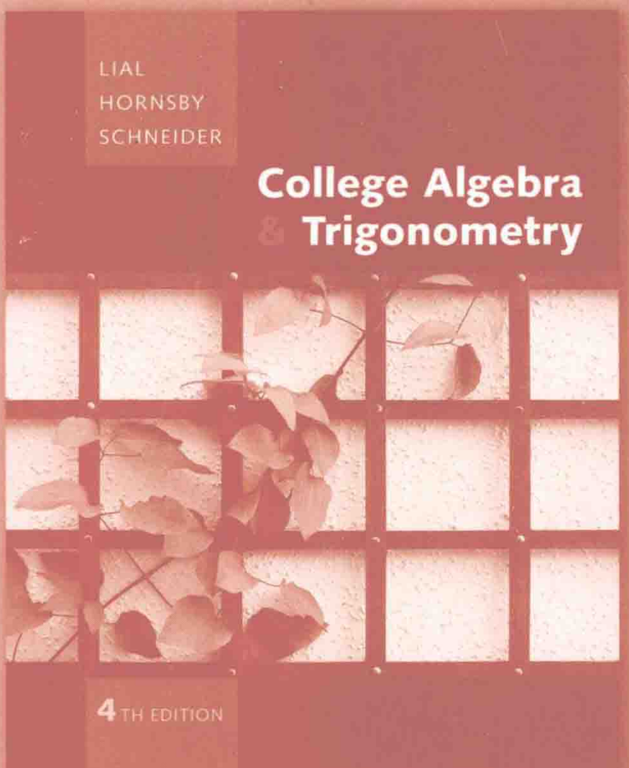
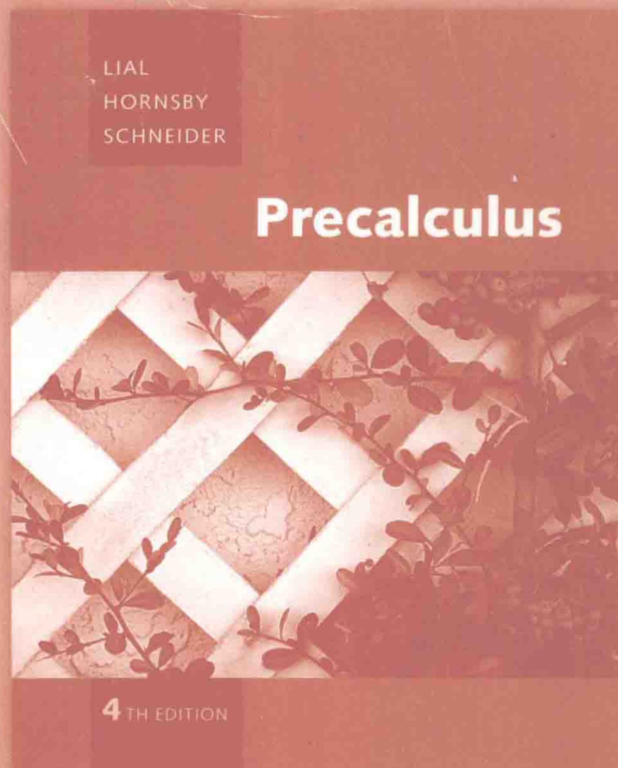
Student's Solutions Manual

Beverly Fusfield





Lial • Hornsby • Schneider

STUDENT'S SOLUTIONS MANUAL

BEVERLY FUSFIELD

COLLEGE ALGEBRA AND TRIGONOMETRY

FOURTH EDITION

AND

PRECALCULUS

FOURTH EDITION

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CONTENTS

3 Polynomial and Rational Functions

4 Inverse, Exponential, and Logarithmic Functions

5 Trigonometric Functions

6 The Circular Functions and Their Graphs

7 Trigonometric Identities and Equations

8 Applications of Trigonometry

9 Systems and Matrices

10 Analytic Geometry

11 Further Topics in Algebra

Appendices

Chapter R

REVIEW OF BASIC CONCEPTS

Section R.1: Sets

1. The elements of the set $\{12, 13, 14, \ldots, 20\}$ are all the natural numbers from 12 to 20 inclusive. There are 9 elements in the set, $\{12, 13, 14, 15, 16, 17, 18, 19, 20\}$.

3. Each element of the set $\left\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{32}\right\}$ after the first is found by multiplying the preceding number by $\frac{1}{2}$. There are 6 elements in the set, $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}\right\}$.

5. To find the elements of the set $\{17, 22, 27 \ldots, 47\}$, start with 17 and add 5 to find the next number. There are 7 elements in the set, $\{17, 22, 27, 32, 37, 42, 47\}$.

7. When you list all elements in the set {all natural numbers greater than 7 and less than 15}, you obtain $\{8, 9, 10, 11, 12, 13, 14\}$.

9. The set $\{4, 5, 6, \ldots, 15\}$ has a limited number of elements, so it is a finite set.

11. The set $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\right\}$ has an unlimited number of elements, so it is an infinite set.

13. The set $\{x \mid x$ is a natural number larger than $5\}$, which can also be written as $\{6, 7, 8, 9, \ldots\}$, has an unlimited number of elements, so it is an infinite set.

15. There are an infinite number of fractions between 0 and 1, so $\{x \mid x$ is a fraction between 0 and $1\}$ is an infinite set.

17. 6 is an element of the set $\{3, 4, 5, 6\}$, so we write $6 \in \{3, 4, 5, 6\}$.

19. –4 is not an element of $\{4, 6, 8, 10\}$, so we write $-4 \notin \{4, 6, 8, 10\}$.

21. 0 is an element of $\{2, 0, 3, 4\}$, so we write $0 \in \{2, 0, 3, 4\}$.

23. $\{3\}$ is a subset of $\{2, 3, 4, 5\}$, not an element of $\{2, 3, 4, 5\}$, so we write $\{3\} \notin \{2, 3, 4, 5\}$.

25. $\{0\}$ is a subset of $\{0, 1, 2, 5\}$, not an element of $\{0, 1, 2, 5\}$, so we write $\{0\} \notin \{0, 1, 2, 5\}$.

