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FIFTH EDITION

FIFTH FDITION

CALCUIS AND AND ANALYIC GROWRING

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PREFACE

The Fifth Edition has been adapted primarily from the Alternate Edition, but in addition to some new applications of calculus to engineering and the physical sciences, the present book contains a variety of interesting professional applications of calculus to business, economics, and the life sciences. The chapter on infinite series (Chapter 16) has been completely rewritten, and now contains twice the number of examples and exercises. Sequences are treated first, and separately from series, and more attention is paid to estimation. A chapter on vector analysis (Chapter 15), including line and surface integrals, Green's theorem, Stokes's theorem, and the Divergence Theorem, has been added. A section on Lagrange multipliers is included in Chapter 13 (Partial Differentiation). The presentation of vectors (Chapters 11 and 12) has been reorganized, and now the treatment of vector geometry precedes the treatment of vector functions and their derivatives. The book also incorporates some hand-held calculator exercises. More art has been included, and the art has been captioned throughout.

At the request of many users, we have moved several topics toward the front of the book. Trigonometric functions are introduced briefly in Chapter 1, and the review of trigonometry and the presentation of differentiation of sines and cosines that used to be in Chapter 4 now appear in Chapter 2. L'Hôpital's rule is presented in Chapter 3, and Newton's method has been moved to Chapter 2. Simpson's rule is now included with the trapezoidal rule in Chapter 4.

The level of rigor is about the same as in earlier editions of the Thomas books. For example, we do not prove that a function that is continuous on a closed and bounded interval has a maximum on the interval, but we state that theorem and use it in proving the Mean Value Theorem.

The first three chapters deal with the definition of, formulas for, and applications of derivatives of functions of one variable. Chapters 4 and 5 are on integration, with applications. Among the new applications are: estimating cardiac output, calculating light output from flashbulbs, determining the average daily inventory of a business, and using Delesse's rule to analyze tissue composition.

Chapter 6 deals with the derivatives of the remaining trigonometric functions, and with the logarithmic and exponential functions. If one wished to include some of the material on hyperbolic functions immediately after Chapter 6, then the material on methods of integration in Chapter 7 could be expanded to include the integration formulas XXVII' to XXXII' from Article 9-5. Otherwise, it is possible to skip Chapter 9 if one also omits subsequent problems that involve hyperbolic functions.

Chapter 8 (Plane Analytic Geometry) and Chapter 10 (Polar Coordinates) treat properties of the conic sections, cardioids, and other standard topics, including areas and arc length.

Chapters 11 and 12 introduce coordinates in space, vector algebra, parametric equations, and motion on a space curve. Chapter 12 concludes with a derivation of Kepler's second law of planetary motion in a central force field.

The remaining chapters include partial differentiation, multiple integration, vector analysis, infinite series, complex numbers and functions, differential equations, an appendix on matrices and determinants (useful in its own right and for the formal expansion of a third-order determinant used in Article 11–7), assorted formulas from elementary mathematics, and brief tables of sines, cosines, tangents, exponential functions, and natural logarithms. The endpapers of the book contain a brief table of integrals for convenient reference.

Answers for most of the problems are given and are keyed to the text by page, as well as by problem number, article, and chapter. The answers that are new to this edition were provided by Paul H. Siegel and Daniel W. Litwhiler. We are grateful to them for this valuable help, and we continue to be grateful to our friends and colleagues who contributed solutions and problems to earlier editions.

Many students, colleagues, and friends have given us the benefit of their criticism and suggestions. We would especially like to mention the valuable contributions of Carl W. R. de Boor, Fred A. Franklin, William A. Ferguson, Solomon Garfunkel, Andrew D. Jorgensen, William Ted Martin, Arthur P. Mattuck, Eric Reissner, J. Barkley Rosser, Oliver G. Selfridge, Donald R. Sherbert, Norton Starr, William U. Walton, and Felicia de May Weitzel. But even with the addition of the reviewers mentioned separately, this list is far from complete. To each and every person who has at any time contributed helpful suggestions, comments, or criticisms, whether or not we have been able to incorporate these into the book, we say "Thank you very much."

