Option Pricing and Estimation of Financial Models with R

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Stefano M. Iacus

Department of Economics, Business and Statistics
University of Milan, Italy

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Preface

Why another book on option pricing and why the choice of R language? The R language is increasingly accepted by so-called 'quants' as the basic infrastructure for financial applications. A growing number of projects, papers and conferences are either R-centric or, at least, deal with R solutions. R itself may not initially be very friendly, but, on the other hand, are stochastic integrals, martingales, and the Lévy process that friendly? In addition to this argument, we should take into account the famous quote from the R community by Greg Snow which describes the correct approach to R but equally applies to the Itô integral and to mathematical finance in general:

> When talking about user friendliness of computer software I like the analogy of cars vs. busses: Busses are very easy to use, you just need to know which bus to get on, where to get on, and where to get off (and you need to pay your fare). Cars on the other hand require much more work, you need to have some type of map or directions (even if the map is in your head), you need to put gas in every now and then, you need to know the rules of the road (have some type of driver's licence). The big advantage of the car is that it can take you a bunch of places that the bus does not go and it is quicker for some trips that would require transferring between busses. R is a 4-wheel drive SUV (though environmentally friendly) with a bike on the back, a kayak on top, good walking and running shoes in the passenger seat, and mountain climbing and spelunking gear in the back. R can take you anywhere you want to go if you take time to learn how to use the equipment, but that is going to take longer than learning where the bus stops are in a point-and-click GUI.

This book aims to present an indication of what is going on in modern finance and how this can be quickly implemented in a general computational framework like R rather than providing extra optimized lower-level programming languages' ad hoc solutions. For example, this book tries to explain how to simulate and calibrate models describing financial assets by general methods and generic functions rather than offering a series of highly specialized functions. Of course, the code in the book tries to be efficient while being generalized and some hints are given in the direction of further optimization when needed.

The choice of the R language is motivated both by the fact that the author is one of the developers of the R Project but also because R, being open source, is transparent in that it is always possible to see how numerical results are obtained without being deterred by a 'black-box' of a commercial product. And the R community, which is made by users and developers who in many cases correspond to researchers in the field, do a continuous referee process on the code. This has been one of the reasons why R has gained so much popularity in the last few years, but this is not without cost (the 'no free lunch' aspect of R), in particular because most R software is given under the disclaimer 'use at your own risk' and because, in general, there is no commercial support for R software, although one can easily experience peer-to-peer support from public mailing lists. This situation is also changing nowadays because an increasing number of companies are selling services and support for R-related software, in particular in finance and genetics.

When passing from the theory of mathematical finance to applied finance, many details should be taken into account such as handling the dates and times, the source of the time series in use, the time spent in running a simulation etc. This books tries to keep this level rather than a very abstract formulation of the problems and solutions, while still trying to present the mathematical models in a proper form to stimulate further reading and study.

The mathematics in this book is necessarily kept to a minimum for reasons of space and to keep the focus on the description and implementation of a wider class of models and estimation techniques. Indeed, while it is true that most mathematical papers contain a section on numerical results and empirical analysis, very few textbooks discuss these topics for models outside the standard Black and Scholes world.

The first chapters of the book provide a more in-depth treatment with exercises and examples from basic probability theory and statistics, because they rely on the basic instruments of calculus an average student (e.g. in economics) should know. They also contain several results without proof (such as inequalities), which will be used to sketch the proofs of the more advanced parts of the book. The second part of the book only touches the surface of mathematical abstraction and provides sketches of the proofs when the mathematical details are too technical, but still tries to give the correct indication of why the level of mathematical abstraction is needed. So the first part can be used by students in finance as a summary and the second part as the main section of the book. It is assumed that readers are familiar with R, but a summary of what they need to know to understand this book is contained in the two appendices as well as some general description of what is available and up-to-date in R in the context of finance.

So, back to Snow's quote: this book is more a car than a bus, but maybe with automatic gears and a solar-power engine, rather than a sport car with completely manual gears that requires continuous refueling and tuning.

