Analysis and Control of Dynamic Economic Systems

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John Wiley & Sons
New York · London · Sydney · Toronto

Library of Congress Cataloging in Publication Data

Chow, Gregory C 1929–

Analysis and control of dynamic economic systems.

(Wiley series in probability and mathematical statistics)

"A Wiley-Interscience publication."

Includes index.

1. Econometrics. 2. Economic policy—Mathematical models. I. Title.

HB141.C5 1975 338.9 74-22433

ISBN 0-471-15616-7

Printed in the United States of America

10 9 8 7 6 5

Preface

This book presents a set of related techniques for analyzing the properties of dynamic stochastic models in economics and for applying these models in the determination of quantitative economic policy. It also illustrates them with a number of examples and applications contained, in particular, in Chapters 5 and 9. Its contents fall naturally into two parts: Part I on the analysis of dynamic econometric systems and Part II on techniques of control in the use of such systems.

I have written this book primarily for graduate students and possibly for advanced undergraduates whose backgrounds include a course in econometric methods at the level of J. Johnston's *Econometric Methods* or Part II of R. J. Wonnacott and T. H. Wonnacott's *Econometrics*. Thus a broad enough knowledge of matrix algebra to master the treatment of econometrics at the level of these texts is required. The book can also serve as a reference work. To fill its multiple purpose more material has been included than can possibly be completed in a one-semester course by advanced undergraduate students. Each section that can be skipped without loss of continuity is starred, for only the nonstarred sections in Chapters 1 to 4 and 7 need be read to obtain a basic understanding of the two related topics of analysis and control. (Sections 8.1 and 8.2 can replace Sections 7.3, 7.4, and 7.6.)

By referring to the Contents the reader can observe the logical organization of the book. The first part contains six chapters. After an introduction (Chapter 1), it discusses methods of analyzing deterministic systems (Chapter 2) and stochastic systems (Chapters 3 and 4). Applications are treated in Chapter 5 and Chapter 6 deals with the analysis of nonlinear and nonstationary models. The second part on optimal control methods begins with two chapters, 7 and 8, on two basic techniques for obtaining optimal policies from *linear* models with *known* parameters. Chapter 9 discusses applications to economic policy problems. Chapters 10 and 11 are concerned with generalizations that apply to linear models with unknown parameters, and Chapter 12 presents methods of controlling nonlinear systems. In addition to the core chapters, 1 to 4 and 7 (or alternatively Sections 8.1 and 8.2), Chapters 5 and 9 on applications are highly recommended to all readers. The remaining chapters, 8 on dynamic programming, 6 and 12 on nonlinear systems, and 10 and 11 on control techniques for unknown linear systems, are optional. Chapters 10 and 11 are not required for the study of Chapter 12 but Chapter 10 is a prerequisite of Chapter 11. Chapters 6 and 12 are mathematically less advanced than Chapters 8 (excepting Sections 8.1 and 8.2), 10 and 11.

It is possible to treat the variety of topics covered in this book only by limiting the discussion to systems in discrete time. The mathematics for dealing with dynamic systems in continuous time is varied and more difficult. Methods for discrete time models are not only more useful because econometric models in the main take this form but are basic to a grasp of the elements of analysis and control without making unreasonable demands on the mathematical training of the reader.

The reader should understand that this book is concerned mainly with methods. He should not expect to master the art and science of applying them to practical economic problems by simply reading this book and not actually carrying out at least one substantial application, in the same way that studying an econometrics text is not a sufficient qualification for a practicing econometrician. Familiarity with the techniques discussed in this book, however, is a necessary step toward their application. It will also enable the reader with no intention of being a practitioner to appreciate the works of others that rely on these techniques and are becoming more and more important in the literature of economics.

I should like to express my sincere thanks to Andrew B. Abel, Ray C. Fair and Richard E. Quandt for having read the entire manuscript and for giving me their valuable comments. Abel has also prepared the index. Stephen M. Goldfeld, A. C. Harberger, and Edwin S. Mills read parts of the manuscript and contributed ideas to its development. Students who have used the drafts in various forms in class and raised questions also deserve my deep appreciation. Betty Kaminski typed most of the manuscript efficiently and in good spirit, her task having been shared by Diana Hauver and Grace Lilley.