27. 0 is not an element of $\varnothing$, since the empty set contains no elements. Thus, $0 \notin \varnothing$.

29. $3 \in \{2, 5, 6, 8\}$
Since 3 is not one of the elements in $\{2, 5, 6, 8\}$, the statement is false.

31. $1 \in \{3, 4, 5, 11, 1\}$
Since 1 is one of the elements of $\{3, 4, 5, 11, 1\}$, the statement is true.

33. $9 \notin \{2, 1, 5, 8\}$
Since 9 is not one of the elements of $\{2, 1, 5, 8\}$, the statement is true.

35. $\{2, 5, 8, 9\} = \{2, 5, 9, 8\}$
This statement is true because both sets contain exactly the same four elements.

37. $\{5, 8, 9\} = \{5, 8, 9, 0\}$
These two sets are not equal because $\{5, 8, 9, 0\}$ contains the element 0, which is not an element of $\{5, 8, 9\}$. Therefore, the statement is false.

39. $\{x \mid x$ is a natural number less than $3\} = \{1, 2\}$
Since 1 and 2 are the only natural numbers less than 3, this statement is true.

41. $\{5, 7, 9, 19\} \cap \{7, 9, 11, 15\} = \{7, 9\}$
Since 7 and 9 are the only elements belonging to both sets, the statement is true.

43. $\{2, 1, 7\} \cup \{1, 5, 9\} = \{1\}$
$\{2, 1, 7\} \cup \{1, 5, 9\} = \{1, 2, 5, 7, 9\}$, while $\{2, 1, 7\} \cap \{1, 5, 9\} = \{1\}$. Therefore, the statement is false.

45. $\{3, 2, 5, 9\} \cap \{2, 7, 8, 10\} = \{2\}$
Since 2 is the only element belonging to both sets, the statement is true.

47. $\{3, 5, 9, 10\} \cap \varnothing = \{3, 5, 9, 10\}$
In order to belong to the intersection of two sets, an element must belong to both sets. Since the empty set contains no elements, $\{3, 5, 9, 10\} \cap \varnothing = \varnothing$, so the statement is false.

49. $\{1, 2, 4\} \cup \{1, 2, 4\} = \{1, 2, 4\}$
Since the two sets are equal, their union contains the same elements, namely 1, 2, and 4. Thus, the statement is true.

51. $\varnothing \cup \varnothing = \varnothing$
Since the empty set contains no elements, the statement is true.

For Exercises 53–63, $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{2, 4, 8, 10\}$, $C = \{4, 10, 12\}$, $D = \{2, 10\}$, and $U = \{2, 4, 6, 8, 10, 12, 14\}$.

53. $A \subseteq U$
This statement says "A is a subset of U." Since every element of A is also an element of U, the statement is true.

55. $D \subseteq B$
Since both elements of D, 2 and 10, are also elements of B, D is a subset of B. The statement is true.

57. $A \subseteq B$
Set A contains two elements, 6 and 12, that are not elements of B. Thus, A is not a subset of B. The statement is false.

59. $\varnothing \subseteq A$
The empty set is a subset of every set, so the statement is true.

61. $\{4, 8, 10\} \subseteq B$
Since 4, 8, and 10 are all elements of B, $\{4, 8, 10\}$ is a subset of B. The statement is true.

63. $B \subseteq D$
Since B contains two elements, 4 and 8, that are not elements of D, B is not a subset of D. The statement is false.

65. Every element of $\{2, 4, 6\}$ is also an element of $\{3, 2, 5, 4, 6\}$, so $\{2, 4, 6\}$ is a subset of $\{3, 2, 5, 4, 6\}$.
We write $\{2, 4, 6\} \subseteq \{3, 2, 5, 4, 6\}$.

67. Since 0 is an element of $\{0, 1, 2\}$, but is not an element of $\{1, 2, 3, 4, 5\}$, $\{0, 1, 2\}$ is not a subset of $\{1, 2, 3, 4, 5\}$. We write $\{0, 1, 2\} \not\subseteq \{1, 2, 3, 4, 5\}$.

69. The empty set is a subset of every set, so $\varnothing \subseteq \{1, 4, 6, 8\}$.

For Exercises 71–93, $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$, $M = \{0, 2, 4, 6, 8\}$, $N = \{1, 3, 5, 7, 9, 11, 13\}$, $Q = \{0, 2, 4, 6, 8, 10, 12\}$, and $R = \{0, 1, 2, 3, 4\}$.

71. $M \cap R$
The only elements belonging to both M and R are 0, 2, and 4, so $M \cap R = \{0, 2, 4\}$.

73. $M \cup N$
The union of two sets contains all elements that belong to either set or to both sets.
$M \cup N = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13\}$

75. $M \cap N$
There are no elements which belong to both M and N, so $M \cap N = \varnothing$. M and N are disjoint sets.

77. $N \cup R = \{0, 1, 2, 3, 4, 5, 7, 9, 11, 13\}$

79. N'
The set N' is the complement of set N, which means the set of all elements in the universal set U that do not belong to N.
$N' = Q$ or $\{0, 2, 4, 6, 8, 10, 12\}$

81. $M' \cap Q$
First form M', the complement of M. M' contains all elements of U that are not elements of M. Thus, $M' = \{1, 3, 5, 7, 9, 10, 11, 12, 13\}$. Now form the intersection of M' and Q. Thus, we have $M' \cap Q = \{10, 12\}$.

83. $\varnothing \cap R$
Since the empty set contains no elements, there are no elements belonging to both $\varnothing$ and R. Thus, $\varnothing$ and R are disjoint sets, and $\varnothing \cap R = \varnothing$.

85. $N \cup \varnothing$
Since $\varnothing$ contains no elements, the only elements belonging to N or $\varnothing$ are the elements of N. Thus, $\varnothing$ and N are disjoint sets, and $N \cup \varnothing = N$ or $\{1, 3, 5, 7, 9, 11, 13\}$.

