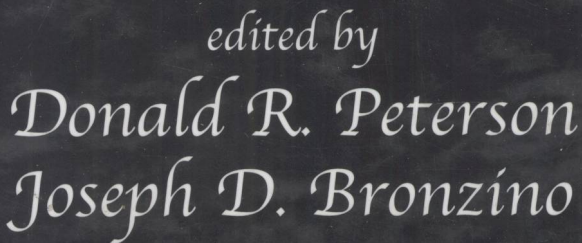


Principles and Applications



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BIOMECHANICS

Principles and Applications

edited by
Donald R. Peterson
Joseph D. Bronzino



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BIOMECHANICS

Principles and Applications

Preface

Engineering is the integration of art and science and involves the use of systematic knowledge based on the principles of mathematics and the physical sciences to design and develop systems that have direct practical applicability for the benefit of mankind and society. With this philosophy in mind, the importance of the engineering sciences becomes obvious, and this is especially true for the biomedical aspects, where the implications are easily identifiable. Of all the engineering sciences, biomedical engineering is considered to be the broadest. Its practice frequently involves the direct combination of the core engineering sciences, such as mechanical, electrical, and chemical engineering, and requires a functional knowledge of other nonengineering disciplines, such as biology and medicine, to achieve effective solutions. It is a multidisciplinary science with its own core aspects, such as biomechanics, bioinstrumentation, and biomaterials, which can be further characterized by a triage of subject matter. For example, the study of biomechanics, or biological mechanics, employs the principles of mechanics, which is a branch of the physical sciences that investigates the effects of energy and forces on matter or material systems. It often embraces a broad range of subject matter that may include aspects of classical mechanics, material science, fluid mechanics, heat transfer, and thermodynamics, in an attempt to model and predict the mechanical behaviors of any living system. As such, it may be called the “liberal arts” of the biomedical engineering sciences.

Biomechanics is deeply rooted throughout scientific history and has been influenced by the research work of early mathematicians, engineers, physicists, biologists, and physicians. Not one of these disciplines can claim sole responsibility for maturing biomechanics to its current state; rather, it has been a conglomeration and integration of these disciplines, involving the application of mathematics, physical principles, and engineering methodologies, that has been responsible for its advancement. Several examinations exist that offer a historical perspective on biomechanics in dedicated chapters within a variety of biomechanics textbooks. For this reason, a historical perspective is not presented within this introduction and it is left to the reader to discover the material within one of these textbooks. As an example, Y.C. Fung (1993) provides a reasonably detailed synopsis of those who were influential to the progress of biomechanical understanding. A review of this material and similar material from other authors commonly shows that biomechanics has occupied the thoughts of some of the most conscientious minds involved in a variety of the sciences.

Leonardo da Vinci, one of the early pioneers of biomechanics, was the first to introduce the principle of “cause and effect” in scientific terms as he firmly believed that “there is no result in nature without a cause; understand the cause and you will have no need of the experiment” (1478–1518). Leonardo understood that experimentation is an essential tool for developing an understanding of nature’s causes and the results they produce, especially when the cause is not immediately obvious. The contemporary approach to understand and solve problems in engineering expands upon Leonardo’s principle and typically follows a sequence of fundamental steps that are commonly defined as observation, experimentation, theorization, validation, and application. These steps are the basis of the engineering methodologies and their significance is emphasized within a formal engineering education, especially in biomedical engineering. Each step is considered to be equally important, and an iterative relationship between steps, with mathematics serving

as the common link, is often necessary in order to converge on a practical understanding of the system in question. An engineering education that ignores these interrelated fundamentals will produce engineers who are ignorant of the ways in which real-world phenomena differ from mathematical models. Since most biomechanical systems are inherently complex and cannot be adequately defined using only theory and mathematics, biomechanics should be considered a discipline whose progress relies heavily on research and experimentation and the careful implementation of the sequence of steps. When a precise solution is not obtainable, utilizing this approach will assist with identifying critical physical phenomena and obtaining approximate solutions that may provide a deeper understanding as well as improvements to the investigative strategy. Not surprisingly, the need to identify critical phenomena and obtain approximate solutions seems to be more significant in biomedical engineering than any other engineering discipline, which can be attributed to the complex biological processes involved.

