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FOREWORD

With the increased use of advanced computer equipment and sophisticated programs in analyzing the inelastic behavior of pressure vessels and piping, significant R&D efforts have been directed at assuring that the analytical results are valid for hardware subjected to broad combinations of thermal and mechanical loads. One accepted method for increasing the confidence in analytical results has been to use structural benchmark problems as bases for comparisons. Here, the first step is to verify that the computer program correctly solves the programmed analytical model (equations) for relatively simple benchmark problems that have known closed-form solutions. Having been assured that the computer program does what it is supposed to do, one then qualifies the computer results by comparing them with results obtained either from another verified program or from an experiment. The latter provides bases for an acceptable solution, recognizing that the design analysis methodology involves a considerable number of variables, such as mathematical modeling, geometric discretization, description of material properties, representation of the load and temperature histories, boundary conditions, etc.

A large part of this bound volume is devoted to efforts that have been carried out to evaluate and compare various analysis methods through the use of specific structural benchmark problems. It is very gratifying to note that international specialists, well versed in the development and application of analytical techniques, have shared the concern for qualifying analytical design tools and have cooperated vigorously in bringing forth the benchmark studies included in this volume.

This volume also includes studies of specialized subjects dealing with advances that have been made in design analysis methods for elbows with end effects, perforated plates, and improved supports for Class 1 piping. On behalf of the ASME, we express sincere appreciation to the authors for their efforts in promulgating their knowledge for the benefit of their fellow engineers.

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CONTENTS

PVRC INTERNATIONAL BENCHMARK PROJECT ON SIMPLIFIED METHODS FOR ELEVATED TEMPERATURE DESIGN AND ANALYSIS: PROBLEM I – THE OAK RIDGE PIPE RATCHETTING EXPERIMENT

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ABSTRACT

Solutions are presented to the first of three benchmark problems that the Pressure Vessel Research Committee (PVRC) included in an international effort to develop comparative bases for various available high-temperature structural analysis methods. This first problem is referred to as the Oak Ridge Pipe Ratchetting Experiment and involves a pipe segment that undergoes repeated thermal transients that are separated by hold periods at an elevated temperature. The other two problems selected by the PVRC are discussed in the next three papers in this publication. A working group within the PVRC Subcommittee on Elevated Temperature Design developed a program plan and the solutions were solicited from organizations in various countries working in the elevated temperature design area. This paper summarizes the contributed solutions and comments on the comparisons. Both detailed inelastic structural analyses and simplified inelastic analyses were performed. Comparisons with experimental results are also included.

INTRODUCTION

The complexity and high cost of detailed analysis of elevated temperature equipment by the digital computer have led the pressure vessel industry to initiate an effort to develop simplified approaches which would permit the designer to carry out the necessary analyses without resorting to large scale general purpose computer programs except in unusual circumstances. As part of this effort the Pressure Vessel Research Committee's Subcommittee on Elevated Temperature Design conceived the idea of an international benchmark project in which simplified methods would be applied by various investigators to several typical problems for which experimental and/or detailed computer results were available. Three problems were selected: (I) the Oak Ridge Pipe Ratchetting Experiment, (II) the Saclay Fluctuating Sodium Level Experiment, and (III) the Oak Ridge Nozzle to Sphere Attachment Analysis.

The first problem considered and discussed here was the Oak Ridge National Laboratory's (ORNL) Pipe Ratchetting Experiment (designated TTT-1) [1].* Results for the other problems appear in the next three papers in this publication. The pipe ratchetting problem was analyzed with a finite element method by ORNL [2]

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*Numbers in brackets refer to the references at the end of this paper.

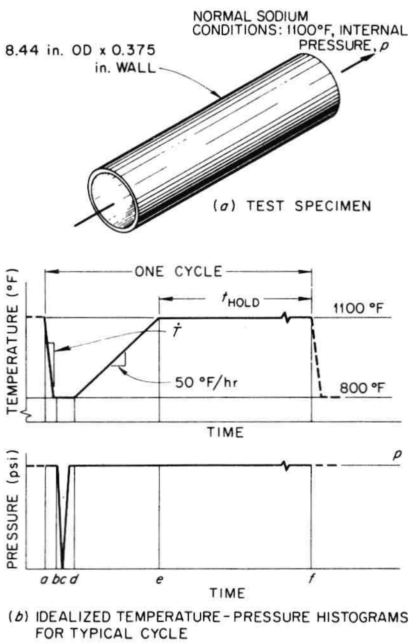
and Sandia Laboratories [3] in the United States, and by the EPICC-A Committee (Elastic Plastic Creep Analysis) of the JSME in Japan [4]. It was also analyzed with simplified approaches by O'Donnell and Porowski [5] in the United States and by Goodall and others at the CEGB in the United Kingdom [6]. The results of these studies are summarized and compared.