A big and long-lasting smile is dedicated to my beloved Ilia, Ludovico and Lucia, for the time I spent away from them during the preparation of this manuscript. As V. Borges said once, 'a smile is the shortest distance between two persons'.

S.M. Iacus
November 2010

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APPENDICES

1

A synthetic view

Mathematical finance has been an exponentially growing field of research in the last decades and is still impressively active. There are also many directions and subfields under the hat of 'finance' and researchers from very different fields, such as economics (of course), engineering, mathematics, numerical analysis and recently statistics, have been involved in this area.

This chapter is intended to give a guidance on the reading of the book and to provide a better focus on the topics discussed herein. The book is intended to be self-contained in its exposition, introducing all the concepts, including very preliminary ones, which are required to better understand more complex topics and to appreciate the details and the beauty of some of the results.

This book is also very computer-oriented and it often moves from theory to applications and examples. The R statistical environment has been chosen as a basis. All the code presented in this book is free and available as an R statistical package called **opefimor** on CRAN.[1]

There are many good publications on mathematical finance on the market. Some of them consider only mathematical aspects of the matter at different level of complexity. Other books that mix theoretical results and software applications are usually based on copyright protected software. These publications touch upon the problem of model calibration only incidentally and in most cases the focus is on discrete time models mainly (ARCH, GARCH, etc.) with notable exceptions.

The main topics of this book are the description of models for asset dynamics and interest rates along with their statistical calibration. In particular, the attention is on continuous time models observed at discrete times and calibration techniques for them in terms of statistical estimation. Then pricing of derivative contracts on a single underlining asset in the Black and Scholes-Merton framework (Black and Scholes 1973; Merton 1973), pricing of basket options, volatility, covariation and regime switching analysis are considered. At the same

[1] CRAN, Comprehensive R Archive Network, http::/cran.r-project.org.

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time, the book considers jump diffusions and telegraph process models and pricing under these dynamics.

1.1 The world of derivatives

There are many kinds of financial markets characterized by the nature of the financial products exchanged rather than their geographical or physical location. Examples of these markets are:

- **stock markets**: this is the familiar notion of stock exchange markets, like New York, London, Tokyo, Milan, etc.;
- **bond markets**: for fixed return financial products, usually issued by central banks, etc.;
- **currency markets or foreign exchange markets**: where currencies are exchanged and their prices are determined;
- **commodity markets**: where prices of commodities like oil, gold, etc. are fixed;
- **futures and options markets**: for derivative products based on one or more other underlying products typical of the previous markets.

The book is divided into two parts (although some natural overlapping occurs). In the first part the modelling and analysis of dynamics of assets prices and interest rates are presented (Chapters 3, 4 and 5). In the second part, the theory and practice on derivatives pricing are presented (Chapters 6 and 7). Chapter 2 and part of Chapter 3 contain basic probabilistic and statistical infrastructure for the subsequent chapters. Chapter 4 introduces the numerical basic tools which, usually in finance, complement the analytical results presented in the other parts. Chapter 8 presents an introduction to recently introduced models which go beyond the standard model of Black and Scholes and the Chapter 9 presents accessory results for the analysis of financial time series which are useful in risk analysis and portfolio choices.

1.1.1 Different kinds of contracts

Derivatives are simply contracts applied to financial products. The most traded and also the object of our interest are the *options*. An option is a contract that gives the right to sell or buy a particular financial product at a given price on a predetermined date. They are clearly asymmetric contracts and what is really sold is the 'option' of exercise of a given right. Other asymmetric contracts are so-called *futures* or *forwards*. Forwards and futures are contracts which oblige one to sell or buy a financial product at a given price on a certain date to another party. Options and futures are similar in that, e.g., prices and dates are prescribed but clearly in one case what is traded is an opportunity of trade and in the other an obligation. We mainly focus on option pricing and we start with an example.

1.1.2 Vanilla options

Vanilla options[2] is a term that indicates the most common form of options. An option is a contract with several ingredients:

- the *holder*: who subscribes the financial contract;
- the *writer*: the seller of the contract;
- the *underlying asset*: the financial product, usually but not necessarily a stock asset, on which the contract is based;
- the *expiry date*: the date on which the right (to sell or buy) the underlying asset can be exercised by the holder;
- the *exercise or strike price*: the predetermined price for the underlying asset at the given date.