Applications of biomechanics have traditionally focused on modeling the system-level aspects of the human body, such as the musculoskeletal system, the respiratory system, and the cardiovascular and cardiopulmonary systems. Technologically, most of the progress has been made on system-level device development and implementation, with obvious influences on athletic performance, work environment interaction, clinical rehabilitation, orthotics, prosthetics, and orthopaedic surgery. However, more recent biomechanics initiatives are now focusing on the mechanical behaviors of the biological subsystems, such as tissues, cells, and molecules, in order to relate subsystem functions across all levels by showing how mechanical function is closely associated with certain cellular and molecular processes. These initiatives have a direct impact on the development of biological nano- and microtechnologies involving polymer dynamics, biomembranes, and molecular motors. The integration of system and subsystem models will advance our overall understanding of human function and performance and further develop the principles of biomechanics. Even still, our modern understanding about certain biomechanic processes is limited, but through ongoing biomechanics research, new information that influences the way we think about biomechanics is generated and important applications that are essential to the betterment of human existence are discovered. As a result, our limitations are reduced and our understanding becomes more refined. Recent advances in biomechanics can also be attributed to advances in experimental methods and instrumentation, such as computational power and imaging capabilities, which are also subject to constant progress.

The rapid advance of biomechanics research continues to yield a large amount of literature that exists in the form of various research and technical papers and specialized reports and textbooks that are only accessible in various journal publications and university libraries. Without access to these resources, collecting the publications that best describe the current state of the art would be extremely difficult. With this in mind, this textbook offers a convenient collection of chapters that present current principles and applications of biomechanics from respected published scientists with diverse backgrounds in biomechanics research and application. A total of 20 chapters is presented, 12 of which have been substantially updated and revised to ensure the presentation of modern viewpoints and developments. The chapters within this text have been organized in an attempt to present the material in a systematic manner. The first group of chapters is related to musculoskeletal mechanics and includes hard and soft tissue mechanics, joint mechanics, and applications related to human function. The next group of chapters covers several aspects of biofluid mechanics and includes a wide range of circulatory dynamics, such as blood vessel and blood cell mechanics, and transport. It is followed by a chapter that introduces current methods and strategies for modeling cellular mechanics. The next group consists of two chapters introducing the mechanical functions and significance of the human ear. Finally, the remaining two chapters introduce performance characteristics of the human body system during exercise and exertion. It is the overall intention of this text to serve as a reference to the skilled professional as well as an introduction to the novice or student of biomechanics. An attempt was made to incorporate material that covers a bulk of the biomechanics field; however, as biomechanics continues to grow, some topics may be inadvertently omitted causing a

disproportionate presentation of the material. Suggestions and comments from readers are welcomed on subject matter that should be considered in future editions of this textbook.

Through the rationalization of biomechanics, I find myself appreciating the complexity and beauty of all living systems. I hope that this textbook helps your understanding of biomechanics and your discovery of life.

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The Editors

Donald R. Peterson, Ph.D., M.S., an assistant professor in the Schools of Medicine, Dental Medicine, and Engineering at the University of Connecticut, and director of the Biodynamics Laboratory and the Bioengineering Facility at the University of Connecticut Health Center, offers graduate-level courses in biomedical engineering in the fields of biomechanics, biodynamics, biofluid mechanics, and ergonomics, and teaches in medicine in the subjects of gross anatomy and occupational biomechanics. He earned a B.S. in both aerospace and biomedical engineering from Worcester Polytechnic Institute, a M.S. in mechanical engineering from the University of Connecticut, and a Ph.D. in biomedical engineering also from the University of Connecticut. Dr. Peterson's current research work is focused on the development of laboratory and field techniques for accurately assessing and modeling human-device interaction and human and/or organism performance, exposure, and response. Recent applications of these protocols model human interactions with existing and developmental devices such as powered and nonpowered tools, spacesuits and spacetools for NASA, surgical and dental instruments, musical instruments, sports equipment, and computer input devices. Other research initiatives focus on cell biomechanics, the acoustics of hearing protection and communication, hand-arm vibration exposure, advanced physiological monitoring methods, advanced vascular imaging techniques, and computational biomechanics.

Joseph D. Bronzino received the B.S.E.E. degree from Worcester Polytechnic Institute, Worcester, MA, in 1959, the M.S.E.E. degree from the Naval Postgraduate School, Monterey, CA, in 1961, and the Ph.D. degree in electrical engineering from Worcester Polytechnic Institute in 1968. He is presently the Vernon Roosa Professor of Applied Science, an endowed chair at Trinity College, Hartford, CT, and president of the Biomedical Engineering Alliance and Consortium (BEACON), which is a nonprofit organization consisting of academic and medical institutions as well as corporations dedicated to the development and commercialization of new medical technologies (for details visit www.beaconalliance.org).