The presentation begins with a brief description of the problem and of the experimental results. This is followed by a presentation of the finite element results and then by a description of the simplified approaches and the results obtained by each. Finally, all of the results are compared and discussed and conclusions are drawn.

PROBLEM DESCRIPTION

The ORNL pipe ratchetting test TTT-1 was described in detail by Corum, Young, and Grindell [1] while the various mechanical properties were presented by Corum [7]. Test TTT-2 which is mentioned in Figs. 1 and 2 is not discussed here. A brief summary of this PVRC Problem I taken from Ref. 1 and is as follows.

> "The test specimen was a 30-in.-long section of pipe having an outside diameter of 8.44 in. and a wall thickness of 0.375 in. It was taken from Heat 9T2796 of type 304 stainless steel. The specimen was welded, after final annealing, to adjacent extensions of 8-in. stainless steel pipe of the same dimensions. The annealing heat treatment consisted of heating the specimens to 2000°F for 1/2 hr and then forced-air cooling rapidly to room temperature.
>
> "The *nominal* sodium temperature and pressure histories for the test are depicted in Fig. 1. The normal sodium temperature and pressure were 1100°F and 700 psi respectively during the long-term hold periods. The thermal down shock was from 1100 to 800°F at a *nominal* rate of 30°F/sec. Near adiabatic conditions were maintained on the outer surface of the specimen during each thermal transient. At 800°F, the



(c) SPECIFIC TEST CONDITIONS

$\dot{T}$ (°F/sec)	p, PRESSURE (psi)	t_{HOLD} (hr)	NO. OF CYCLES
SPECIMEN TTT-1:			
30	700	160	13
SPECIMEN TTT-2:			
22.8	400	328	5
30	700	160	13
22.8	400	328	5

Fig. 1. Specimen and nominal test conditions for pipe ratchetting tests [1].

internal pressure was removed, reapplied, and the temperature was slowly returned to 1100°F at a rate of 50°F/hr. The specimen was then held at 1100°F and subjected to the internal pressure of 700 psi for a period of 160 hr before the next transient was initiated. The total cycle time was 168 hr (1 week), with approximately 2 hr spent at 800°F and 6 hr spent in heating from 800 to 1100°F. The first specimen was subjected to a total of 13 cycles.

"The idealized ramp transient shown in Fig. 1 was not actually obtained in the tests. The measured sodium thermal transients imposed on the pipe specimens were more realistic and are shown in Fig. 2. These transients were measured at the center of the test sections. They were reproducible from cycle to cycle and test to test, and they should be used as the thermal transients in inelastic analyses. There was a slight temperature gradient along the specimen axis during the actual transients (about 1.3°F/in. in the most severe transient). It is believed that this gradient which was approximately linear, can be neglected in benchmark analyses.

"The test facility in which the specimens were tested was designed to virtually eliminate piping reactions in the pipe ratchetting test assemblies, and measurements of strains during heatup of the system indicated that if any net reactions did exist, they were very small. Thus, each test specimen can be analyzed as an infinitely long straight pipe with only the internal pressure loading (due to the pressure acting on the inside area of the specimen) in the axial direction.

"The measured ratchetting behavior for the first pipe specimen is shown in Fig. 3, where the circumferential strain on the outer surface is plotted as a function of the accumulated hold time at 1100°F. Data points from two capacitive gages located 90 deg apart near the midlength of the 30-in. test section are shown. Initial pressurization data and data corresponding to the minimum strain during each transient are included.

