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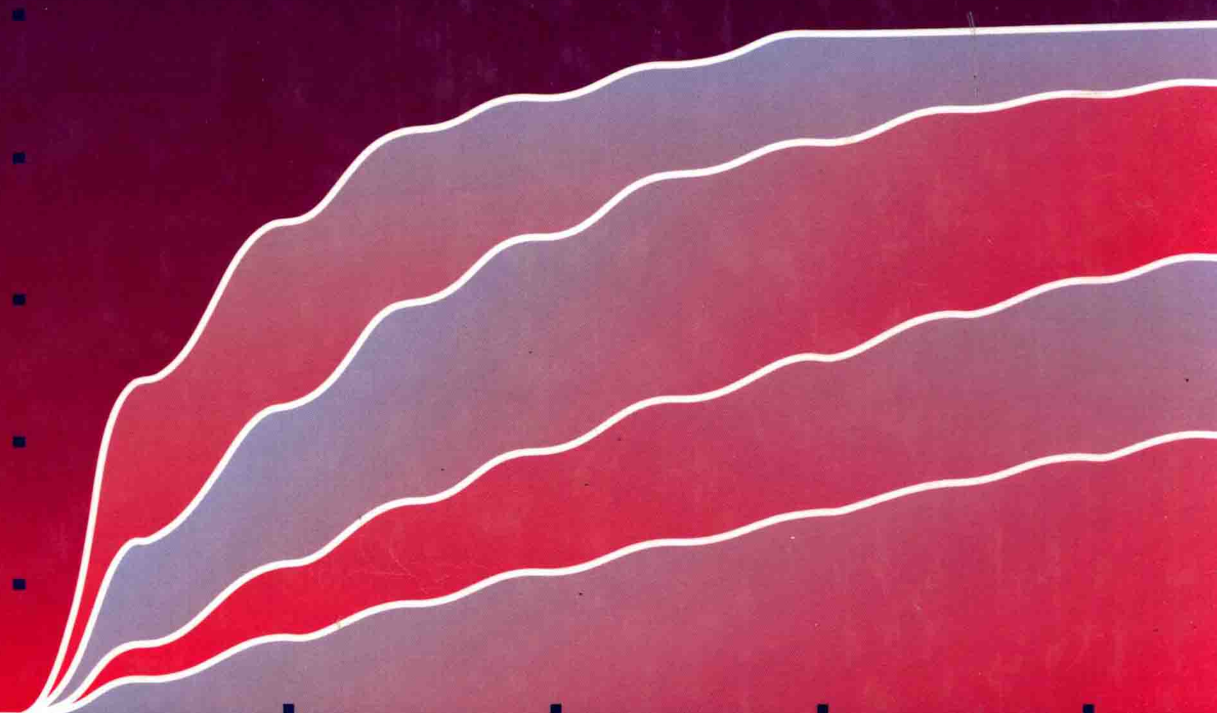
# RANDOM VIBRATIONS

## Analysis of Structural and Mechanical Systems



LOREN D. LUTES • SHAHRAM SARKANI

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## Analysis of Structural and Mechanical Systems

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# Random Vibrations

## Analysis of Structural and Mechanical Systems

To our children
Douglas, Daniel, Alan, David, Richard,
Laura, Steven, Rebekah, and Caroline

# PREFACE

This book has much in common with our earlier book (Lutes and Sarkani, 1997). In fact, a few of the chapters are almost unchanged. At the same time, we introduce several concepts that were not included in the earlier book and reorganize and update the presentation on several other topics.

The book is designed for use as a text for graduate courses in random vibrations or stochastic structural dynamics, such as might be offered in departments of civil engineering, mechanical engineering, aerospace engineering, ocean engineering, and applied mechanics. It is also intended for use as a reference for graduate students and practicing engineers with a similar level of preparation. The focus is on the determination of response levels for dynamical systems excited by forces that can be modeled as stochastic processes.