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Mechanics of Hard Tissue

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Hard tissue, *mineralized tissue*, and *calcified tissue* are often used as synonyms for bone when describing the structure and properties of bone or tooth. The *hard* is self-evident in comparison with all other mammalian tissues, which often are referred to as *soft tissues*. Use of the terms mineralized and calcified arises from the fact that, in addition to the principle protein, collagen, and other proteins, glycoproteins, and protein-polysaccharides, comprising about 50% of the volume, the major constituent of bone is a calcium phosphate (thus the term *calcified*) in the form of a crystalline carbonate **apatite** (similar to naturally occurring minerals, thus the term *mineralized*). Irrespective of its biological function, bone is one of the most interesting materials known in terms of structure–property relationships. Bone is an anisotropic, heterogeneous, inhomogeneous, nonlinear, thermorheologically complex viscoelastic material. It exhibits electromechanical effects, presumed to be due to streaming potentials, both *in vivo* and *in vitro* when wet. In the dry state, bone exhibits piezoelectric properties. Because of the complexity of the structure–property relationships in bone, and the space limitation for this chapter, it is necessary to concentrate on one aspect of the mechanics. Currey [1984] states unequivocally that he thinks, “the most important feature of bone material is its stiffness.” This is, of course, the premiere consideration for the weight-bearing long bones. Thus, this chapter will concentrate on the elastic and viscoelastic properties of compact **cortical bone** and the elastic properties of trabecular bone as exemplar of mineralized tissue mechanics.

1.1 Structure of Bone

The complexity of bone's properties arises from the complexity in its structure. Thus it is important to have an understanding of the structure of mammalian bone in order to appreciate the related properties. Figure 1.1 is a diagram showing the structure of a human femur at different levels [Park, 1979]. For convenience, the structures shown in Figure 1.1 will be grouped into four levels. A further subdivision of structural organization of mammalian bone is shown in Figure 1.2 [Wainwright et al., 1982]. The individual figures within this diagram can be sorted into one of the appropriate levels of structure shown on Figure 1.1 as described in the following. At the smallest unit of structure we have the *tropocollagen* molecule and the associated apatite crystallites (abbreviated Ap). The former is approximately 1.5 by 280 nm, made up of three individual left-handed helical polypeptide (α) chains coiled into a right-handed triple helix. Ap crystallites have been found to be carbonate-substituted hydroxyapatite, generally thought to be nonstoichiometric. The crystallites appear to be about $4 \times 20 \times 60$ nm in size. This level is denoted the *molecular*. The next level we denote the *ultrastructural*. Here, the collagen and Ap are intimately associated and assembled into a microfibrillar composite, several of which are then assembled into fibers from approximately 3 to 5 μm thick. At the next level, the *microstructural*, these fibers are either randomly arranged (woven bone) or organized into concentric lamellar groups (**osteons**) or linear lamellar groups (**plexiform bone**). This is the level of structure we usually mean when we talk about bone *tissue* properties. In addition to the differences in lamellar organization at this level, there are also two different types of architectural structure. The dense type of bone found, for example, in the shafts of long bone is known as compact or *cortical bone*. A more porous or spongy type of bone is found, for example, at the articulating ends of long bones. This is called **cancellous bone**. It is important to note that the material and structural organization of collagen–Ap making up osteonic or *haversian* bone and plexiform bone are the same as the material comprising cancellous bone.