Much of the research reported in this book was supported by grants from the National Science Foundation to the Econometric Research Program at Princeton University for which I am grateful. Part of my work is based on research completed and published while I was associated with the IBM Research Center at which I was provided with an extremely

congenial environment. I am also indebted to the *American Economic Review*, *Annals of Economic and Social Measurement*, *International Economic Review*, *Journal of the American Statistical Association*, *Quarterly Journal of Economics*, and *Review of Economics and Statistics* for permission to use materials published in their respective journals.

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Princeton, New Jersey
September 1974

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Part 1

Analysis of Dynamic Economic Systems

CHAPTER 1

Problems of Stochastic Dynamic Economics

1.1 THE SUBJECT MATTER OF THIS BOOK IN ECONOMICS

Since the late 1940s much progress has been made in devising methods of estimating systems of interdependent relationships in macroeconomics and in applying them to construct econometric models of national economies. What are the main uses of these econometric models? To provide economic forecasts, to explain the dynamic behavior of the economy in question, and to help make better quantitative economic policies. This book is concerned with the second and third of these uses.

Besides being a natural development of econometrics, our subject matter is also an extension of macroeconomic theory and policy. Macroeconomic theory has become more quantitative in recent years, and sound macroeconomic policy will be based on the knowledge of quantitative economic relationships and on methods of utilizing that knowledge to achieve desired objectives. Furthermore, our subject is a natural development of economic analysis in general, from the simpler static, deterministic form to the more complicated dynamic, stochastic form that can be used to understand and control a changing economy. The evolution from static to dynamic and from deterministic to stochastic economics needs further elaboration.

A large portion of economic theory, especially microeconomic but to a lesser extent macroeconomic, is static. A static theory determines the values of certain endogenous variables, given the values of the exogenous variables and the parameters without specifying how the variables evolve through time. A theory is called a model if it is expressed in mathematical form; for example, a simple static theory or model of the macroeconomy consists of two equations:

$$Y = C + A,$$

$$C = \alpha_0 + \alpha_1 Y,$$

where Y is aggregate expenditures or Gross National Product, C is consumption expenditures, and A is autonomous expenditures; C and Y are endogenous variables and A is an exogenous variable; α_0 and α_1 in the consumption function are parameters whose values are taken as given. The model determines C and Y for any specified value of A.

Three related assumptions are implicit in the use of a static model like the one just mentioned, provided that unique equilibrium values of the endogenous variables can be determined by the equations in the model. First, if the exogenous variables are constant, the associated endogenous variables will take constant equilibrium values. Second, following from the first, if the exogenous variables were to take a new set of values, the associated endogenous variables would also take a new set of constant equilibrium values. The method of comparative statics is precisely to study the changes in the equilibrium values of the endogenous variables with respect to changes in the values of the exogenous variables. Third, a static model can be used for either a "short-run" or "long-run" analysis in the following sense. For a short-run analysis many other variables in the economy such as capital stock and population are assumed to remain constant. The effect of government expenditures (a part of autonomous expenditures) on GNP may then be studied by following the first two assumptions. For a long-run analysis some of the variables held constant in the short-run analysis are allowed to vary, and the model determines a larger number of endogenous variables, still obeying the first two assumptions. Short-run and long-run effects are assumed to differ.

Although static models may serve as a useful first approximation to explain or predict the effects of certain exogenous variables on the endogenous variables, holding constant the less relevant variables (depending on whether the short run or the long run is being considered) and ignoring timing relationships, their shortcomings in the study of economic fluctuations and growth should be obvious. Under the first assumption a static model cannot explain the phenomena of cyclical fluctuations in economic variables or their growth or decline. Furthermore, thinking in static terms, an economist might fall into the trap of believing that the economy is intrinsically stable in the sense of producing nearly constant or very smooth time paths for GNP, employment, the price level, and so on, only if the exogenous variables such as money supply are kept constant. This may be so, but a dynamic model (to be defined more precisely later in this chapter) is needed to ascertain its empirical validity. It is dangerous to identify the properties, especially the dynamic properties, of a static model used in abstract theorizing with the properties of the real economy.