Because many readers will be new to the subject, our primary goal is clarity, particularly regarding the fundamental principles and relationships. At the same time, we seek to make the presentation sufficiently thorough and rigorous that the reader will be able to move on to more advanced work. We believe that the book can meet the needs of both those who wish to apply existing stochastic procedures to practical problems and those who wish to prepare for research that will extend the boundaries of knowledge.

In the hopes of meeting the needs of a broad audience, we have made this book relatively self-contained. We begin with a fairly extensive review of probability, random variables, and stochastic processes before proceeding to the analysis of dynamics problems. We do presume that the reader has a background in deterministic structural dynamics or mechanical vibration, but we also give a brief review of these methods before extending them for use in stochastic problems. Some knowledge of complex functions is necessary for the understanding of important frequency domain concepts. However, we also present time domain integration techniques that provide viable alternatives to the calculus of residues. Because of this, the book should also be useful to engineers who do not have a strong background in complex analysis.

The choice of prerequisites, as well as the demands of brevity, sometimes makes it necessary to omit mathematical proofs of results. We do always try to give mathematically rigorous definitions and results even when mathematical details are omitted. This approach is particularly important for the reader who wishes to pursue further study. An important part of the book is the inclusion of a number of worked examples that illustrate the modeling of physical problems as well as the proper application of theoretical solutions. Similar problems are also presented as exercises to be solved by the reader.

We attempt to introduce engineering applications of the material at the earliest possible stage, because we have found that many engineering students become impatient with lengthy study of mathematical procedures for which they do not know the application. Thus, we introduce linear vibration problems immediately after the introductory chapter on the modeling of stochastic problems. Time-domain interpretations are emphasized throughout the book, even in the presentation of important frequency-domain concepts. This includes, for example, the time history implications of bandwidth, with situations varying from narrowband to white noise.

One new topic added in this book is the use of evolutionary spectral density and the necessary time-domain and frequency-domain background on modulated processes. The final chapter is also new, introducing the effect of uncertainty about parameter values. Like the rest of the book, this chapter focuses on random vibration problems. The discussion of fatigue has major revisions and is grouped with first passage in an expanded chapter on the analysis of failure.

We intentionally include more material than can be covered in the typical one-semester or one-quarter format, anticipating that different instructors will choose to include different topics within an introductory course. To promote this flexibility, the crucial material is concentrated in the early portions of the book. In particular, the fundamentals of stochastic modeling and analysis of vibration problems are presented by the end of Chapter 6. From this point the reader can proceed to most topics in any of the other chapters. The book, and a modest number of readings on current research, could also form the basis for a two-semester course. It should be noted that Chapters 9–12 include topics that are the subjects of ongoing research, with the intent that these introductions will equip the reader to use the current literature, and possibly contribute to its future.

*Loren D. Lutes*
*Shahram Sarkani*

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**Chapter 4 Analysis of Stochastic Processes**



**Chapter 5 Time Domain Linear Vibration Analysis**



**Chapter 6 Frequency Domain Analysis**

# Chapter 1
# Introduction

## 1.1 Why Study Random Vibration?

Most structural and mechanical engineers who study probability do so specifically so that they may better estimate the likelihood that some engineering system will provide satisfactory service. This is often stated in the complementary way as estimating the probability of unsatisfactory service or failure. Thus, the study of probability generally implies that the engineer accepts the idea that it is either impossible or infeasible to devise a system that is absolutely sure to perform satisfactorily. We believe that this is an honest acceptance of the facts in our uncertain world, but it is somewhat of a departure from the philosophy of much of past engineering education and of the explicit form of many engineering design codes. Of course, engineers have always known that there was a possibility of failure, but they have not always made an effort to quantify the likelihood of that event and to use it in assessing the adequacy of a design. We believe that more rational design decisions will result from such explicit study of the likelihood of failure, and that is our motivation for the study of probabilistic methods.