87. $(M \cap N) \cup R$
First form the intersection of M and N. Since M and N have no common elements (they are disjoint), $M \cap N = \varnothing$. Thus, $(M \cap N) \cup R = \varnothing \cup R$. Now, since $\varnothing$ contains no elements, the only elements belonging to R or $\varnothing$ are the elements of R. Thus, $\varnothing$ and R are disjoint sets, and $\varnothing \cup R = R$ or $\{0, 1, 2, 3, 4\}$.

89. $(Q \cap M) \cup R$
First form the intersection of Q and M. We have $Q \cap M = \{0, 2, 4, 6, 8\} = M$. Now form the union of this set with R. We have $(Q \cap M) \cup R = M \cup R = \{0, 1, 2, 3, 4, 6, 8\}$.

91. $(M' \cup Q) \cap R$
First, find M', the complement of M. We have $M' = \{1, 3, 5, 7, 9, 10, 11, 12, 13\}$. Next, form the union of M' and Q. We have $M' \cup Q = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\} = U$. Thus, we have $(M' \cup Q) \cap R = U \cap R = R$ or $\{0, 1, 2, 3, 4\}$.

93. $Q' \cap (N' \cap U)$
First, find Q', the complement of Q. We have $Q' = \{1, 3, 5, 7, 9, 11, 13\} = N$. Now find N', the complement of N. We have $N' = \{0, 2, 4, 6, 8, 10, 12\} = Q$. Next, form the intersection of N' and U. We have $N' \cap U = Q \cap U = Q$ Finally, we have $Q' \cap (N' \cap U) = Q' \cap Q = \varnothing$ Since the intersection of Q' and $(N' \cap U)$ is $\varnothing$, Q' and $(N' \cap U)$ are disjoint sets.

95. M' is the set of all students in this school who are not taking this course.

97. $N \cap P$ is the set of all students in this school who are taking both calculus and history.

99. $M \cup P$ is the set of all students in this school who are taking this course or history or both.

Section R.2: Real Numbers and Their Properties

1. **(a)** 0 is a whole number. Therefore, it is also an integer, a rational number, and a real number. 0 belongs to B, C, D, F.

(b) 34 is a natural number. Therefore, it is also a whole number, an integer, a rational number, and a real number. 34 belongs to A, B, C, D, F.

(c) $-\frac{9}{4}$ is a rational number and a real number. $-\frac{9}{4}$ belongs to D, F.

(d) $\sqrt{36} = 6$ is a natural number. Therefore, it is also a whole number, an integer, a rational number, and a real number. $\sqrt{36}$ belongs to A, B, C, D, F.

(e) $\sqrt{13}$ is an irrational number and a real number. $\sqrt{13}$ belongs to E, F.

(f) $2.16 = \frac{216}{100} = \frac{54}{25}$ is a rational number and a real number. 2.16 belongs to D, F.

3. False. Positive integers are whole numbers, but negative integers are not.

5. False. No irrational numbers are integers.

7. True. Every natural number is a whole number.

9. True. Some rational numbers are whole numbers.

11. 1 and 3 are natural numbers.

13. -6, $-\frac{12}{4}$ (or -3), 0, 1, and 3 are integers.

15. $-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2) = -16$

17. $(-2)^4 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 16$

19. $(-3)^5 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3) = -243$

21. $-2 \cdot 3^4 = -2 \cdot (3 \cdot 3 \cdot 3 \cdot 3) = -2 \cdot 81 = -162$

23.
$$\begin{aligned}-2 \cdot 5 + 12 \div 3 &= -10 + 12 \div 3 \\ &= -10 + 4 = -6\end{aligned}$$

25.
$$\begin{aligned}-4(9-8) + (-7)(2)^3 &= -4(1) + (-7)(2)^3 \\ &= -4(1) + (-7) \cdot 8 \\ &= -4 + (-7) \cdot 8 \\ &= -4 + (-56) = -60\end{aligned}$$

27.
$$\begin{aligned}(4 - 2^3)(-2 + \sqrt{25}) &= (4-8)(-2+5) \\ &= (-4)(3) = -12\end{aligned}$$

29.
$$\begin{aligned}&\left(-\frac{2}{9} - \frac{1}{4}\right) - \left[-\frac{5}{18} - \left(-\frac{1}{2}\right)\right] \\ &= \left(-\frac{8}{36} - \frac{9}{36}\right) - \left(-\frac{5}{18} + \frac{9}{18}\right) \\ &= \left(-\frac{17}{36}\right) - \left(\frac{4}{18}\right) = -\frac{17}{36} - \frac{8}{36} = -\frac{25}{36}\end{aligned}$$

31.
$$\begin{aligned}\frac{-8 + (-4)(-6) \div 12}{4 - (-3)} &= \frac{-8 + 24 \div 12}{4 + 3} \\ &= \frac{-8 + 2}{7} \\ &= \frac{-6}{7} = -\frac{6}{7}\end{aligned}$$

33. Let $p = -4$, $q = 8$, and $r = -10$.
$$\begin{aligned}2p - 7q + r^2 &= 2(-4) - 7 \cdot 8 + (-10)^2 \\ &= 2(-4) - 7 \cdot 8 + 100 \\ &= -8 - 7 \cdot 8 + 100 \\ &= -8 - 56 + 100 \\ &= -64 + 100 = 36\end{aligned}$$

35. Let $p = -4$, $q = 8$, and $r = -10$.
$$\frac{q+r}{q+p} = \frac{8+(-10)}{8+(-4)} = \frac{-2}{4} = -\frac{1}{2}$$

37. Let $p = -4$, $q = 8$, and $r = -10$.
$$\frac{3q}{r} - \frac{5}{p} = \frac{3\cdot 8}{-10} - \frac{5}{-4} = \frac{24}{-10} - \frac{5}{-4} = -\frac{12}{5} - \frac{5}{-4}$$
$$= -\frac{12}{5} + \frac{5}{4} = -\frac{48}{20} + \frac{25}{20} = -\frac{23}{20}$$