"The response of the specimen indicated substantial ratchetting in the early cycles, but the incremental growth per cycle decreased continually with an increasing number of cycles. The net strains from the two circumferential capacitive gages on the specimen agree reasonably well with each other although there does seem to be a consistent difference [gage 513 generally indicates more plastic (time-independent) strain and less creep (time-dependent) strain than gage 514]. These differences are thought to be real. A possible explanation is the

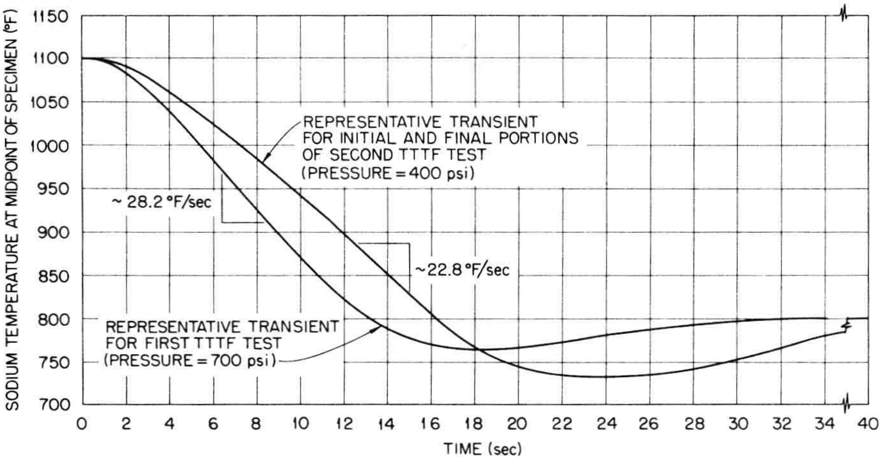


Fig. 2. Actual measured sodium temperature history [1].

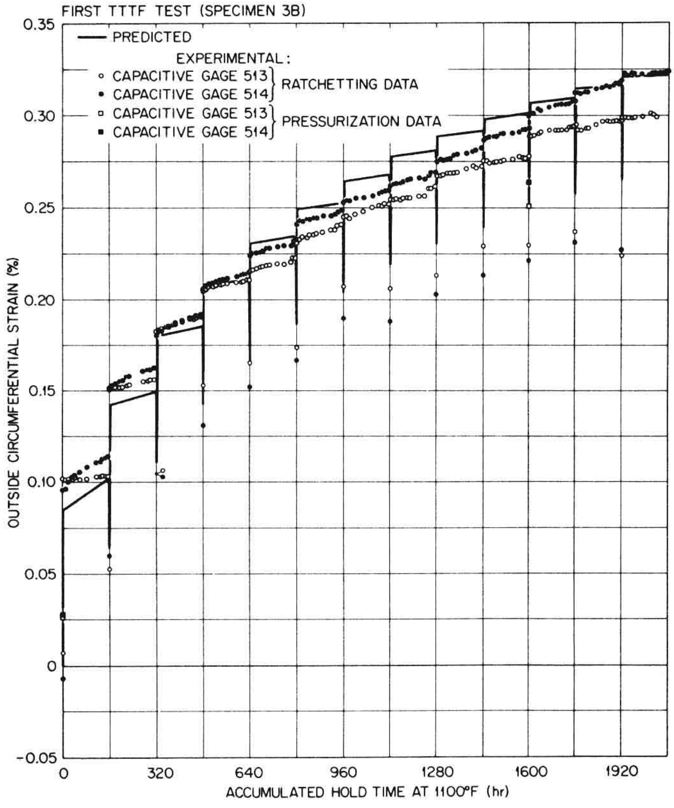


Fig. 3. Comparison of measured and predicted circumferential strains at the outer surface of the pipe by ORNL [2].

significant variation in properties that has been found around the circumference of the 8-in. pipe product from which the ratchetting specimens were obtained. Also, although extreme care was taken to support the specimens so that negligible end reactions existed, it is possible that very small reactions did reach the specimens. Although small, these could have a significant influence, particularly during creep periods."

FINITE ELEMENT RESULTS

The problem described above was analyzed with the finite element method by three groups. These were Clinard, Corum, and Sartory of ORNL [2], Swenson of the Sandia Laboratories [3], and the EPICC-A Committee of the JSME [4]. All of these groups were given the mechanical properties published for the heat of material by Corum [7]. Measurements of the actual *specimen* material properties were not used in this study.

