atoms have a hard core that is short-ranged (~ 1 Å) and strongly repulsive. At a longer distance (~ 100 Å), the atoms are attracted to each other because of the van der Waals force. When two atoms collide, they experience both these forces, and the net interaction can be either repulsive or attractive depending on the hyperfine and translational states of the colliding atoms. Under normal conditions, a dilute-gas BEC system can be treated as a weakly interacting Bose gas. The Bogoliubov theory of a weakly interacting Bose gas and related topics are described in Chapter 2.

One of the remarkable aspects of a dilute gas BEC system is the great success of the mean-field theory governed by the Gross–Pitaevskii (GP) equation [Gross (1961); Pitaevskii (1961)]. The GP equation describes the mean-field ground state as well as the linear and nonlinear response of the system. Various nonlinear matter-wave phenomena including four-wave mixing [Deng, *et al.* (1999); Rolston and Phillips (2002)] and topological excitations such as solitons [Denschlag, *et al.* (2000)] and vortices [Matthews, *et al.* (1999); Madison, *et al.* (2000)], have been successfully described by the GP equation. This remarkable success of the mean-field theory is due to the high ($> 99\%$) degree of condensation of bosons into a single-particle state, which in turn originates in an extremely low density ($\sim 10^{11} - 10^{15}$ cm^{-3}) of the system operating at ultralow temperatures ($\lesssim 10^{-6}$ K). The Gross–Pitaevskii theory together with its various applications is discussed in Chapter 3.

The linear response theory provides a general theoretical framework to investigate collective modes of Bose–Einstein condensates and superfluids. A sum-rule approach is also very useful for this purpose because the ground state for a dilute-gas Bose–Einstein condensate can be obtained very accurately. These subjects are discussed in Chapter 4.

Superfluidity manifests itself as a response of the system to its moving container. A statistical-mechanical theory to tackle such problems and some basic properties of superfluidity are described in Chapter 5.

Alkali atoms have both electronic spin $\mathbf{s}$ and nuclear spin $\mathbf{i}$, and these two spins interact with each other via the hyperfine interaction. When the energy of the hyperfine coupling exceeds the electronic and nuclear Zeeman energies as well as the thermal energy, the total spin $\mathbf{f} = \mathbf{s} + \mathbf{i}$, which is called the hyperfine spin, is a conserved quantum number. When atoms are confined in a magnetic potential, the spin of each atom points in the direction of an external magnetic field. The spin degrees of freedom are therefore frozen and the mean-field properties of the system are described by a scalar order parameter. When the system is confined in an optical trap, the frozen degrees of freedom are liberated, yielding a rich variety of phenomena arising from the magnetic moment of the atom. Since the magnetic moments of alkali atoms originate primarily from the electronic spin, this system's response to an external magnetic field is much greater than that of superfluid helium-3. We can expect interesting interplay between superfluidity and magnetism with the possibility of new ground states, spin domains, and vortex structures. Spinor condensates are discussed in Chapter 6.

When the rotational speed of the container of the system is faster than the critical frequency, vortices enter the system and form a vortex lattice. The direct observation of vortex lattice formation [Madison, *et al.* (2000); Abo-Shaeer, *et al.* (2001)] has attracted considerable interest in the equilibrium and nonequilibrium dynamics of condensates. The effect of rotation on neutral particles is equivalent to that of a magnetic field on charged particles. Therefore, the properties of a vortex lattice of neutral particles are similar to those of superconductors. Furthermore, it is pointed out that in systems containing neutral bosons that are subject to very fast rotation, the vortex lattice melts, and a new vortex liquid state similar to the Laughlin state in the fractional quantum Hall system may be realized. A brief overview of these subjects is presented in Chapter 7.

Almost every bosonic atom has its fermionic counterpart. Fermions and bosons of the same species exhibit the same properties at high temperature, but they exhibit remarkably different behavior when quantum degeneracy sets in. Bosons undergo BEC below the transition temperature; in contrast, fermions become degenerate below the Fermi temperature, where almost every quantum state below the Fermi energy is occupied by one fermion and most quantum states above the Fermi energy are empty. At even lower temperatures, fermionic systems may exhibit superfluidity by forming Cooper pairs via the Bardeen–Cooper–Schrieffer transition. This is a rapidly developing field that has relevance to high-temperature superconductivity. We describe the basics and some of the recent developments of ultracold fermionic systems in Chapter 8.

It is known that BEC does not occur at finite temperature in one- or two-dimensional infinite systems because thermal fluctuations destroy the off-diagonal long-range order (ODLRO). In one-dimensional systems, BEC does not occur even at absolute zero because quantum fluctuations wash out the ODLRO. However, confined low dimensional systems can exhibit BEC because long-wavelength fluctuations are cut off by confinement. We may thus investigate interesting phenomena associated with low-dimensional BEC, such as solitons and the Berezinskii–Kosterlitz–Thouless transition. These subjects are discussed in Chapter 9.