39. Let $p = -4$, $q = 8$, and $r = -10$.
$$\frac{-(p+2)^2 - 3r}{2-q} = \frac{-(-4+2)^2 - 3(-10)}{2-8}$$
$$= \frac{-(-2)^2 - 3(-10)}{-6}$$
$$= \frac{-4-3(-10)}{-6} = \frac{-4-(-30)}{-6}$$
$$= \frac{-4+30}{-6} = \frac{26}{-6} = -\frac{13}{3}$$

41. $A = 451$, $C = 281$, $Y = 3049$, $T = 22$, $I = 6$
Passing Rating
$$= 85.68\left(\frac{C}{A}\right) + 4.31\left(\frac{Y}{A}\right) + 326.42\left(\frac{T}{A}\right) - 419.07\left(\frac{I}{A}\right)$$
$$\approx 85.68\left(\frac{281}{451}\right) + 4.31\left(\frac{3049}{451}\right) + 326.42\left(\frac{22}{451}\right) - 419.07\left(\frac{6}{451}\right)$$
$$\approx 53.38 + 29.14 + 15.92 - 5.58 \approx 92.9$$

43. $A = 610$, $C = 375$, $Y = 4359$, $T = 24$, $I = 15$
Passing Rating
$$= 85.68\left(\frac{C}{A}\right) + 4.31\left(\frac{Y}{A}\right) + 326.42\left(\frac{T}{A}\right) - 419.07\left(\frac{I}{A}\right)$$
$$\approx 85.68\left(\frac{375}{610}\right) + 4.31\left(\frac{4359}{610}\right) + 326.42\left(\frac{24}{610}\right) - 419.07\left(\frac{15}{610}\right)$$
$$\approx 52.67 + 30.80 + 12.84 - 10.31 = 86.0$$

45. $\text{BAC} = 48 \times 3.2 \times .075 \div 190 - 2 \times .015 \approx .031$

47. Exercise 45:
$\text{BAC} = 48 \times 3.2 \times .075 \div 215 - 2 \times .015 \approx .024$
Exercise 46:
$\text{BAC} = 36 \times 4.0 \times .075 \div 160 - 3 \times .015 = 0.023$
The increased weight results in lower BACs.

49. distributive

51. inverse

53. identity

55. No; in general $a - b \neq b - a$. Examples will vary, i.e. if $a = 15$ and $b = 0$, then $a - b = 15 - 0 = 15$, but $b - a = 0 - 15 = -15$.

57. $8p - 14p = (8-14)p = -6p$

59. $-4(z-y) = -4z - (-4y) = -4z + 4y$

61. $\frac{10}{11}(22z) = \left(\frac{10}{11}\cdot 22\right)z = 20z$

63. $(m+5)+6 = m+(5+6) = m+11$

65.
$$\frac{3}{8}\left(\frac{16}{9}y + \frac{32}{27}z - \frac{40}{9}\right)$$
$$= \frac{3}{8}\left(\frac{16}{9}y\right) + \frac{3}{8}\left(\frac{32}{27}z\right) - \frac{3}{8}\left(\frac{40}{9}\right)$$
$$= \left(\frac{3}{8}\cdot\frac{16}{9}\right)y + \left(\frac{3}{8}\cdot\frac{32}{27}\right)z - \frac{5}{3}$$
$$= \frac{2}{3}y + \frac{4}{9}z - \frac{5}{3}$$

67. The process in your head should be the following:
$$72\cdot 17 + 28\cdot 17 = (72+28)(17)$$
$$= (100)(17) = 1700$$

69. The process in your head should be the following:
$$123\frac{5}{8}\cdot 1\frac{1}{2} - 23\frac{5}{8}\cdot 1\frac{1}{2} = \left(123\frac{5}{8} - 23\frac{5}{8}\right)\left(1\frac{1}{2}\right)$$
$$= (100)\left(1\frac{1}{2}\right) = (100)(1.5)$$
$$= 150$$

71. This statement is false since $|6-8| = |-2| = 2$ and $|6| - |8| = 6 - 8 = -2$. A corrected statement would be $|6-8| \neq |6| - |8|$ or $|6-8| = |8| - |6|$

73. This statement is true since $|-5|\cdot|6| = 5\cdot 6 = 30$ and $|-5\cdot 6| = |-30| = 30$.

75. This statement is false. For example if you let $a = 2$ and $b = 6$ then $|2-6| = |-4| = 4$ and $|a| - |b| = |2| - |6| = 2 - 6 = -4$. A corrected statement is $|a-b| = |b| - |a|$, if $b > a > 0$.

77. $|-10|=10$

79. $-\left|\frac{4}{7}\right|=-\frac{4}{7}$

81. Let $x=-4$ and $y=2$.
$|x-y|=|-4-2|=|-6|=6$

83. Let $x=-4$ and $y=2$.
$|3x+4y|=|3(-4)+4(2)|$
$=|-12+8|=|-4|=4$

85. Let $x=-4$ and $y=2$.
$$\frac{2|y|-3|x|}{|xy|}=\frac{2|2|-3|-4|}{|-4(2)|}=\frac{2(2)-3(4)}{|-8|}=\frac{4-12}{8}=\frac{-8}{8}=-1$$

87. Let $x=-4$ and $y=2$.
$$\frac{|-8y+x|}{-|x|}=\frac{|-8(2)+(-4)|}{-|-4|}=\frac{|-16+(-4)|}{-|-4|}=\frac{|-20|}{-(4)}=\frac{20}{-4}=-5$$

89. Property 2

91. Property 3

93. Property 1

95. Since $|-3-5|=|-8|=8$ and $|5-(-3)|=|8|=8,$ the number of strokes between their scores is 8.

97. $P_d=|P-125|=|116-125|=|-9|=9$
The P_d value for a woman whose actual systolic pressure is 116 and whose normal value should be 125 is 9.

99. The absolute value of the difference in wind-chill factors for wind at 15 mph with a 30°F temperature and wind at 10 mph with a –10°F temperature is $|19°-(-28°)|=|47°|=47°$ F.

101. The absolute value of the difference in wind-chill factors for wind at 30 mph with a –30°F temperature and wind at 15 mph with a –20°F temperature is
$|-67°-(-45°)|=|-22°|=22°$ F.