The characterization of uncertainty in this book will always be done by methods based on probability theory. This is a purely pragmatic choice, based on the fact that these methods have been shown to be useful for a great variety of problems. Methods based on fundamentally different concepts, such as fuzzy sets, have also been demonstrated for some problems, but they will not be investigated here.

The engineering systems studied in this book are dynamical systems. Specifically, they are systems for which the dynamic motion can be modeled by a differential or integral equation or a set of such equations. Such systems usually consist of elements having mass, stiffness, and damping and exhibiting vibratory dynamic behavior. The methods presented are general and can be applied to a great variety of problems of structural and mechanical vibration. Examples will vary from simple mechanical oscillators to buildings or other large structures, with excitations that can be either forces or base motion. The primary emphasis will be on problems with linear models, but we will also include some study of nonlinear problems. For nonlinear problems, we will particularly emphasize methods that are direct extensions of linear methods.

Throughout most of this book, the uncertainty studied will be limited to that in the excitation of the system. Only in Chapter 12 will we introduce the topic of uncertainty about the parameters of the system. Experience has shown that there are indeed many problems in which the uncertainty about the input to the system is a key factor determining the probability of system failure. This is particularly true when the inputs are such environmental loads as earthquakes, wind, or ocean waves, but it also applies to numerous other situations such as the pressure variations in the exhaust from a jet engine. Nonetheless, there is almost always additional uncertainty about the system parameters, and this also can affect the probability of system failure.

The response of a dynamical system of the type studied here is a time history defined on a continuous set of time values. The field of probability that is applicable to such problems is called stochastic (or random) processes. Thus, the applications presented here involve the use of stochastic processes to model problems involving the behavior of dynamical systems. An individual whose background includes both a course in stochastic processes and a course in either structural dynamics or mechanical vibrations might be considered to be in an ideal situation to study stochastic vibrations, but this is an unreasonably high set of prerequisites for the beginning of such study. In particular, we will not assume prior knowledge of stochastic processes and will develop the methods for analysis of such processes within this book. The probability background needed for the study of stochastic processes is a fairly thorough understanding of the fundamental methods for investigating probability and, especially, random variables. This is because a stochastic process is generally viewed as a family of random variables. For the benefit of readers lacking the necessary random variable background, Chapters 2 and 3 give a relatively comprehensive introduction to the topic, focusing on the aspects that are most important for the understanding of stochastic processes. This material may be bypassed by readers with a strong background in probability and random variables, although some review of the notation used may be helpful, because it is also the notation of the remainder of this book.

We expect the reader to be familiar with deterministic approaches to vibration problems by using superposition methods such as the Duhamel convolution integral and, to a lesser extent, the Fourier transform. We will present brief reviews of the principal ideas involved in these methods of vibration analysis, but the reader without a solid background in this area will probably need to do some outside reading on these topics.

## 1.2 Probabilistic Modeling and Terminology

Within the realm of probabilistic methods, there are several terms related to uncertainty that warrant some comment. The term *random* will be used here for any variable about which we have some uncertainty. This does not mean that no knowledge is available but rather that we have less than perfect knowledge. As indicated in the previous section, we will particularly use results from the area of random variables. The word *stochastic* in common usage is essentially synonymous with *random*, but we will use it in a somewhat more specialized way. In particular, we will use the term *stochastic* to imply that there is a time history involved. Thus, we will say that the dynamic response at one instant of time $t$ is a random variable $X(t)$ but that the uncertain history of response over a range of time values is a stochastic process $\{X(t)\}$. The practice of denoting a stochastic process by putting the notation for the associated random variables in braces will be used to indicate that the stochastic process is a family of random variables—one for each $t$ value. The term *probability*, of course, will be used in the sense of fundamental probability theory. The probability of any event is a number in the range of zero to unity that models the likelihood of the event occurring. We can compute the probabilities of events that are defined in terms of random variables having certain values, or in terms of stochastic processes behaving in certain ways.