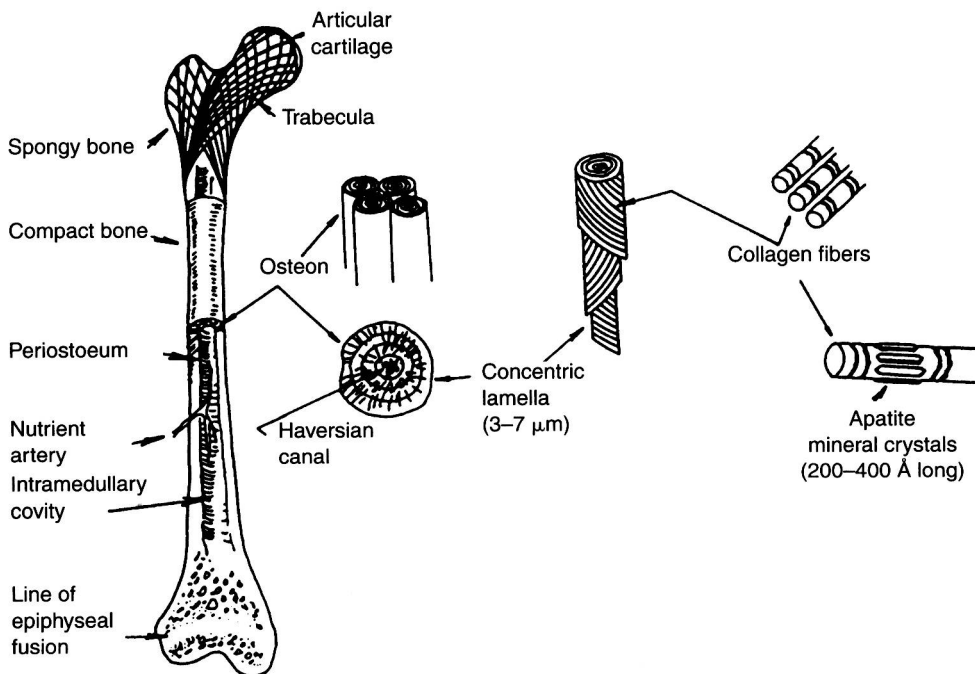


FIGURE 1.1 Hierarchical levels of structure in a human femur [Park, 1979]. (Courtesy of Plenum Press and Dr. J.B. Park.)