103. $d(P,Q)=|-1-(-4)|=|-1+4|=|3|=3$ or
$d(P,Q)=|-4-(-1)|=|-4+1|=|-3|=3$

105. $d(Q,R)=|8-(-1)|=|8+1|=|9|=9$ or
$d(Q,R)=|-1-8|=|-9|=9$

107. $xy>0$ if x and y have the same sign.

109. $\frac{x}{y}<0$ if x and y have different signs.

111. Since x^3 has the same sign as x, $\frac{x^3}{y}>0$ if x and y have the same sign.

Section R.3: Polynomials

1. Incorrect: $(mn)^2=m^2n^2$

3. Incorrect: $\left(\frac{k}{5}\right)^3=\frac{k^3}{5^3}=\frac{k^3}{125}$

5. Incorrect: $4^5\cdot 4^2=4^{5+2}=4^7$

7. Incorrect: $cd^0=c\cdot 1=c$

9. Correct: $\left(\frac{1}{4}\right)^5=\frac{1}{4^5}$

11. $9^3\cdot 9^5=9^{3+5}=9^8$

13. $(-4x^5)(4x^2)=(-4\cdot 4)(x^5x^2)$
$=-16x^{5+2}=-16x^7$

15. $n^6\cdot n^4\cdot n=n^{6+4+1}=n^{11}$

17. $(-3m^4)(6m^2)(-4m^5)$
$=[(-3)(6)(-4)](m^4m^2m^5)$
$=72m^{4+2+5}=72m^{11}$

19. $(2^2)^5=2^{2\cdot 5}=2^{10}$

21. $(-6x^2)^3=(-6)^3(x^2)^3=(-6)^3x^6=-216x^6$

23. $-(4m^3n^0)^2=-\left[4^2(m^3)^2(n^0)^2\right]$
$=-4^2m^{3\cdot 2}n^{0\cdot 2}$
$=-4^2m^6n^0=-(4^2)m^6\cdot 1$
$=-4^2m^6=-16m^6$

25. $\left(\frac{r^8}{s^2}\right)^3=\frac{(r^8)^3}{(s^2)^3}=\frac{r^{8\cdot 3}}{s^{2\cdot 3}}=\frac{r^{24}}{s^6}$

27. $\left(\frac{-4m^2}{t}\right)^4 = \frac{(-4)^4(m^2)^4}{t^4} = \frac{(-4)^4m^8}{t^4} = \frac{256m^8}{t^4}$

29. **(a)** $6^0 = 1$; B **(b)** $-6^0 = -1$; C

(c) $(-6)^0 = 1$; B **(d)** $-(-6)^0 = -1$; C

31. Answers will vary.
$x^2 + x^2 = 2x^2$

33. $-5x^{11}$ is a polynomial. It is a monomial since it has one term. It has degree 11 since 11 is the highest exponent.

35. $18p^5q + 6pq$ is a polynomial. It is a binomial since it has two terms. It has degree 6 because 6 is the sum of the exponents in the term $18p^5q$, and this term has a higher degree than the term $6pq$.

37. $\sqrt{2}x^2 + \sqrt{3}x^6$ is a polynomial. It is a binomial since it has two terms. It has degree 6 since 6 is the highest exponent.

39. $\frac{1}{3}r^2s^2 - \frac{3}{5}r^4s^2 + rs^3$ is a polynomial. It is a trinomial since it has three terms. It has degree 6 because the sum of the exponents in the term $-\frac{3}{5}r^4s^2$ is 6, and this term has the highest degree.

41. $\frac{5}{p} + \frac{2}{p^2} + \frac{5}{p^3}$ is not a polynomial since positive exponents in the denominator are equivalent to negative exponents in the numerator.

43. $(5x^2 - 4x + 7) + (-4x^2 + 3x - 5)$
$= [5 + (-4)]x^2 + (-4 + 3)x + [7 + (-5)]$
$= 1 \cdot x^2 + (-1)x + 2 = x^2 - x + 2$

45. $2(12y^2 - 8y + 6) - 4(3y^2 - 4y + 2)$
$= 2(12y^2) - 2(8y) + 2(6) - 4(3y^2)$
$\quad - 4(-4y) - 4 \cdot 2$
$= 24y^2 - 16y + 12 - 12y^2 + 16y - 8$
$= 12y^2 + 4$

47. $(6m^4 - 3m^2 + m) - (2m^3 + 5m^2 + 4m) + (m^2 - m)$
$= 6m^4 - 3m^2 + m - 2m^3 - 5m^2 - 4m + m^2 - m$
$= 6m^4 - 2m^3 + (-3 - 5 + 1)m^2 + (1 - 4 - 1)m$
$= 6m^4 - 2m^3 + (-7)m^2 + (-4)m$
$= 6m^4 - 2m^3 - 7m^2 - 4m$

49. $(4r - 1)(7r + 2) = 4r(7r) + 4r(2) - 1(7r) - 1(2)$
$= 28r^2 + 8r - 7r - 2$
$= 28r^2 + r - 2$

51. $x^2\left(3x - \frac{2}{3}\right)\left(5x + \frac{1}{3}\right)$
$= x^2\left[\left(3x - \frac{2}{3}\right)\left(5x + \frac{1}{3}\right)\right]$
$= x^2\left[(3x)(5x) + (3x)\left(\frac{1}{3}\right) - \frac{2}{3}(5x) - \frac{2}{3}\left(\frac{1}{3}\right)\right]$
$= x^2\left(15x^2 + x - \frac{10}{3}x - \frac{2}{9}\right)$
$= x^2\left(15x^2 + \frac{3}{3}x - \frac{10}{3}x - \frac{2}{9}\right)$
$= x^2\left(15x^2 - \frac{7}{3}x - \frac{2}{9}\right) = 15x^4 - \frac{7}{3}x^3 - \frac{2}{9}x^2$

53. $4x^2(3x^3 + 2x^2 - 5x + 1)$
$= 4x^2(3x^3) + 4x^2(2x^2) - 4x^2(5x) + 4x^2 \cdot 1$
$= 12x^5 + 8x^4 - 20x^3 + 4x^2$