It is a pleasure to acknowledge the superb assistance in illustration, editing, design, and composition that the staff of Addison-Wesley Publishing Company has given to the preparation of this edition. The senior author also acknowledges with gratitude the special talent and productivity beyond his expectation that his co-author has brought to this new edition.

The text is available in one complete volume, which can be covered in three or four semesters, depending on the pace, or as two separate parts. The first part treats functions of one variable, analytic geometry in two dimensions, and infinite series (Chapters 1 through 10, and Chapter 16). It also contains the appendix on matrices and determinants. The second part begins with Chapter 11 on vectors and parametric equations, and contains all subsequent chapters, including Chapter 16, and the appendix on matrices and determinants. Both parts include answers.

Any errors that may appear are the responsibility of the authors. We will appreciate having these brought to our attention.

G. B. T., Jr.

October 1978 R. L. F.

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AVAILABLE SUPPLEMENTS

The following supplementary materials are available for use by students:

Pocket Calculator Supplement J. Barkley Rosser and Carl W. R. for Calculus

de Boor (University of Wisconsin

at Madison)

Self-Study Manual Maurice D. Weir (Naval

Postgraduate School)

Student Supplement Gurcharan S. Gill (Brigham Young

University)

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THE RATE OF CHANGE OF A FUNCTION



Calculus is the mathematics of change and motion. Where there is motion or growth, where forces are at work producing acceleration, calculus is the right mathematical tool. This was true in the beginnings of the subject, and it is true today. Calculus is used to predict the orbits of earth satellites; to design inertial navigation systems, cyclotrons, and radar systems; to explore problems of space travel; to test scientific theories about ocean currents and the dynamics of the atmosphere; and to model economic, social, and psychological behavior. Calculus is used increasingly to model problems in the fields of business, biology, medicine, animal husbandry, and political science. Of course, a scientist needs a great deal more than mathematical competence, and needs more mathematics than calculus. But calculus is a tool of great importance and usefulness and is a prerequisite for further study in nearly all branches of higher mathematics.

One of the great mathematicians of the twentieth century, John von Neumann (1903–57), wrote: "The calculus was the first achievement of modern mathematics, and it is difficult to overestimate its importance. I think it defines more unequivocally than anything else the inception of modern mathematics; and the system of mathematical analysis, which is its logical development, still constitutes the greatest technical advance in exact thinking."*

Calculus provides methods for solving two large classes of problems. The first of these involves finding the rate at which a variable quantity is changing. When a body travels in a straight line, its distance from its starting point changes with time and we may ask *how fast* it is moving at any given instant. *Differential calculus* is the branch of calculus that treats such problems.

On the other hand, if we are given the *velocity* of a moving body at every instant of time, we may seek to find the distance it has moved as a function of time. This second type of problem, that of finding a function when its rate of change is known, belongs to the domain of *integral calculus*.

Modern science and engineering use both branches of calculus to express physical laws in precise mathematical terms, and to study the consequences of those laws. It was with calculus that Sir Isaac Newton (1642–1727) was able to explain the motion of the planets about the sun as a consequence of the physical assumption known today as the law of gravitational attraction. Kepler (1571–1630) spent some twenty years studying observational data and using empirical methods to discover the three laws now known as Kepler's laws:

- a) Each planet traces an orbit about the sun which is an ellipse with the sun at one focus.
- b) The line joining the planet and the sun sweeps over equal areas in equal intervals of time.
- c) The squares of the periods of revolution of the planets about the sun are proportional to the cubes of their mean distances from the sun.

^{*} World of Mathematics, Vol. 4, "The Mathematician," by John von Neumann, pp. 2053-2063.

