Recent Advances In Stochastic Operations Research II

Tadashi Dohi
Shunji Osaki
Katsushige Sawaki
Editors

Recent Advances In Stochastic Operations Research II

Editors

Tadashi Dohi

Hiroshima University, Japan

Shunji Osaki

Katsushige Sawaki

Nanzan University, Japan

World Scientific

NEW JERSEY · LONDON · SINGAPORE · BEIJING · SHANGHAI · HONG KONG · TAIPEI · CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

RECENT ADVANCES IN STOCHASTIC OPERATIONS RESEARCH II

ISBN-13 978-981-279-166-5
ISBN-10 981-279-166-3

Printed in Singapore by World Scientific Printers

Recent Advances In Stochastic Operations Research II

PREFACE

Operations Research uses quantitative models to analyze and predict the behavior of systems, and to provide information for decision makers. Two key concepts in Operations Research are Optimization and Uncertainty. Uncertainty is emphasized in Operations Research that could be called "Stochastic Operations Research" in which uncertainty is described by stochastic models. The typical models in Stochastic Operations Research are queueing models, inventory models, financial engineering models, reliability models, and simulation models.

International Workshop on Recent Advances in Stochastic Operations Research (2005 RASOR Canmore) was held in Canmore, Alberta, Canada, on August 25-26, 2005. At that time, a local proceedings was published and distributed to all the participants, where 40 papers were presented. After the conference, through the peer reviewing process, we published a book "Recent Advances in Stochastic Operations Research," edited by T. Dohi, S. Osaki and K. Sawaki, from World Scientific Publishing Co. Pte. Ltd., Singapore in 2007.

Following 2005 RASOR Canmore, we hosted International Workshop on Recent Advances in Stochastic Operations Research II (2007 RASOR Nanzan) at Nanzan University, Nagoya, Japan, on March 5-6, 2007. Again a local proceedings containing 43 papers was published and distributed to all the participants. After a careful peer reviewing process, this time, we are publishing a book "Recent Advances in Stochastic Operations Research II," edited by T. Dohi, S. Osaki and K. Sawaki, from World Scientific Publishing Co. Pte. Ltd., Singapore.

This conference was sponsored by the Research Center for Mathematical Sciences and Information Engineering, Nanzan University, 27 Seirei-cho, Seto-shi, Aichi 489-0863, Japan, to whom we would like to express our appreciation for their financial support. We also appreciated the financial support we received in the form of Grant-in-Aid for Scientific Research from the Ministry of Education, Sports, Science and Culture of Japan under Grant Nos. 16201035 and 18510138. Our special thanks are due to Professor Hiroyuki Okamura, Hiroshima University and Dr. Koichiro Rinsaka, Kobe Gakuin University, Japan, for their continual support from the initial planning of the conference to the final stage of editing the proceedings. Finally, we would like to thank Chelsea Chin, World Scientific Publishing Co., Singapore, for her warm help and patience.

Tadashi Dohi — Hiroshima University
Shunji Osaki — Nanzan University
Katsushige Sawaki — Nanzan University

August 2008

LIST OF CONTRIBUTORS

I. H. Chung	— Hyundai Rotem Company, Korea
T. Dohi	— Hiroshima University, Japan
E. A. Elsayed	— Rutgers, The State University of New Jersey, USA
F. Ferreira	— University of Trás-os-Montes e Alto Douro, Portugal
T. Furuta	— Nanzan University, Japan
M. Fushimi	— Nanzan University, Japan
P. Hagmark	— Tampere University of Technology, Finland
R. Hohzaki	— National Defence Academy, Japan
Y. Ida	— Ministry of Defence, Japan
S. Inoue	— Tottori University, Japan
H. Ishii	— Osaka University, Japan
A. Ito	— Nanzan University, Japan
K. Ito	— Kinjo Gakuin University, Japan
N. Kaio	— Hiroshima Shudo University, Japan
H. Kawai	— Tottori University, Japan
H. G. Kim	— Dong-Eui University, Korea
H. Kondo	— Nanzan University, Japan
H. Kono	— Osaka University, Japan
J. Koyanagi	— Tottori University, Japan
H. Liao	— Wichita State University, USA
S. Maeji	— Kinjo Gakuin University, Japan
S. Mizutani	— Aichi University of Technology, Japan
T. Nakagawa	— Aichi Institute of Technology, Japan
T. Nakai	— Chiba University, Japan
S. Nakamura	— Kinjo Gakuin University, Japan
D. Nanba	— Tottori University, Japan
K. Naruse	— Nagoya Sangyo University, Japan
S. Osaki	— Nanzan University, Japan
A. Pacheco	— Technical University of Lisbon, Portugal
H. Ribeiro	— Polytechnic Instutute of Leiria, Portugal
K. Rinsaka	— Kobe Gakuin University, Japan
K. Sato	— Nanzan University, Japan
K. Sawaki	— Nanzan University, Japan