Atoms with magnetic moments and polar molecules undergo dipole–dipole interactions, which are long-ranged and anisotropic and yield a wealth of novel phenomena. The magnetic dipole–dipole interaction is by far the weakest of the relevant interactions in cold atom systems; yet it plays a dominant role in forming spin textures and magnetic ordering and produces a spectacular effect in the course of the collapsing dynamics. The electric dipole–dipole interaction between polar molecules, in contrast, is very strong and may cause instabilities of the system; at the same time, it has the potential to yield several exotic phases and for use in quantum information processing. Some basic properties of the dipolar condensates are reviewed in Chapter 10.

An optical lattice is a periodic potential created by interference between two counterpropagating laser beams. Atoms in an optical lattice behave like electrons in a crystal. An optical lattice can host bosons as well as fermions, and it offers an ideal testbed to simulate quantum many-body physics and quantum information processing. Chapter 11 provides a brief overview of some basic properties of this artificial condensed matter system.

Superfluids host a rich variety of topological defects such as vortices, monopoles, and skyrmions. Those topological excitations are best described by the homotopy theory. Chapter 12 is devoted to an introduction of the homotopy theory, classfication of topological excitations, and an account of am how to calculate various topological charges.

Fifteen years after its first experimental realization, the field of ultracold atomic gases is still growing at a remarkable speed, such that coverage of every topic of importance far exceeds the range of this or perhaps any book. Rather, I have chosen a small number of important issues and tried to discuss their physical aspects as engagingly as possible. Many of the phenomena that have been observed in the past decade and those that will possibly be observed in the near future are of fundamental importance because of the very fact that they are being "seen" on a macroscopic scale.

If this book succeeds in conveying even a portion of the fascination inherent in this field, it will have well served its intended purpose.

This book derives from a set of lecture notes delivered at several universities over the past decade or so. I have benefited greatly from students and colleagues who actively participated in the class and collaboration. Special thanks are due to Rina Kanamoto, Yuki Kawaguchi, Michikazu Kobayashi, Tony Leggett, Hiroki Saito, and Masaki Tezuka. I would like to thank all of them for their questions, comments, and criticisms that helped me clarify my thoughts and improve the presentation of the material in this book. I am grateful to A. Koda, Y. Ookawara, and A. Yoshida for their efficient editing and preparation of the figures.

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Contents

Chapter 1

Fundamentals of Bose–Einstein Condensation

1.1 Indistinguishability of Identical Particles

Quantum statistics is governed by the principle of indistinguishability of identical particles. Particles with integer (half-integer) spin (in multiples of $\hbar$, where $\hbar$ is the Planck constant divided by 2π) are called bosons (fermions). Bosons obey Bose–Einstein statistics in which there is no restriction on the occupation number of any single-particle state. Fermions obey Fermi–Dirac statistics in which not more than one particle can occupy any single-particle state. The many-body wave function of identical bosons (fermions) must be symmetric (antisymmetric) under the exchange of any two particles. This symmetry requirement drastically reduces the number of available quantum states of the system, resulting in highly nonclassical phenomena at low temperature.

To understand this, let us suppose that we obtain a wave function $\Phi(\xi_1, \xi_2)$ of a two-particle system by solving the Schrödinger equation, where ξ_1 and ξ_2 represent the space and possibly spin coordinates of the two particles. For identical bosons (fermions), the symmetrized (antisymmetrized) wave function is given by

$$\Psi(\xi_1, \xi_2) = \frac{1}{\sqrt{2}}[\Phi(\xi_1, \xi_2) \pm \Phi(\xi_2, \xi_1)], \tag{1.1}$$

where the plus (minus) sign indicates bosons (fermions). The joint probability of finding the two particles at ξ_1 and ξ_2 is given by

$$\begin{aligned} |\Psi(\xi_1, \xi_2)|^2 = \frac{1}{2}\{&|\Phi(\xi_1, \xi_2)|^2 + |\Phi(\xi_2, \xi_1)|^2 \\ &\pm 2\mathrm{Re}[\Phi^*(\xi_1, \xi_2)\Phi(\xi_2, \xi_1)]\}, \end{aligned} \tag{1.2}$$

where Re denotes the real part. Because of the last interference term in Eq. (1.2), the probability of finding the two identical bosons at the same coordinate, $|\Psi(\xi,\xi)|^2$, is twice as high as $|\Phi(\xi,\xi)|^2$, which gives the corresponding probability for distinguishable particles. In contrast, for fermions, $|\Psi(\xi,\xi)|^2$ vanishes in accordance with Pauli's exclusion principle.