55. $(2z - 1)(-z^2 + 3z - 4)$
$= (2z - 1)(-z^2) + (2z - 1)(3z) - (2z - 1)(4)$
$= 2z(-z^2) - 1(-z^2) + 2z(3z) - 1(3z)$
$\quad - (2z)(4) - (-1)(4)$
$= -2z^3 + z^2 + 6z^2 - 3z - 8z - (-4)$
$= -2z^3 + 7z^2 - 11z + 4$

We may also multiply vertically.

$$\begin{array}{rl} -z^2 + 3z - 4 & \\ 2z - 1 & \\ \hline z^2 - 3z + 4 & \leftarrow -1(-z^2 + 3z - 4) \\ -2z^3 + 6z^2 - 8z \quad & \leftarrow 2z(-z^2 + 3z - 4) \\ \hline -2z^3 + 7z^2 - 11z + 4 & \end{array}$$

57. $(m-n+k)(m+2n-3k)$
$= (m-n+k)(m)+(m-n+k)(2n) - (m-n+k)(3k)$
$= m^2 - mn + km + 2mn - 2n^2 + 2kn - 3km + 3kn - 3k^2$
$= m^2 + mn - 2n^2 - 2km + 5kn - 3k^2$
We may also multiply vertically.

$$\begin{array}{r} m - \quad n + \quad k \\ \underline{m + 2n - 3k} \\ -3km + 3kn - 3k^2 \\ 2mn - 2n^2 \qquad + 2kn \qquad\qquad \\ \underline{m^2 - mn \qquad + km \qquad\qquad\qquad} \\ m^2 + mn - 2n^2 - 2km + 5kn - 3k^2 \end{array}$$

59. $(2m+3)(2m-3) = (2m)^2 - 3^2$
$= 4m^2 - 9$

61. $(4x^2-5y)(4x^2+5y) = (4x^2)^2 - (5y)^2$
$= 16x^4 - 25y^2$

63. $(4m+2n)^2 = (4m)^2 + 2(4m)(2n) + (2n)^2$
$= 16m^2 + 16mn + 4n^2$

65. $(5r-3t^2)^2 = (5r)^2 - 2(5r)(3t^2) + (3t^2)^2$
$= 25r^2 - 30rt^2 + 9t^4$

67. $[(2p-3)+q]^2$
$= (2p-3)^2 + 2(2p-3)(q) + q^2$
$= (2p)^2 - 2(2p)(3) + (3)^2 + 4pq - 6q + q^2$
$= 4p^2 - 12p + 9 + 4pq - 6q + q^2$

69. $[(3q+5)-p][(3q+5)+p]$
$= (3q+5)^2 - p^2$
$= [(3q)^2 + 2(3q)(5) + 5^2] - p^2$
$= 9q^2 + 30q + 25 - p^2$

71. $[(3a+b)-1]^2 = (3a+b)^2 - 2(3a+b)(1) + 1^2$
$= (9a^2 + 6ab + b^2) - 2(3a+b) + 1$
$= 9a^2 + 6ab + b^2 - 6a - 2b + 1$

73. $(y+2)^3 = (y+2)^2(y+2)$
$= (y^2+4y+4)(y+2)$
$= y^3 + 4y^2 + 4y + 2y^2 + 8y + 8$
$= y^3 + 6y^2 + 12y + 8$

75. $(q-2)^4 = (q-2)^2(q-2)^2$
$= (q^2-4q+4)(q^2-4q+4)$
$= q^4 - 4q^3 + 4q^2 - 4q^3 + 16q^2 - 16q + 4q^2 - 16q + 16$
$= q^4 - 8q^3 + 24q^2 - 32q + 16$

77. $(p^3 - 4p^2 + p) - (3p^2 + 2p + 7)$
$= p^3 - 4p^2 + p - 3p^2 - 2p - 7$
$= p^3 - 7p^2 - p - 7$

79. $(7m+2n)(7m-2n) = (7m)^2 - (2n)^2$
$= 49m^2 - 4n^2$

81. $-3(4q^2 - 3q + 2) + 2(-q^2 + q - 4)$
$= -12q^2 + 9q - 6 - 2q^2 + 2q - 8$
$= -14q^2 + 11q - 14$

83. $p(4p-6) + 2(3p-8) = 4p^2 - 6p + 6p - 16$
$= 4p^2 - 16$

85. $-y(y^2-4) + 6y^2(2y-3)$
$= -y^3 + 4y + 12y^3 - 18y^2$
$= 11y^3 - 18y^2 + 4y$

87.
$$\begin{array}{r} 2x^5 + 7x^4 - 5x^2 + 7 \\ -2x^2 \overline{\big) -4x^7 - 14x^6 + 10x^4 - 14x^2} \\ \underline{-4x^7} \qquad\qquad\qquad\qquad\qquad \\ -14x^6 \qquad\qquad\qquad \\ \underline{-14x^6} \qquad\qquad\qquad \\ 10x^4 \qquad\quad \\ \underline{10x^4} \qquad\quad \\ -14x^2 \\ \underline{-14x^2} \\ 0 \end{array}$$

$$\frac{-4x^7 - 14x^6 + 10x^4 - 14x^2}{-2x^2} = 2x^5 + 7x^4 - 5x^2 + 7$$

89.
$$\begin{array}{r|l} & -5x^2+8 \\ \hline -2x^6 & 10x^8-16x^6-4x^4 \\ & \underline{10x^8} \\ & \quad -16x^6 \\ & \quad \underline{-16x^6} \\ & \qquad -4x^4 \end{array}$$

$$\frac{10x^8-16x^6-4x^4}{-2x^6} = -5x^2+8-\frac{4x^4}{-2x^6} = -5x^2+8+\frac{2}{x^2}$$