With the calculus as the main mathematical tool, all three of these laws can be derived from Newton's laws of gravitation and motion.

COORDINATES

German mathematician and philosopher Gottfried Wilhelm Leibniz (1646–1716) independently developed a large part of the calculus. His notation has been widely adopted in preference to Newton's.

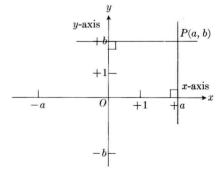
Analytic geometry, which forms a third division of the subject matter of this book, was the creation of several mathematicians.* Two French mathematicians, René Descartes (1596–1650) and Pierre de Fermat (1601–1665), are the chief inventors of analytic geometry as we now know it. The idea of locating a point in the plane by means of its directed distances from two perpendicular axes is Descartes', and his name is commemorated in the terminology "Cartesian coordinates." We discuss coordinates in Article 1–2. The distinguishing characteristic of analytic geometry is that it uses algebraic methods and equations to gain information about geometric problems. Conversely, it lets us portray algebraic equations as geometric curves, and thus bring the tools of geometry to bear on algebraic problems. Most of the theory of calculus can be presented in geometrical terms, and calculus and analytic geometry may be profitably united and studied as a whole.

The connection between algebra and geometry in analytic geometry is made by setting up a one-to-one correspondence between the points of a plane and ordered pairs of numbers (x, y). There are many ways to establish such a correspondence. The one most commonly used is the one described here.

A line in the plane, extending indefinitely to the left and to the right, is chosen, defined to be horizontal, and called the x-axis or axis of abscissas. (See Fig. 1–1.) A point of origin O on this line and a unit of length are then chosen. The axis is scaled off in terms of this unit. The number zero is attached to O, the number +a is attached to the point which is a units to the right of O, and the number -a is attached to the point located symmetrically to the left of O. This establishes a one-to-one correspondence between the points of the x-axis and the set of all real numbers (numbers which may be represented by terminating or nonterminating decimals).

Now through O take a second, vertical line in the plane, extending indefinitely up and down. This line is to be the y-axis, or axis of ordinates. The unit of length used to represent +1 on the y-axis need not be the one used to represent +1 on the x-axis. The y-axis is scaled off in terms of the unit of length adopted for it, with the positive number +b attached to the point b units above O and the negative number -b attached to the point located symmetrically b units below O.

We are now ready to assign number pairs to points. If a line perpendicular to the x-axis is drawn through the point marked a, and another line is



1-1 The line perpendicular to the x-axis at a and the line perpendicular to the y-axis at b cross at P(a, b).

COORDINATES

^{*} See World of Mathematics, Vol. 1, "Commentary on Descartes and Analytical Geometry," pp. 235-37. Also the article "The Invention of Analytic Geometry," by Carl B. Boyer, Scientific American, January, 1949.

drawn perpendicular to the y-axis at b, their point of intersection is labeled P(a, b). The number a is the x-coordinate of P, and the number b is the y-coordinate of P. The pair (a, b) is called the coordinate pair of the point P. Note that (a, b) is an ordered pair; we list the x-coordinate first and the y-coordinate second.

To be sure that none of the points in the plane has been missed, we can start with any point P and draw lines through it perpendicular to the two axes. If these perpendiculars cross the x- and y-axes at a and b, then P has already been assigned the coordinate pair (a, b).

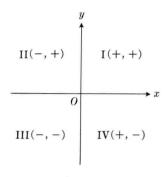
Points can never be assigned more than one coordinate pair; when we drop perpendiculars from P(a, b) to the axes the perpendiculars must meet the axes at a and b. Only one perpendicular can be drawn from a point to a line.

The two axes divide the plane into four quadrants, called the first quadrant, second quadrant, and so on, as in Fig. 1-2.