S. Shiode — Kobe Gakuin University, Japan
M. Tamaki — Aichi University, Japan
Y. Tamura — Hiroshima Institute of Technology, Japan
K. Tokuno — Tottori University, Japan
S. Virtanen — Tampere University of Technology, Finland
S. Yamada — Tottori University, Japan
W. Y. Yun — Pusan National University, Korea
H. Zhang — Rutgers, The State University of New Jersey, USA
Y. Zhu — Rutgers, The State University of New Jersey, USA

CONTENTS

Part A Foundation of Stochastic Operations Research

A PROBABILISTIC PROOF OF AN IDENTITY RELATED TO THE STIRLING NUMBER OF THE FIRST KIND

MITSUSHI TAMAKI
Faculty of Business Administration
Aichi University
Miyoshi, Aichi, Japan
tamaki@vega.aichi-u.ac.jp

The basic assumption of the infinite formulation of the secretary problem, originally studied by Gianini and Samuels, is that, if $U_j, j = 1, 2, \ldots,$ is defined as the arrival time of the jth best from an infinite sequence of rankable items, then $U_1, U_2, \ldots,$ are i.i.d., uniform on the unit interval $(0, 1)$. An item is referred to as a record if it is relatively best. It can be shown that a well known identity related to the Stirling number of the first kind, as given in Eq.(3) in this note, is just the identity obtained through the derivation of the probability mass function of the number of records that appear on time interval $(s, t), 0 < s < t < 1$, in two ways in the infinite formulation.

1. Introduction

A set of n rankable items (1 being the best and n the worst) appear before us one at a time in random order with all $n!$ permutations equally likely. That is, each of the successive ranks of n items constitutes a random permutation. Suppose that all that can be observed are the relative ranks of the items as they appear. If X_j denotes the relative rank of the jth item among the first j items, the sequentially observed random variables are $X_1, X_2, \ldots, X_n$. Renyi[8] has shown that

(a) $X_1, X_2, \ldots, X_n$ are independent random variables.
(b) $P\{X_j = i\} = 1/j, \quad 1 \leq i \leq j, 1 \leq j \leq n.$

The reader is advised to check the case $n = 3$ or 4, if he/she is not familiar to these properties of the relative ranks.

The jth item is called *candidate* if it is relatively best, i.e., $X_j = 1$ and we introduce an indicator defined as

$$I_j = \begin{cases} 1, \text{ if } X_j = 1 \\ 0, \text{ otherwise.} \end{cases}$$

Thens

$$N_n = I_1 + I_2 + \cdots + I_n \tag{1}$$

denotes the total number of candidates. It is well known(see, e.g., Eq(2.5.9) of Arnold et al.[1] or Sec. 6.2, 6.3 and 9.5 of Blom et al.[2]) that the probability mass function of N_n is expressed as

$$p_n(k) = P\{N_n = k\} = \frac{1}{n!}\begin{bmatrix} n \\ k \end{bmatrix}, \qquad 1 \le k \le n,$$

where the notation $\begin{bmatrix} n \\ k \end{bmatrix}$, $1 \le k \le n,\ 1 \le n$ is a real number called *Stirling number of the first kind* (see an interesting paper by Knuth[7] for this notation). This number can be simply calculated from the following recursive relation

$$\begin{bmatrix} n \\ k \end{bmatrix} = \begin{bmatrix} n-1 \\ k-1 \end{bmatrix} + (n-1)\begin{bmatrix} n-1 \\ k \end{bmatrix} \quad 1 \le k \le n,\ 2 \le n$$

with $\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1$ and $\begin{bmatrix} n \\ k \end{bmatrix} = 0$ for $k = 0$ or $k > n$, or directly from

$$\begin{bmatrix} n \\ k \end{bmatrix} = (n-1)! \sum_{i_{k-1}=k-1}^{n-1} \frac{1}{i_{k-1}} \sum_{i_{k-2}=k-2}^{i_{k-1}-1} \frac{1}{i_{k-2}} \cdots \sum_{i_1=1}^{i_2-1} \frac{1}{i_1}.$$

It is noted that $\begin{bmatrix} n \\ k \end{bmatrix}$ is also interpreted as the number of permutations of n elements having k cycles(see, e.g., Graham et al.[5] or Blom et al.[2]).