91.
$$\begin{array}{r|l} & 2m^2+m-2 \\ \hline 3m+2 & 6m^3+7m^2-4m+2 \\ & \underline{6m^3+4m^2} \\ & \quad 3m^2-4m \\ & \quad \underline{3m^2+2m} \\ & \qquad -6m+2 \\ & \qquad \underline{-6m-4} \\ & \qquad\qquad 6 \end{array}$$

$$\frac{6m^3+7m^2-4m+2}{3m+2} = 2m^2+m-2+\frac{6}{3m+2}$$

93.
$$\begin{array}{r|l} & x^3-x^2-x+4 \\ \hline 3x+3 & 3x^4-0x^3-6x^2+9x-5 \\ & \underline{3x^4+3x^3} \\ & \quad -3x^3-6x^2 \\ & \quad \underline{-3x^3-3x^2} \\ & \qquad -3x^2+9x \\ & \qquad \underline{-3x^2-3x} \\ & \qquad\qquad 12x-5 \\ & \qquad\qquad \underline{12x+12} \\ & \qquad\qquad\quad -17 \end{array}$$

$$\frac{3x^4-6x^2+9x-5}{3x+3} = x^3-x^2-x+4+\frac{-17}{3x+3}$$

or $x^3-x^2-x+4-\frac{17}{3x+3}$.

95. $99\times 101 = (100-1)(100+1) = 100^2-1^2$
$= 10{,}000-1 = 9999$

97. $102^2 = (100+2)^2 = 100^2+2(100)(2)+2^2$
$= 10{,}000+400+4 = 10{,}404$

99. **(a)** The area of the largest square is $s^2 = (x+y)^2$.

(b) The areas of the two squares are x^2 and y^2. The area of each rectangle is xy. Therefore, the area of the largest square can be written as $x^2+2xy+y^2$.

(c) Answers will vary. The total area must equal the sum of the four parts.

(d) It reinforces the special product for squaring a binomial: $(x+y)^2 = x^2+2xy+y^2$.

101. **(a)** The volume is

$$V = \frac{1}{3}h(a^2+ab+b^2)$$
$$= \frac{1}{3}(200)(314^2+314\times 756+756^2)$$
$$\approx 60{,}501{,}000\text{ ft}^3$$

(b) The shape becomes a rectangular box with a square base. Its volume is given by length × width × height or b^2h.

(c) If we let $a = b$, then $V = \frac{1}{3}h(a^2+ab+b^2)$ becomes $V = \frac{1}{3}h(b^2+bb+b^2)$, which simplifies to $V = hb^2$. Yes, the Egyptian formula gives the same result.

103. $x = 1940$

$$.000020591075(1940)^3 - .1201456829(1940)^2 + 233.5530856(1940) - 151{,}249.8184 \approx 6.2$$

The formula is .1 high.

105. $x = 1978$

$$.000020591075(1978)^3 - .1201456829(1978)^2 + 233.5530856(1978) - 151{,}249.8184 \approx 2.3$$

The formula is exact.

107. $(.25^3)(400^3) = [(.25)(400)]^3$
$= 100^3 = 1{,}000{,}000$

109. $\frac{4.2^5}{2.1^5} = \left(\frac{4.2}{2.1}\right)^5 = 2^5 = 32$

Section R.4: Factoring Polynomials

1. The greatest common factor is 12.
$12m+60=12(m)+12(5)=12(m+5)$

3. The greatest common factor is $8k$.
$8k^3+24k=8k\left(k^2\right)+8k(3)=8k\left(k^2+3\right)$

5. The greatest common factor is xy.
$xy-5xy^2=xy\cdot 1-xy(5y)=xy(1-5y)$

7. The greatest common factor is $-2p^2q^4$.
$-4p^3q^4-2p^2q^5$
$=\left(-2p^2q^4\right)(2p)+\left(-2p^2q^4\right)(q)$
$=-2p^2q^4(2p+q)$

9. The greatest common factor is $4k^2m^3$.
$4k^2m^3+8k^4m^3-12k^2m^4$
$=\left(4k^2m^3\right)(1)+\left(4k^2m^3\right)\left(2k^2\right)-\left(4k^2m^3\right)(3m)$
$=4k^2m^3\left(1+2z^2-3m\right)$

11. The greatest common factor is $2(a+b)$
$2(a+b)\ +\ 4m(a+b)$
$=[2(a+b)](1)+[2(a+b)](2m)$
$=\ 2(a+b)(1\ +\ 2m)$

13. $(5r-6)(r+3)-(2r-1)(r+3)$
$=(r+3)[(5r-6)-(2r-1)]$
$=(r+3)[5r-6-2r+1]=(r+3)(3r-5)$

15. $2(m-1)-3(m-1)^2+2(m-1)^3$
$=(m-1)\left[2-3(m-1)+2(m-1)^2\right]$
$=(m-1)\left[2-3m+3+2(m^2-2m+1)\right]$
$=(m-1)(2-3m+3+2m^2-4m+2)$
$=(m-1)(2m^2-7m+7)$

17. The completely factored form of $4x^2y^5-8xy^3$ is $4xy^3\left(xy^2-2\right)$.

19. $6st+9t-10s-15=(6st+9t)-(10s+15)$
$=3t(2s+3)-5(2s+3)$
$=(2s+3)(3t-5)$

21. $2m^4+6-am^4-3a$
$=(2m^4+6)-(am^4+3a)$
$=2(m^4+3)-a(m^4+3)=(m^4+3)(2-a)$

23. $p^2q^2-10-2q^2+5p^2$
$=p^2q^2-2q^2+5p^2-10$
$=q^2(p^2-2)+5(p^2-2)$
$=(p^2-2)(q^2+5)$

25. The positive factors of 6 could be 2 and 3, or 1 and 6. Since the middle term is negative, we know the factors of 4 must both be negative. As factors of 4, we could have -1 and -4, or -2 and -2. Try different combinations of these factors until the correct one is found.
$6a^2-11a+4=(2a-1)(3a-4)$

27. The positive factors of 3 are 1 and 3. Since the middle term is positive, we know the factors of 8 must both be positive. As factors of 8, we could have 1 and 8, or 2 and 4. Try different combinations of these factors until the correct one is found.
$3m^2+14m+8=(3m+2)(m+4)$

29. The positive factors of 15 are 1 and 15, or 3 and 5. Since the middle term is positive, we know the factors of 8 must both be positive. As factors of 8, we could have 1 and 8, or 2 and 4. Trying different combinations of these factors we find that $15p^2+24p+8$ is prime.