Such a bunching effect of bosons becomes increasingly pronounced when the number of bosons is large. For N number of bosons, the symmetrized wave function is given by

$$\Psi(\xi_1,\xi_2,\cdots,\xi_N) = \frac{1}{\sqrt{N!}} \sum_{(i_1,i_2,\cdots,i_N)} \Phi(\xi_{i_1},\xi_{i_2},\cdots,\xi_{i_N}), \tag{1.3}$$

where the summation over $i_1, i_2, \cdots, i_N$ is to be taken over all $N!$ permutations of $1, 2, \cdots, N$. The joint probability of finding all N bosons at the same coordinate is thus $N!$ times that for distinguishable bosons, $|\Phi(\xi,\xi,\cdots,\xi)|^2$, due to the constructive interference of the permuted probability amplitudes:

$$|\Psi(\xi,\xi,\cdots,\xi)|^2 = N!|\Phi(\xi,\xi,\cdots,\xi)|^2. \tag{1.4}$$

The constructive interference of the probability amplitudes is effective only when the wave packets of bosons overlap each other. At temperature T, each wave packet has a spatial extent of $\Delta x \sim \hbar/\sqrt{Mk_{\rm B}T}$, where M is the mass of the boson and $k_{\rm B}$ is the Boltzmann constant. By setting Δx equal to the average interparticle distance $n^{-\frac{1}{3}}$, where n is the particle number density, we can estimate the transition temperature T_0 of Bose–Einstein condensation (BEC) to be

$$k_{\rm B}T_0 \sim \frac{\hbar^2}{M} n^{\frac{2}{3}}. \tag{1.5}$$

Because of the large enhancement factor of $N!$ in Eq. (1.4), a large number of particles suddenly begin to condense into a single-particle state below T_0. When N is macroscopic, the onset of this condensation becomes prominent, endowing BEC with a conspicuous trait of quantum phase transition. Substituting $n = N/V$, where V is the volume of the system, in Eq. (1.5) gives

$$k_{\rm B}T_0 \sim \frac{\hbar^2}{MV^{\frac{2}{3}}} N^{\frac{2}{3}}. \tag{1.6}$$

Here, $\hbar^2/(MV^{\frac{2}{3}})$ gives an estimate of the energy gap between the ground state and the first excited state. Classical particles would condense into the ground state below the corresponding temperature $T_{\rm cl} \sim \hbar^2(k_{\rm B}MV^{\frac{2}{3}})$. Equation (1.6) shows that BEC occurs at a considerably higher temperature; further, the large enhancement factor $N^{\frac{2}{3}}$ can be attributed to the interference effect as discussed above. A more quantitative treatment described in Sec. 1.2 will validate Eq. (1.5).

1.2 Ideal Bose Gas in a Uniform System

The grand partition function Ξ of a system of particles with the Hamiltonian $\hat{H}$ and particle-number operator $\hat{N}$ is given by

$$\Xi = \mathrm{Tr} e^{-\beta(\hat{H}-\mu\hat{N})}, \tag{1.7}$$

where $\beta \equiv (k_B T)^{-1}$, Tr denotes a trace operation, and the chemical potential μ serves as a Lagrange multiplier that is to be determined so as to fix the average number of particles to a prescribed value.

For ideal (*i.e.*, noninteracting) identical bosons with the dispersion relation $\epsilon_{\mathbf{k}} = \hbar^2\mathbf{k}^2/2M$, $\hat{H} - \mu\hat{N}$ is given by

$$\hat{H} - \mu\hat{N} = \sum_{\mathbf{k}} (\epsilon_{\mathbf{k}} - \mu)\hat{n}_{\mathbf{k}}, \tag{1.8}$$

where $\hat{n}_{\mathbf{k}}$ denotes the number operator of particles with wave vector $\mathbf{k}$. Substituting Eq. (1.8) in Eq. (1.7) gives

$$\Xi = \prod_{\mathbf{k}} \sum_{n_{\mathbf{k}}=0}^{\infty} (e^{\beta(\mu-\epsilon_{\mathbf{k}})})^{n_{\mathbf{k}}}. \tag{1.9}$$

For the geometric series in Eq. (1.9) to converge, $e^{\beta(\mu-\epsilon_{\mathbf{k}})}$ must be less than one. It follows from $\epsilon_k \geq 0$ that

$$\mu < 0. \tag{1.10}$$

Then, Eq. (1.9) gives

$$\Xi = \prod_{\mathbf{k}} \frac{1}{1 - e^{\beta(\mu-\epsilon_{\mathbf{k}})}}.$$

The thermodynamic potential Ω is defined in terms of Ξ as

$$\Omega \equiv -\frac{1}{\beta} \ln \Xi = \frac{1}{\beta} \sum_{\mathbf{k}} \ln(1 - e^{\beta(\mu-\epsilon_{\mathbf{k}})}) = \sum_{\mathbf{k}} \Omega_{\mathbf{k}}, \tag{1.11}$$

where

$$\Omega_{\mathbf{k}} = \frac{1}{\beta} \ln(1 - e^{\beta(\mu-\epsilon_{\mathbf{k}})}). \tag{1.12}$$

The average number of particles with wave vector $\mathbf{k}$ is given by

$$\bar{n}_{\mathbf{k}} = -\frac{\partial\Omega_{\mathbf{k}}}{\partial\mu} = \frac{1}{e^{\beta(\epsilon_{\mathbf{k}}-\mu)} - 1}, \tag{1.13}$$

which is referred to as the Bose–Einstein distribution function. The average total number of bosons is expressed in terms of the chemical potential μ as

$$N = \sum_{\mathbf{k}} \frac{1}{e^{\beta(\epsilon_{\mathbf{k}}-\mu)} - 1}. \tag{1.14}$$

For a given N, μ is determined such that it satisfies Eq. (1.14).