ORNL Results

Clinard et al. [2] used the ORNL program PLACRE. The mathematical basis of the plasticity and creep models that are used in this program are also described in Ref. 2. Briefly, the plasticity model uses linear kinematic hardening with the von Mises yield condition and associated flow rule. Cyclic growth of the yield surface is also accommodated. This makes for combined kinematic and isotropic hardening. The creep is handled by an equation of state formulation using rules for cyclic hardening. The computer results for the circumferential strains at the outer surface are shown in Fig. 3 along with the experimental data. The agreement is excellent, particularly at the end of the test.

Sandia Results

Swenson [3] of Sandia used the MARC [8] finite element program with an axisymmetric (type 10) element. This element is an arbitrary quadrilateral element with a linear displacement variation along each edge. It was also used for the heat conduction analysis whose results are shown in Fig. 4 along with the temperature measured during the test. The agreement is excellent. For the mechanical problem generalized plane strain boundary conditions were imposed. For the plasticity calculation kinematic hardening was used without the cyclic growth of the yield surface that ORNL used. The uniaxial material stress-strain response was bilinear and defined by Young's modulus, the yield stress and the plastic work hardening slope as reported by Corum [7]. The creep data for the material were also obtained from Ref. 7 and were programmed directly into the MARC CRPLAW subroutine without fitting the data to an equation. A strain hardening creep response was assumed. For times above 10,000 h, the slope between 5000 and 10,000 h was extended linearly.

The results for the circumferential strain at the outer surface are shown as a function of time in Fig. 5 along with the experimental data and the ORNL calculations. The comparison between the Sandia results and the ORNL results is generally good. However, the Sandia results fall consistently below the ORNL test data and calculated results. These differences can be accounted for by the differences in the finite element model (ORNL used a finer mesh), in the material hardening rule (ORNL used cyclic growth of the yield surface), and the programming of the creep data (ORNL apparently used a smaller creep rate above 10,000 h).

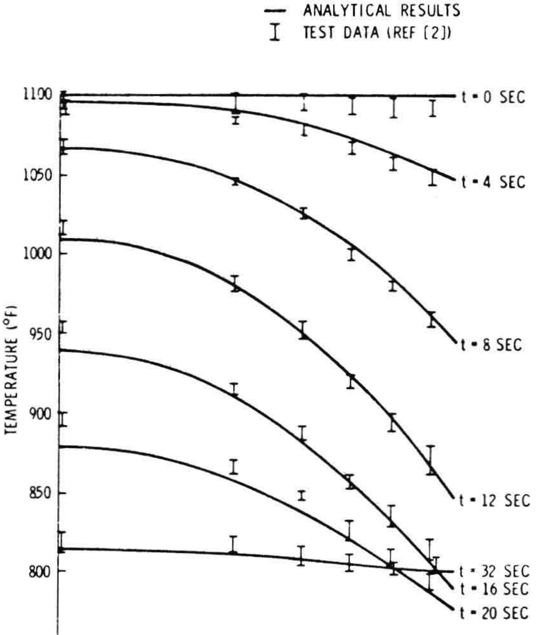


Fig. 4. Measured and calculated (Sandia) temperatures through the wall of the pipe [3].

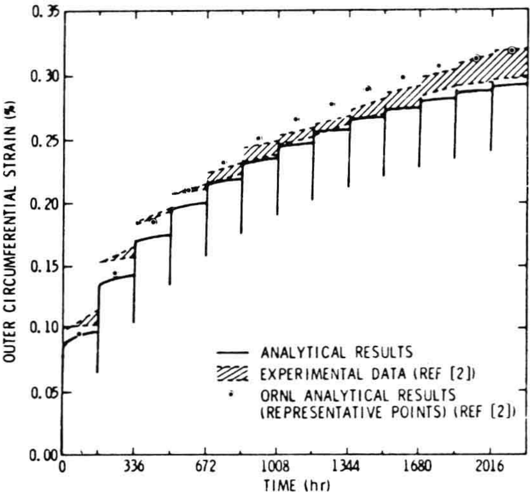


Fig. 5. Comparison of measured and calculated (ORNL and Sandia) circumferential strains at the outer surface of the pipe [3].