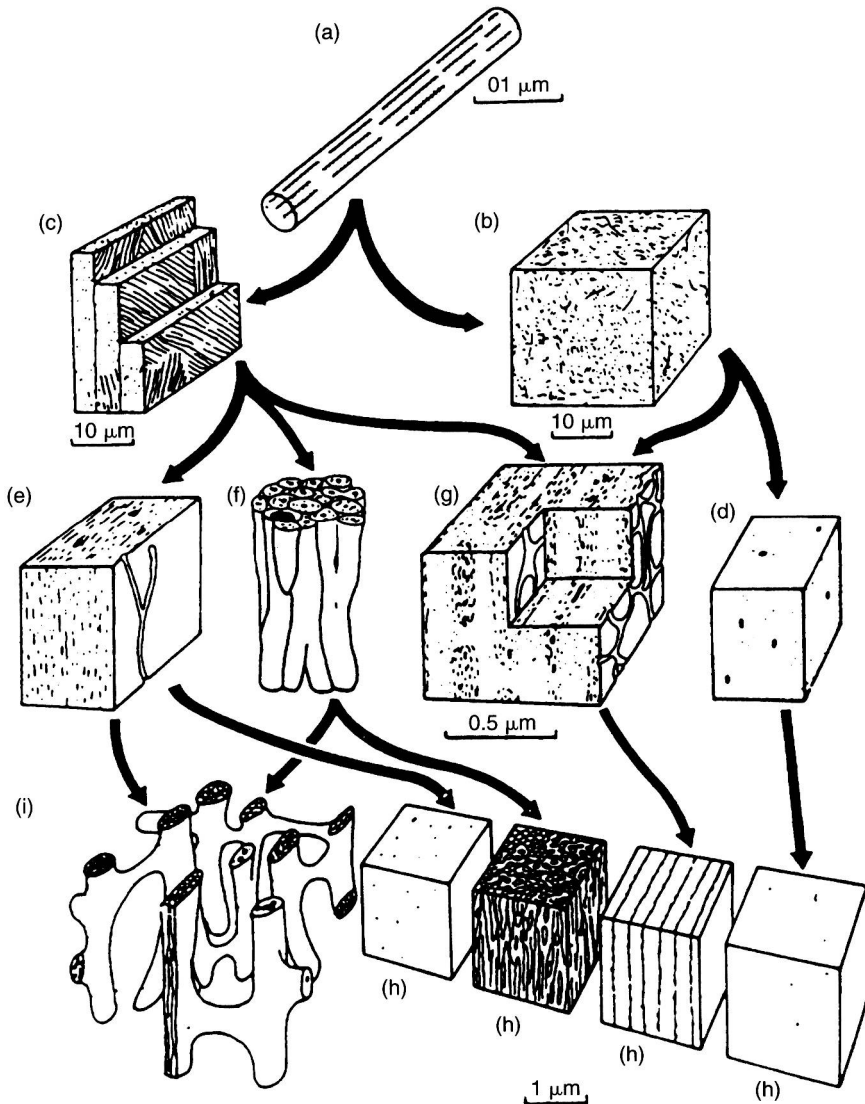


FIGURE 1.2 Diagram showing the structure of mammalian bone at different levels. Bone at the same level is drawn at the same magnification. The arrows show what types may contribute to structures at higher levels [Wainwright et al., 1982]. (Courtesy Princeton University Press.) (a) Collagen fibril with associated mineral crystals. (b) Woven bone. The collagen fibrils are arranged more or less randomly. Osteocytes are not shown. (c) Lamellar bone. There are separate lamellae, and the collagen fibrils are arranged in “domains” of preferred fibrillar orientation in each lamella. Osteocytes are not shown. (d) Woven bone. Blood channels are shown as large black spots. At this level woven bone is indicated by light dotting. (e) Primary lamellar bone. At this level lamellar bone is indicated by fine dashes. (f) **Haversian bone**. A collection of Haversian systems, each with concentric lamellae round a central blood channel. The large black area represents the cavity formed as a cylinder of bone is eroded away. It will be filled in with concentric lamellae and form a new Haversian system. (g) Laminar bone. Two blood channel networks are exposed. Note how layers of woven and lamellar bone alternate. (h) Compact bone of the types shown at the lower levels. (i) Cancellous bone.

Finally, we have the whole bone itself constructed of osteons and portions of older, partially destroyed osteons (called **interstitial lamellae**) in the case of humans or of osteons and/or plexiform bone in the case of mammals. This we denote the *macrostructural level*. The elastic properties of the whole bone results from the hierarchical contribution of each of these levels.

TABLE 1.1 Composition of Adult Human and Bovine Cortical Bone

Species	% H ₂ O	Ap	% Dry Weight		Reference
			Collagen	GAG ^a	
Bovine	9.1	76.4	21.5	N.D. ^b	Herring, 1977
Human	7.3	67.2	21.2	0.34	Pellagrino and Blitz, 1965; Vejlens, 1971

^a Glycosaminoglycan.

^b Not determined.

1.2 Composition of Bone

The composition of bone depends on a large number of factors: the species, which bone, the location from which the sample is taken, and the age, sex, and type of bone tissue, for example, woven, cancellous, cortical. However, a rough estimate for overall composition by volume is one-third Ap, one-third collagen and other organic components, and one-third H₂O. Some data in the literature for the composition of adult human and bovine cortical bone are given in Table 1.1.

1.3 Elastic Properties

Although bone is a viscoelastic material, at the quasi-static strain rates in mechanical testing and even at the ultrasonic frequencies used experimentally, it is a reasonable first approximation to model cortical bone as an anisotropic, linear elastic solid with Hooke’s law as the appropriate constitutive equation. Tensor notation for the equation is written as:

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \tag{1.1}$$

where σ_{ij} and ϵ_{kl} are the second-rank stress and infinitesimal second-rank strain tensors, respectively, and C_{ijkl} is the fourth-rank elasticity tensor. Using the reduced notation, we can rewrite Equation 1.1 as

$$\sigma_i = C_{ij}\epsilon_j \quad i, j = 1 \text{ to } 6 \tag{1.2}$$

where C_{ij} are the stiffness coefficients (elastic constants). The inverse of the C_{ij} , the S_{ij} , are known as the *compliance coefficients*.

The anisotropy of cortical bone tissue has been described in two symmetry arrangements. Lang [1969], Katz and Ukraincik [1971], and Yoon and Katz [1976a, b] assumed bone to be transversely isotropic with the bone axis of symmetry (the 3 direction) as the unique axis of symmetry. Any small difference in elastic properties between the radial (1 direction) and transverse (2 direction) axes, due to the apparent gradient in porosity from the periosteal to the endosteal sides of bone, was deemed to be due essentially to the defect and did not alter the basic symmetry. For a transverse isotropic material, the stiffness matrix $[C_{ij}]$ is given by

$$[C_{ij}] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \tag{1.3}$$

where $C_{66} = \frac{1}{2}(C_{11} - C_{12})$. Of the 12 nonzero coefficients, only 5 are independent.

However, Van Buskirk and Ashman [1981] used the small differences in elastic properties between the radial and tangential directions to postulate that bone is an **orthotropic** material; this requires that 9 of