A typical identity of the Stirling number of the first kind is

$$\sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix} z^k = z(z+1)\cdots(z+n-1), \tag{2}$$

which is immediate from $(a), (b)$ and (1) if we observe that the probability generating function of the sum of the independent random variables is the product of the individual probability generating functions, i.e., $E[z^{N_n}] = \prod_{j=1}^{n} E[z^{I_j}]$. The identity with which we are concerned here is, for any positive integer k,

$$\sum_{n=k}^{\infty} \begin{bmatrix} n \\ k \end{bmatrix} \frac{z^n}{n!} = \frac{1}{k!}\left(\log \frac{1}{1-z}\right)^k, \qquad 0 < z < 1 \tag{3}$$

as listed in Graham et al.[5](see Eq.(7.50), p.337). Multiply both sides of (2) by $v^n/n!$ $(0 < v < 1)$ and then add up over n. Then

$$\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix} z^k\right)\frac{v^n}{n!} = \sum_{n=0}^{\infty}\binom{z+n-1}{n} v^n = (1-v)^{-z},$$

where the last equality follows from the binomial theorem. Expanding $(1-v)^{-z} = \exp\{z \log(\frac{1}{1-v})\}$ into powers of z and comparing the coefficient of z^k on both sides yields the identity (3) with z replaced by v. Our objective is to give a probabilistic proof of this identity.

2. Probabilistic Proof

We employ the framework of the infinite secretary problem as defined and originally studied by Gianini and Samuels[3]. Let the best, second best, etc., of an infinite sequence of rankable items arrive at times $U_1, U_2, \ldots$, which are i.i.d., uniform on the unit interval $(0, 1)$. For each t in this interval, let $V_i(t)$ be the arrival time of the item which is ith best among all those that arrive before time t. Then as a familiar property of random samples from a uniform distribution on $(0, 1)$, we find that

$$V_i(t)\text{'s are i.i.d., uniform on } (0, t). \tag{4}$$

An item is called *record* if it is relatively best when it appears. We denote by $N(s, t)$ the number of records that appear on time interval $(s, t), 0 < s < t < 1$, and derive the probability mass function of $N(s, t)$ in two ways.

2.1. *Derivation by a forward-looking argument*

One way is to relate $N(s, t)$ with a random variable defined as

$$M(s, t) = \min\{i \geq 1 : V_i(t) < s\}.$$

That is, $M(s, t)$ represents the rank of the best item that appears before s relative to all those that appear before t. Focus our attention on the arrival times of the first $m+1$ bests that appear before t. Then $M(s,t)$ takes on a value $m+1$ if and only if m bests appear after s, whereas the $(m+1)th$ best appears before s. Thus we have from the property (4)

$$P\{M(s,t)=m+1\}=\frac{s}{t}\left(1-\frac{s}{t}\right)^m, \qquad m=0,1,2,\ldots$$

implying that $M(s,t)$ has a geometric distribution. Conditioning on $M(s,t)$ yields

$$\begin{aligned} P\{N(s,t)=k\} &= \sum_{m=k}^{\infty} P\{N(s,t)=k|M(s,t)=m+1\} \\ &\quad \times P\{M(s,t)=m+1\} \\ &= \sum_{m=k}^{\infty} p_m(k)P\{M(s,t)=m+1\} \\ &= \sum_{m=k}^{\infty} \frac{1}{m!}\begin{bmatrix} m \\ k \end{bmatrix} \frac{s}{t}\left(1-\frac{s}{t}\right)^m, \end{aligned} \tag{5}$$

where the second equality follows because, given $M(s,t)=m+1$, the arrival orders of m bests are equally likely and each of the records is identified as a candidate.

2.2. *Derivation by a backward-looking argument*

Another way to obtain $P\{N(s,t)=k\}$ is to trace the arrival epochs of the records backwards in time. The following lemma is crucial, which can be seen as a refinement of Theorem 1 of Gilbert and Mosteller[4](see also problem 32 of Chap.13 of Karlin and Taylor[6]).

Lemma. Let $Z_1, Z_2, \ldots$ be a sequence of random variables with Z_k uniformly distributed on time interval $(0, Z_{k-1})$, $Z_0 \equiv t < 1$. That is, $P\{Z_k \le x \mid Z_{k-1}=a\} = x/a, 0<x<a, k\ge 1$. If we denote by $K(s,t)$ the number of $Z_1, Z_2, \ldots$ whose values exceed s for $0<s<t$, namely, $K(s,t)=\max\{k : Z_k > s\}$ where $\max\{\phi\}=0$, then $K(s,t)$ is distributed as a Poisson random variable with parameter $\log(t/s)$, i.e.,

$$P\{K(s,t)=k\}=\frac{s}{t}\frac{\{\log(t/s)\}^k}{k!}. \tag{6}$$

Proof. We first show by induction on i that the distribution function of