EPICC-A Committee Results

Yamada and Nagato [4] reported on the results obtained by five teams of Japanese investigators who used various computer programs as follows: (1) Y. Yamamoto and H. Matsunaga of the Toyo Information Systems Co., Ltd. (MARC, both [8]), (2) T. Mori and T. Murakami of the Toshiba Corp. (HEACON-1D [9], PCRAP-2 [10]), (3) K. Nagato and A. Minato of Kawasaki Heavy Industries, Ltd. (THETA-2D [11], MINAT-X [12]), (4) M. Ueda and M. Tanikawa of the Hitachi Shipbuilding and Engineering Co., Ltd. (SATEPIC, both [13]), and (5) Y. Yokouchi, A. Ishii, and T. Edamoto of the University of Electro-Communications (ASTUC, both [14]). The programs listed in parentheses are for the thermal transient and elastic-plastic-creep, respectively. Of these MARC, THETA-2D, and ASTUC are finite element programs, SATEPIC is a finite difference program, whereas HEACON-1D is based on a thermal layer approximation.

All of the elastic-plastic-creep analysis programs are based on the finite element method and adopt the tangent stiffness method for the elastic-plastic analysis. MARC, PCRAP-2, and SATEPIC allow an iterative computation at each incremental loading step. The iterative scheme of SATEPIC incorporates a procedure that makes an adjustment of the strain hardening rate so that the computed solution is on the input stress-strain curve at the stage of initial yielding. PCRAP-2 performs iterations twice at each incremental step. MINAT-X includes no scheme for iterative calculation, while ASTUC employs a load or time increment adjustment at the elastic-plastic transition. Translations of the yield surface in case of the kinematic hardening postulate are calculated by Prager's rule in MARC and by Ziegler's rule in the other four Japanese programs.

In the finite element thermal transient analysis, axisymmetric four-node quadrilateral rings (element 40) are used in MARC and axisymmetric triangular elements in THETA-2D.

In the elastic-plastic-creep analysis by MARC, quadrilateral axisymmetric rings (element 10) are used and the outputs of the stresses and strains are given at the centroid of the element. PCRAP-2 employs axisymmetric generalized plane strain quadrilateral ring elements and the stresses and strains are also given at the centroid of the element. In the case of MINAT-X and SATEPIC, isoparametric four-node rings with 9 integration points are used and the stresses and strains are evaluated at the central integration point. When the temperature dependency of the material behavior is considered, temperatures at four nodes of each element are averaged to have the mean temperature at the element centroid and used subsequently in the stress and strain analyses. It merits noting here that ASTUC is a program specially designed for the analysis of stresses and/or temperatures in a uniform cylinder using a simplified element having a single degree of freedom in the radial direction and an axial displacement degree of freedom which is common to all of the elements. This is essentially the same element as used in PLACRE [2].

All of the Japanese groups used a common set of mechanical properties and an equation of state creep representation that were derived from the data reported by Corum [7]. They all used combined kinematic and isotropic hardening for the plasticity calculation.

The results of the thermal transients and elastic-plastic-creep analyses are compared with the measured temperature distributions through the wall of the pipe at various times during the down shock and the measured circumferential ratchetting strains on the outer surface of the pipe of thermal ratchetting test TTT-1 in Figs. 6 and 7. The following conclusions are obtained from the analyses.

Temperature distributions through the wall of the pipe as predicted by the finite element and finite difference method computer programs correlated satisfactorily with the measured ones. On the other hand, the results by HECON-1D using the thermal layer approximation were slightly different from the measured ones at intermediate stages, i.e., 8 and 12 sec, of the transient.

Thermal ratchetting strains at the first loading cycle predicted by the five elastic-plastic-creep analysis programs agree well with the measured ratchetting strain data. The deviation of the numerical results from the experimental data gradually increases on and after the second loading cycle. It is noted, however, that the deviation, which shows a maximum around the sixth loading cycle, diminishes and the results recover coincidence with the experimental data around the tenth loading cycle.