One can view the concepts of event, random variable, and stochastic process as forming a hierarchy, in order of increasing complexity. One can always give all the probabilistic information about an event by giving one number—the probability of occurrence for the event. To have all the information about a random variable generally requires knowledge of the probability of many events. In fact, we will be most concerned with so-called continuous random variables, and one must know the probabilities of infinitely many events to completely describe the probabilities of a continuous random variable. As mentioned before, a stochastic process is a family of random variables, so its probabilistic description will always require significantly more information than does the description of any one of those random variables. We will be most concerned with the case in which the stochastic process consists of infinitely many random variables, so the additional information required will be much more than for a random variable. One can also extend this hierarchy further, with the next step being stochastic fields, which are families of stochastic processes. Within this book we will use events, random variables, and especially stochastic processes, but we will avoid stochastic fields and further generalizations.

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Example 1.1: Let $t$ denote time in seconds and the random variable $X(t)$, for any fixed $t$ value, be the magnitude of the wind speed at a specified location at that time. Furthermore, let the family of $X(t)$ random variables for all nonnegative $t$ values be a stochastic process, $\{X(t)\}$, and let $A$ be the event $\{X(10) \le 5\,\text{m/s}\}$. Review the amount of information needed to give complete probabilistic descriptions of the event $A$, the random variable $X(t)$, and the stochastic process $\{X(t)\}$.

All the probabilistic information about the event $A$ is given by one number—its probability of occurrence. Thus, we might say that $p = P(A)$ is that probability of occurrence, and the only other probabilistic statement that can be made about $A$ is the almost trivial affirmation that $1 - p = P(A^c)$, in which $A^c$ denotes the event of $A$ not occurring, and is read as "$A$ complement" or "not $A$."

We expect there to be many possible values for $X(10)$. Thus, it takes much more information to give its probabilistic description than it did to describe $A$. In fact, one of the simpler comprehensive ways of describing the random variable $X(10)$ is to give the probability of infinitely many events like $A$. That is, if we know $P[X(10) \le u]$ for all possible $u$ values, then we have a complete probabilistic description of the random variable $X(10)$. Thus, in going from an event to a random variable we have moved from needing one number to needing many (often infinitely many) numbers to describe the probabilities.

The stochastic process $\{X(t): t \ge 0\}$ is a family of random variables, of which $X(10)$ is one particular member. Clearly, it takes infinitely more information to give the complete probability description for this stochastic process than it does to describe any one member of the family. In particular, we would need to know the probability of events such as $[X(t_1) \le u_1, X(t_2) \le u_2, \cdots, X(t_j) \le u_j]$ for all possible choices of $j$, $t_1, \cdots, t_j$, and $u_1, \cdots, u_j$.

If one chooses to extend this hierarchy further, then a next step could be a stochastic field giving the wind speed at many different locations, with the speed at any particular location being a stochastic process like $\{X(t)\}$.

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It should also be noted that there exist special cases that somewhat blur the boundaries between the various levels of complexity in the common classification system based on the concepts of event, random variable, stochastic process, stochastic field, and so forth. In particular, there are random variables that can be described in terms of the probabilities of only a few events, or even only one event. Similarly, one can define stochastic processes that are families of only a few random variables. Within this book, we will generally use the concept of a vector random variable to describe any finite family of random variables and reserve the term *stochastic process* for an infinite (usually uncountable) family of random variables. Finally, we will treat a finite family of stochastic processes as a vector stochastic process, even though it could be considered a stochastic field.

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Example 1.2: Let the random variable $X$ denote the maintenance cost for an antenna subjected to the wind, and presume that $X=0$ if the antenna is undamaged and $5,000 (replacement cost) if it is damaged. How much information is needed to describe all probabilities of $X$?

Because $X$ has only two possible values in this simplified situation, one can describe all its probabilities with only one number— $p = P(X = 5{,}000) = P(D)$, in which $D$ denotes the event of antenna damage occurring. The only other information that can be given about the random variable $X$ is $P(X=0) = P(D^c) = 1 - p$.