31. Factor out the greatest common factor, $2a$: $12a^3+10a^2-42a=2a(6a^2+5a-21)$. Now factor the trinomial by trial and error: $6a^2+5a-21=(3a+7)(2a-3)$. Thus, $12a^3+10a^2-42a=2a(3a+7)(2a-3)$.

33. The positive factors of 6 could be 2 and 3, or 1 and 6. As factors of -6, we could have -1 and 6, -6 and 1, -2 and 3, or -3 and 2. Try different combinations of these factors until the correct one is found.
$6k^2+5kp-6p^2=(2k+3p)(3k-2p)$

35. The positive factors of 5 can only be 1 and 5. As factors of -6, we could have -1 and 6, -6 and 1, -2 and 3, or -3 and 2. Try different combinations of these factors until the correct one is found.
$5a^2-7ab-6b^2=(5a+3b)(a-2b)$

37. The positive factors of 12 could be 4 and 3, 2 and 6, or 1 and 12. The factors of $-y^2$ are y and $-y$. Try different combination of these factors until the correct one is found.
$12x^2-xy-y^2=(4x+y)(3x-y)$

39. Factor out the greatest common factor, $2a^2$:

$24a^4+10a^3b-4a^2b^2=2a^2\left(12a^2+5ab-2b^2\right)$

Now factor the trinomial by trial and error:

$12a^2+5ab-2b^2=(4a-b)(3a+2b)$

Thus,

$$\begin{aligned}24a^4+10a^3b-4a^2b^2&=2a^2\left(12a^2+5ab-2b^2\right)\\&=2a^2(4a-b)(3a+2b)\end{aligned}$$

41. $$\begin{aligned}9m^2-12m+4&=(3m)^2-12m+2^2\\&=(3m)^2-2(3m)(2)+2^2\\&=(3m-2)^2\end{aligned}$$

43. $$\begin{aligned}&32a^2+48ab+18b^2\\&=2(16a^2+24ab+9b^2)\\&=2[(4a)^2+24ab+(3b)^2]\\&=2[(4a)^2+2(4a)(3b)+(3b)^2]=2(4a+3b)^2\end{aligned}$$

45. $$\begin{aligned}4x^2y^2+28xy+49&=(2xy)^2+28xy+7^2\\&=(2xy)^2+2(2xy)(7)+7^2\\&=(2xy+7)^2\end{aligned}$$

47. $$\begin{aligned}&(a-3b)^2-6(a-3b)+9\\&=(a-3b)^2-6(a-3b)+3^2\\&=(a-3b)^2-2(a-3b)(3)+3^2\\&=[(a-3b)-3]^2=(a-3b-3)^2\end{aligned}$$

49. **(a)** Since $(x+5y)^2=x^2+10xy+25y^2$, a matches B.

(b) Since $(x-5y)^2=x^2-10xy+25y^2$, b matches C.

(c) Since $(x+5y)(x-5y)=x^2-25y^2$, c matches A.

(d) Since $(5y+x)(5y-x)=25y^2-x^2$, d matches D.

51. $$\begin{aligned}9a^2-16&=(3a)^2-4^2\\&=(3a+4)(3a-4)\end{aligned}$$

53. $$36x^2-\frac{16}{25}=\left(6x-\frac{4}{5}\right)\left(6x+\frac{4}{5}\right)$$

55. $$\begin{aligned}25s^4-9t^2&=(5s^2)^2-(3t)^2\\&=(5s^2+3t)(5s^2-3t)\end{aligned}$$

57. $$\begin{aligned}(a+b)^2-16&=(a+b)^2-4^2\\&=[(a+b)+4][(a+b)-4]\\&=(a+b+4)(a+b-4)\end{aligned}$$

59. $$\begin{aligned}p^4-625&=(p^2)^2-25^2=(p^2+25)(p^2-25)\\&=(p^2+25)(p^2-5^2)\\&=(p^2+25)(p+5)(p-5)\end{aligned}$$

Note that p^2+25 is a prime factor.

61. $$\begin{aligned}8-a^3&=2^3-a^3=(2-a)(2^2+2\cdot a+a^2)\\&=(2-a)(4+2a+a^2)\end{aligned}$$

63. $$\begin{aligned}125x^3-27&=(5x)^3-3^3\\&=(5x-3)\left[(5x)^2+5x\cdot 3+3^2\right]\\&=(5x-3)(25x^2+15x+9)\end{aligned}$$

65. $$\begin{aligned}&27y^9+125z^6\\&=(3y^3)^3+(5z^2)^3\\&=(3y^3+5z^2)\left[(3y^3)^2-(3y^3)(5z^2)+(5z^2)^2\right]\\&=(3y^3+5z^2)(9y^6-15y^3z^2+25z^4)\end{aligned}$$

67. $$\begin{aligned}&(r+6)^3-216\\&=(r+6)^3-6^3\\&=\left[(r+6)-6\right]\left[(r+6)^2+(r+6)(6)+6^2\right]\\&=\left[(r+6)-6\right]\left[r^2+12r+36+(r+6)(6)+6^2\right]\\&=\left[r+6-6\right]\left[r^2+12r+36+6r+36+36\right]\\&=r(r^2+18r+108)\end{aligned}$$

69. $$\begin{aligned}&27-(m+2n)^3\\&=3^3-(m+2n)^3\\&=\left[3-(m+2n)\right]\cdot\\&\qquad\left[3^2+(3)(m+2n)+(m+2n)^2\right]\\&=\left[3-(m+2n)\right]\cdot\\&\qquad\left[3^2+(3)(m+2n)+m^2+4mn+4n^2\right]\\&=(3-m-2n)\left(9+3m+6n+m^2+4mn+4n^2\right)\end{aligned}$$

71. The correct complete factorization of x^4-1 is choice B: $(x^2+1)(x+1)(x-1)$. Choice A is not a complete factorization, since x^2-1 can be factored as $(x+1)(x-1)$. The other choices are not correct factorizations of x^4-1.