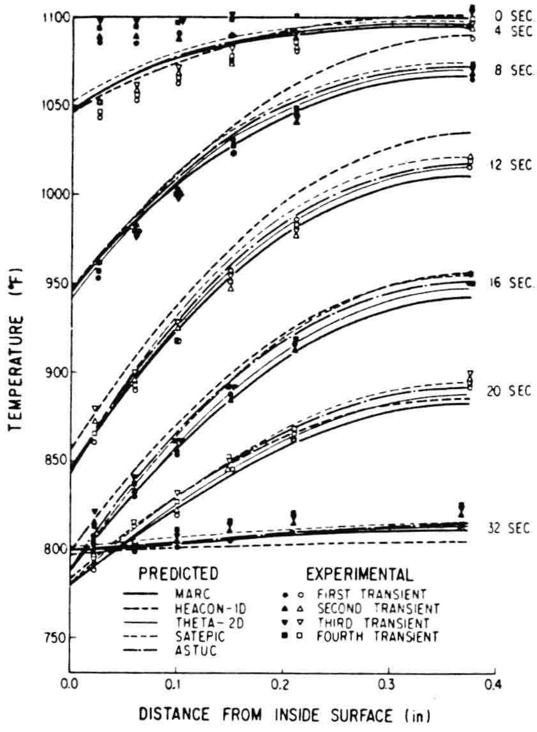


Fig. 6. Measured and calculated (EPICC-A) temperature through the wall of the pipe [4].

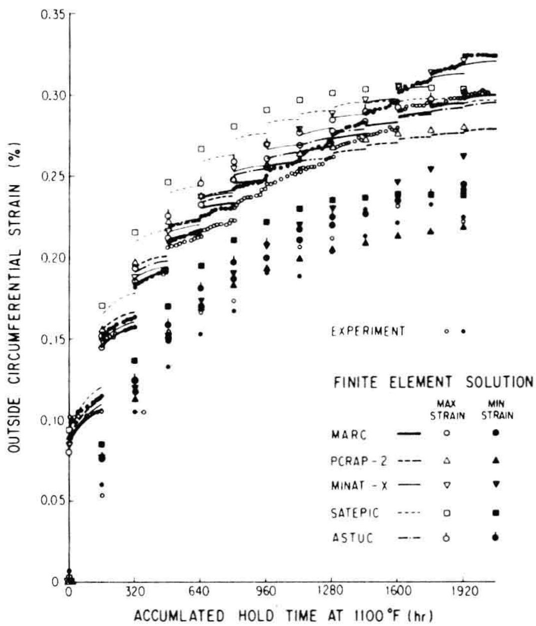


Fig. 7. Comparison of measured and calculated (EPICC-A) circumferential strains at the outer surface of the pipe [4].

SIMPLIFIED ANALYSES AND RESULTS

Two groups presented simplified analyses of this problem: O'Donnell and Porowski [5], of O'Donnell and Associates, and Goodall et al. [6] of the Central Electricity Generating Board. The chief distinction between these, that can be pointed out at the outset, is that the former used elastically calculated stresses while the latter used a cyclic plasticity solution in their estimates of the ratchetting problem. Individual descriptions of each method and the results obtained will now be presented.

O'Donnell and Porowski Solution

This solution is based upon the bounding technique derived by O'Donnell and Porowski [15] which is now a part of ASME Code Case N-47 for Elevated Temperature Design. The authors recently modified this method [16] to obtain better bounds on the creep strains. The analysis is based on a one dimensional elastic solution and proceeds as follows.

The maximum primary stress occurs in the hoop direction and is given for a cylinder by

$$\sigma_p = pr/a ,$$

where p is the pressure, r is the inside radius, and a is the wall thickness. Thus, for the dimensions and pressure of the problem

$$\sigma_p = 7177 \text{ psi} .$$

The maximum linearized (per ASME Code Case) thermal stress was calculated [5] to be

$$\sigma_t = 34{,}044 \text{ psi} .$$

The primary and secondary stress parameters X and Y defined for use with ASME Code Case N-47, Figure T-1324-1 (the so-called "Bree Diagram" [17] which is reproduced here as Fig. 8) are now obtained as

$$X = \frac{\sigma_p}{\overline{S}_y} = \frac{7177}{15,900} = 0.451$$

$$Y = \frac{\sigma_p}{\overline{S}_y} = \frac{34,044}{15,900} = 2.14 \ ,$$

where $\overline{S}_y$ = 15,900 psi is the yield strength at the average temperature in the cycle; i.e., T = 950°F. These coordinates define a point that falls into the "P" region of Fig. 8. From the above the Code Case defines an effective creep stress parameter