There are times when there is no physical relation between the units used to measure x and y. For example, if y is the maximum number of minutes that a diver can stay at a depth of x meters without having to stop to decompress on the way up, then the "1" on the x-axis stands for one meter, while the "1" on the y-axis stands for one minute. Clearly, there is no need to mark the two 1's the same number of millimeters, or whatever, from the origin.

In surveying, on the other hand, one foot measured north-and-south should be the same as one foot measured east-and-west. For this reason, it is usually assumed in trigonometry that the units of length on the two axes are the same. We make this assumption in analytic geometry also.

In this book, if coordinates of points are given without any physical units attached, it is to be assumed that the scales on the two axes are the same. In particular, this assumption is made wherever our work involves angles between lines or lengths of line segments that are not parallel to the axes.



1-2 The four quadrants.

PROBLEMS

In each of the following problems (1-12), first draw a pair of coordinate axes. Then plot the given point P(a, b) and plot:

- a) The point Q such that PQ is perpendicular to the x-axis and is bisected by it. Give the coordinates of Q. (P and Q are symmetric with respect to the x-axis.)
- **b)** The point R such that PR is perpendicular to and is bisected by the y-axis. Give the coordinates of R. (P and R are symmetric with respect to the y-axis.)
- c) The point S such that PS is bisected by the origin. Give the coordinates of S. (P and S are symmetric with respect to the origin.)
- d) The point T such that PT is perpendicular to and is bisected by the 45° line L through the origin bisecting the first quadrant and the third. Give the coordinates of T,

assuming equal units on the axes. (P and T are symmetric with respect to L.)

1.
$$(1, -2)$$
2. $(2, -1)$ 3. $(-2, 2)$ 4. $(-2, 1)$ 5. $(0, 1)$ 6. $(1, 0)$ 7. $(-2, 0)$ 8. $(0, -3)$ 9. $(-1, -3)$ 10. $(\sqrt{2}, -\sqrt{2})$ 11. $(-\pi, -\pi)$ 12. $(-1.5, 2.3)$

13. If P = P(x, y), then the coordinates of the point Q described in (a) above can be expressed in terms of x and y as (x, -y). Express the coordinates of R, S, and T in terms of x and y.

In Problems 14-17, take the units of length on the two axes to be equal.

14. A line is drawn through the point (0, 0) and the point (1, 1). What acute angle does it make with the positive x-axis? Sketch.

15. There are three parallelograms with vertices at (-1, 1), (2, 0), and (2, 3). Sketch them and give the coordinate pairs of the missing vertices.

16. A circle in quadrant II is tangent to both axes. It touches the y-axis at (0, 3).

a) At what point does it meet the x-axis? Sketch.

b) What are the coordinates of the center of the circle?

17. The line through the points (1, 1) and (2, 0) cuts the y-axis at the point (0, b). Find b by using similar triangles.

If a particle starts at a point $P_1(x_1, y_1)$ and goes to a new position $P_2(x_2, y_2)$, we say that its coordinates have changed by increments Δx (read *delta* x) and Δy (read *delta* y). For example, if the particle moves from A(1, -2) to B(6, 7), as in Fig. 1-3, then these increments are

$$\Delta x = 5, \qquad \Delta y = 9.$$

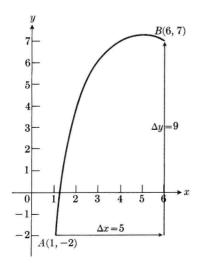
Observe that the increment in a coordinate is the net change, given by

$$\Delta x = (x \text{ of terminal point}) - (x \text{ of initial point})$$

and

$$\Delta y = (y \text{ of terminal point}) - (y \text{ of initial point}).$$

The positions the particle occupies between its initial and terminal locations do not affect these net changes.



1-3 When a particle moves from one point to another, Δx and Δy are computed from the coordinates of the initial and terminal positions:

$$\Delta x = 6 - 1 = 5, \qquad \Delta y = 7 - (-2) = 9.$$