The second assumption for static models is equally dangerous when applied to dynamic phenomena. We might be led, or misled, to believe that if government expenditures or money supply were to change from one level to another the response of GNP or the price level would be a smooth journey toward a new equilibrium level. Thinking in terms of static models might also lead to the conclusion that if money supply is controlled to grow at a constant percentage rate the resulting economic aggregates will also follow smooth growth paths. Anyone with experience in studying the dynamic properties of econometric models realizes that these presuppositions may not be valid. The resulting time path of GNP from a one-step increase in government expenditures is not likely to be smooth. Neither will it necessarily converge to a constant value as time goes on. From a dynamic econometric model one may also search in vain for the long-run multiplier of government expenditures on GNP (the rate of change of GNP with respect to government expenditures). The existence of a multiplier presumes that a new static equilibrium will be reached, a presumption that may not be valid for a dynamic economic model or for the real economy. Neither is the presumption that the paths of GNP and the price level will be smooth if money supply grows at a constant rate.

Now we come to the third assumption in using static models, that short-run and long-run effects may differ and that one can exist without the other. In a comparative static analysis the model may imply that changes in certain exogenous variables do not affect a certain subset of the endogenous variables in either the long or the short run; for example, according to one interpretation of the quantity theory of money, a change in money supply affects only the price level but not real output in the long run, although it may affect real output in the short run. If such a proposition is believed to hold for the real economic data, or for a dynamic econometric model that explains these data, certain difficulties will appear. Can one necessarily expect from the actual economy, or from the econometric model constructed, that if only money supply is increased from one quantity to another within a short period and is kept constant in the remaining periods, all other exogenous variables being kept constant, then real GNP will eventually return exactly to its initial level? If money supply affects economic life in the short run, that is, if real output will respond to money supply, how can the short-run effects be exactly cancelled in the long run? Increase in real output is likely to be associated with increase in capital goods. How can one assume that these capital goods will eventually be depreciated or disappear in some way so that the initial values of real GNP and all its components will prevail?

The shortcomings of static models and of the associated assumptions should be clear. So is the need for dynamic models that can explain the time paths of the endogenous variables. These time paths may show trends and fluctuations. There are no presumptions that they are smooth or will reach equilibrium values as time increases, that their responses to a one-step change in an exogenous variable are smooth and converging, and that their responses to smooth exogenous changes are also smooth. In this book we shall study first the dynamic properties of econometric models to find out whether they resemble the corresponding dynamic properties of the real economy. Constructing theories or models that can explain the phenomena observed is the main purpose of science. Economics is no exception. Second, we shall present quantitative methods for using econometric models for the purpose of improving the dynamic characteristics of the economy. Economic science is intimately related to economic policy. Policy analysis often follows economic analysis in both micro- and macroeconomics. We are mainly concerned with tools of macroeconomic policy, which can be applied to achieve smoother time paths and more desirable trends for the economic variables such as prices, employment, output, and balance of payments. Macroeconometric models are complex. Systematic methods are required for studying their dynamic properties and for improving their properties according to the stated goals of full employment, price stability, equilibrium in the balance of payments, and sufficient growth.

For introductory purposes let me be brief on the importance of introducing stochastic elements in dynamic economic models for the purposes of analysis and control. Anyone who has studied econometrics knows why stochastic models are needed. Economic life is more complicated than can be described completely by any deterministic economic model. Therefore stochastic residuals or disturbances are used partly to summarize the combined effects of the omitted variables in the equations. Furthermore, by using these random distrubances, a much richer theory of economic fluctuations will be obtained, as we demonstrate in Chapters 3, 4, and 5. Third, the use of a stochastic model enables one to bring the techniques of statistics to bear on problems of estimating the parameters in the model and of testing hypotheses concerning the model; or should one say that the techniques of statistical inference have been invented because stochastic models are needed in various branches of sciences? Fourth, to account for uncertainty, in the random disturbances and/or in the model parameters themselves, is important for the quantative study of economic policy simply because uncertainty actually exists.