$$Z = X \cdot Y = 0.967 \ ,$$

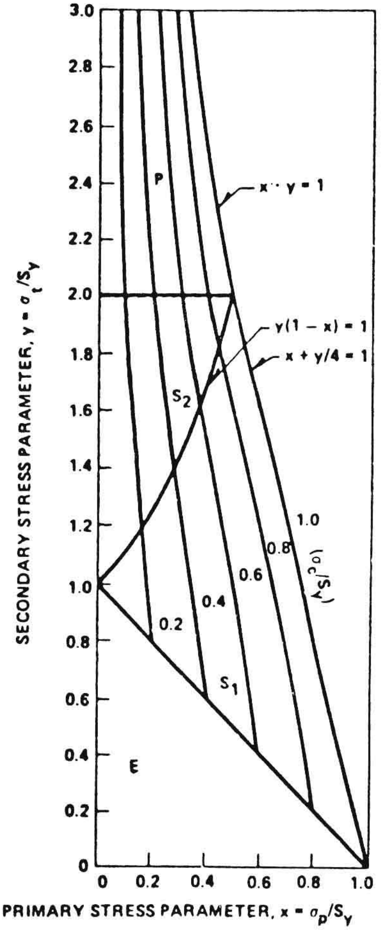


Fig. 8. Interaction diagram for constant primary stress and cyclic secondary stress for a tube (the so-called Bree diagram adopted in ASME Code Case N-47).

and an effective stress in the elastic core of the tube of

$$\sigma_c = Z\overline{S}_y = 0.967(15{,}900) = 15{,}400 \text{ psi} .$$

Since this core stress is larger than the yield stress at the hot extreme of the cycle (S_{yH} = 14,700 @ 1100°F), plastic strain increments will occur and the bounding technique of Ref. 15 is not applicable. For such situations O'Donnell and Porowski have derived an improved bounding technique [16]. This method extends the earlier method to include work hardening and temperature dependent yield properties. It also introduces the idea of partial relaxation of the stress in the elastic core of the wall as opposed to the earlier model used, for example, by Bree [17] in which complete relaxation was postulated. The newer technique permits the designer to take credit for the higher residual stresses which can be retained during shutdown because of the higher yield strength of the material at the low temperature. To begin the application of the new technique, we observe that according to the ASME Code Data:

Yield strength at cold extreme:

$$S_{yL} = \overline{S}_y = 15{,}900 \text{ psi} .$$

Yield strength at hot extreme:

$$S_{yH} = 14{,}700 \text{ psi} .$$

The primary and secondary load parameters and consequently the effective creep parameter Z and core stress at the cold extreme are identical to those obtained previously, or

$$Z_L = Z = 0.967$$

$$\sigma_{cA} = \sigma_{cL} = 15{,}400 \text{ psi} .$$

The corresponding values for the hot extreme are obtained from Fig. 2 of Ref. 16 as:

$$X_H = \sigma_p/S_{yH} = 7177/14{,}700 = 0.4882$$

$$Y_H = \sigma_t/S_{yH} = 34{,}044/14{,}700 = 2.3159$$

$$Z_H = X_H Y_H = 1.13$$

$$\sigma_{cH} = Z_H \cdot S_{yH} = 16{,}620 \text{ psi} .$$

The plastic strain increment per cycle is given by Eq. 18 of Ref. 16 as:

$$\eta_{(n)} = \frac{1}{E}[(\sigma_{cL} - S_{yL}) + (\{\sigma_{cH}\} - S_{yH}) = \frac{1}{21.7 \cdot 10^6}[(15{,}367 - 15{,}900)$$

$$+ (16{,}621 - 14{,}700)] = 0.00639\% ,$$

where the elastic modulus at 1100°F is $E = 21.7(10^6)$ psi.

The mid-plane stress at the beginning of the cycle is equal to the yield strength at the hot extreme:

$$\sigma_{c(n)} = S_{yH} = 14{,}700 \text{ psi} .$$

Assuming an intermediate relaxation stress σ_l = 11,000 psi, the inelastic strain increment due to relaxation and enhanced creep is given by Eq. 20 of Ref. 16 as:

$$\delta(n) = \frac{1}{E}\frac{\sigma_c^2 - \sigma_l^2}{\sigma_l} = \frac{S_{yH} + \sigma_l}{\sigma_l}(S_{yH} - \sigma_l)$$

$$= \frac{1}{21.7}\frac{14,700 + 11,000}{11,000}(14,700 - 11,000)10^{-6} = 0.0398\% \ .$$