Hence, the holder buys a right and not an obligation (to sell or buy), conversely the writer is obliged to honor the contract (sell or buy at a given price) at the expiry date.

The right of this choice has an economical value which has to be paid in advance. At the same time, the writer has to be compensated from the obligation. Hence the problem of fixing a *fair* price for an option contract arises. So, option pricing should answer the following two questions:

- how much should one pay for his right of choice? i.e. how to fix the price of an option in order to be accepted by the holder?
- how to minimize the risk associated with the obligation of the writer? i.e. to which (economical) extent can the writer reasonably support the cost of the contract?

Example 1.1.1 (From Wilmott *et al.* (1995)) *Suppose that there exists an asset on the market which is sold at \$25 and assume we want to fix the price of an option on this asset with an expiry date of 8 months and exercise price of buying this asset at \$25. Assume there are only two possible scenarios: (i) in 8 months the price of the asset rises to \$27 or (ii) in 8 months the price of the asset falls to*

[2] From *Free On-Line Dictionary of Computing*, http://foldoc.doc.ic.ac.uk. Vanilla : *(Default flavour of ice cream in the US) Ordinary flavour, standard. When used of food, very often does not mean that the food is flavoured with vanilla extract! For example, 'vanilla wonton soup' means ordinary wonton soup, as opposed to hot-and-sour wonton soup. This word differs from the canonical in that the latter means 'default', whereas vanilla simply means 'ordinary'. For example, when hackers go to a Chinese restaurant, hot-and-sour wonton soup is the canonical wonton soup to get (because that is what most of them usually order) even though it isn't the vanilla wonton soup.*

\$23. In case (i) the potential holder of the option can exercise the right, pay \$25 to the writer to get the asset, sell it on the market at \$27 to get a return of \$2, i.e.

$$\$27 - \$25 = \$2.$$

In scenario (ii), the option will not be exercised, hence the expected return is \$0. If both scenarios are likely to happen with the same probability of $\frac{1}{2}$, *the expected return for the potential holder of this option will be*

$$\frac{1}{2} \times \$0 + \frac{1}{2} \times \$2 = \$1.$$

So, if we assume no transaction costs, no interest rates, etc., the fair value of this option should be \$1. If this is the fair price, a holder investing \$1 in this contract could gain $-\$1 + \$2 = \$1$, *which means 100% of the invested money in scenario (i) and in scenario (ii)* $-\$1 + \$0 = -\$1$, *i.e. 100% of total loss. Which means that derivatives are extremely risky financial contracts that, even in this simple example, may lead to 100% of gain or 100% of loss.*

Now, assume that the potential holder, instead of buying the option, just buys the asset. In case (i) the return from this investment would be $-\$25 + \$27 = \$2$ *which means* $+2/25 = 0.08$ *(+8%) and in scenario (ii)* $-\$25 + \$23 = -\$2$ *which equates to a loss of value of* $-2/25 = -0.08$ *(−8%).*

From the previous example we learn different things:

- the value of an option reacts quickly (instantaneously) to the variation of the underlying asset;
- to fix the fair price of an option we need to know the price of the underlying asset at the expiry date: either we have a crystal ball or a good predictive model. We try the second approach in Chapters 3 and 5;
- the higher the final price of the underlying asset the larger will be the profit; hence the price depends on the present and future values of the asset;
- the value of the option also depends on the strike price: the lower the strike price, the less the loss for the writer;
- clearly, the expiry date of the contract is another key ingredient: the closer the expiry date, the less the uncertainty on future values of the asset and vice versa;
- if the underlying asset has high volatility (variability) this is reflected by the risk (and price) of the contract, because it is less certainty about future values of the asset. The study of volatility and Greeks will be the subject of Chapters 5, 6 and 9.

It is also worth remarking that, in pricing an option (as any other risky contract) there is a need to compare the potential revenue of the investment against fixed