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Example 1.3: Let the random variable $X$ denote the maintenance cost for an antenna structure subjected to the wind, and presume that there are two possible types of damage. Event $A$ denotes damage to the structure that supports the antenna dish, and it costs $2,000 to repair, while event $B$ denotes damage to the dish itself, and costs $3,000 to repair. Let the random variable $X$ denote the total maintenance cost. How much information is needed to describe all probabilities of $X$?

In this problem, $X$ may take on any of four values: zero if neither the structure or the dish is damaged, $2{,}000$ if only the structure is damaged, $3{,}000$ if only the dish is damaged, and $5{,}000$ if both structure and dish are damaged. Thus, one can give all the probability information about $X$ with no more than the four numbers giving the probability that $X$ takes on each of its possible values. These are easily described by using the events $A$ and $B$ and the operations of complement and intersection. For example, we might write $p_1 = P(X = 5{,}000) = P(AB)$, $p_2 = P(X = 3{,}000) = P(A^c B)$, $p_3 = P(X = 2{,}000) = P(A B^c)$, and $p_4 = P(X=0) = P(A^c B^c)$. Even this is somewhat redundant because we also know

that $p_1 + p_2 + p_3 + p_4 = 1$, so knowledge of only three of the probabilities, such as $p_1$, $p_2$, and $p_3$, would be sufficient to describe the problem.
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Example 1.4: Consider the permanent displacement of a rigid 10 meter square foundation slab during an earthquake that causes some sliding of the underlying soil. Let $X$, $Y$, and $Z$ denote the east-west, north-south, and vertical translations of the center of the slab, and let $\theta_x$, $\theta_y$, and $\theta_z$ be the rotations (in radians) about the three axes. What type of probability information is required to describe this foundation motion?

Because $\{X,Y,Z,\theta_x,\theta_y,\theta_z\}$ is a family of random variables, one could consider this to be a simple stochastic process. The family has only a finite number of members, though, so we can equally well consider it to be a vector random variable. We will denote vectors by putting an arrow over them and treat them as column matrices. Thus we can write $\vec{V} = (X,Y,Z,\theta_x,\theta_y,\theta_z)^T$, in which the $T$ superscript denotes the matrix transpose operation, and this column vector $\vec{V}$ gives the permanent displacement of the foundation. Knowledge of all the probability information about $\vec{V}$ would allow us to write the probability of any event that was defined in terms of the components of $\vec{V}$. That is, we want to be able to give $P(A)$ for any event $A$ that depends on $\vec{V}$ in the sense that we can tell whether $A$ has or has not occurred if we know the value of the vector $\vec{V}$. Clearly we must have information such as $P(X \le 100\,\text{mm})$ and $P(\theta_z > 0.05\,\text{rad})$, but we must also know probabilities of intersections like $P(X \le 100\,\text{mm}, \theta_z < 0.05\,\text{rad}, \theta_y < 0.1\,\text{rad})$, and so forth.
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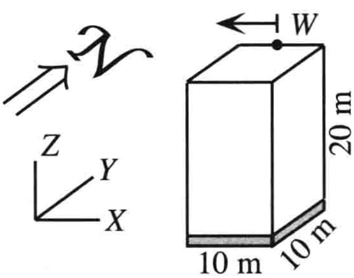
Example 1.5: Consider the permanent deformation of a system consisting of a rigid building 20 meters high resting on the foundation of Example 1.4. Let a new random variable $W$ denote the translation to the west of a point at the top of the north face of the building, as shown in the sketch. Show the relationship between $W$ and the vector $\vec{V}$ of Example 1.4.




In order to describe the random variable $W$, we need to be able to calculate probabilities of the sort $P(W \le 200\,\text{mm})$. We can see, though, that $W$ is related to the components of our vector $\vec{V}$ by $W = -X + 5\theta_z - 20\theta_y$, so $P(W \le 200\,\text{mm}) = P(-X + 5\theta_z - 20\theta_y \le 200\,\text{mm})$. It can be shown that one has sufficient information to compute all such terms as this if one knows $P(X \le u,$