The creep strain at the constant stress σ_l = 11,000 psi within 2080 h of hold time can be obtained from Table 2 in Ref. 7:

$$\nu \le 0.195\% \ .$$

Thus, the total inelastic strain accumulated for test TTT-1 using Eq. 22 of Ref. 16 is

$$\Sigma\varepsilon_{(n)} = \Sigma\nu + n[\eta_{(n)} + \delta_{(n)}] = [0.1950 + 13(0.00639 + 0.03980)]10^{-2} = 0.795\%$$

since the component $\delta_{(n+1)}$ included in Ref. 16 for uniform cycles is zero.

The foregoing analysis pertained to perfect plasticity. For kinematic hardening the analysis had to be carried out cycle by cycle and the resulting accumulated strain was reduced to 0.76% compared to the measured value of 0.32%. Since the resulting strain in the simplified analysis was, nevertheless, below the 1% limit on accumulated strain that is specified in ASME Code Case N-47, the analysis was stopped. Improvements could have been obtained by trying several values of the relaxation stress σ_l, since the authors [16] have shown that the use of any $\sigma_p \le \sigma_l \le \sigma_c$ gives the upper bound strain results. The optimal choice of σ_l minimizes total strain computed with Eq. 22 of Ref. 16 as given above.

Central Electricity Generating Board Solution

The analysis used is the bounding method described by Ainsworth [18]. The general upper bound of Ref. 18 (inequality 21) is for behavior in the steady cyclic state whereas the ORNL test is concerned with behavior prior to reaching the steady cyclic state. The modification of Ref. 18 to include the transitional behavior leads to

$$\int_0^T \int_S R_i \dot{u}_i^* dSdt \le \int_0^T \int_S R_i \dot{u}_i^* dSdt + A(0) - A(T)$$

$$+ \frac{1}{n}\int_0^T \int_V \dot{D}(n\sigma_{ij}^*/(n + 1)\sigma_0)\ dV\ dt$$

where

$$A = \frac{1}{2}\int_V \{(\sigma_{ij}^* - \sigma_{ij})(e_{ij}^* - e_{ij}) + m(p_{ij}^* - p_{ij})(p_{ij}^* - p_{ij})\}dV \ .$$

The notation is that of Ref. 18. Here the actual displacements u_i in the line of additional loads R_i are bounded by the displacements u_i^* and stresses σ_{ij}^* which occur in a structure subject to the actual load/temperature history plus the loads R_i. This fictitious structure has the same elastic-plastic behavior as the material of the actual structure but is assumed not to creep. The load history need not be cyclic and the bound can be evaluated for any time, T, of interest. For cyclic loading the stress and plastic strain histories eventually became cyclic so that identifying times 0, T as the beginning and end of the cycle leads to $A(0) = A(T)$ and inequality 21 of Ref. 18 is recovered.

In general $A(T)$ is undetermined, but since A is positive-definite, the term $A(T)$ can be omitted from the bound. The actual behavior is then determined from the initial conditions and from the fictitious elastic-plastic history. As in Ref. 18 it is possible to optimize the bound for both the magnitude of additional loading and the initial state $\sigma_{ij}^*(0)$, $p_{ij}^*(0)$.

For the present problem the additional loading R_i is taken as a constant pressure R. Optimization for the initial state is not attempted and $\sigma_{ij}^*(0)$ is taken simply as that resulting elastically from the pressure R. Since the actual initial state is $\sigma_{ij}(0) = p_{ij}(0) = 0$

$$A(0) = \frac{1}{2} \int_V \sigma_{ij}^* e_{ij}^* dV = \frac{1}{2} \int_S R_i u_i^*(0) dS \ .$$

The initial displacement $u(0) = 0$ so that the displacement at time T is bounded by

$$u(T) \leq u^*(T) - \frac{1}{2} u^*(0) + (1/nR) \int_0^T \int_V \dot{D}(n\sigma_{ij}^*/(n + 1)\sigma_0) dV\, dt \ .$$

The stress σ_{ij}^* is constant during the hold period so that for any cycle $\int \dot{D}\, dt$ is the product of σ_{ij}^* and the creep strain due to σ_{ij}^* in the hold time. The value of σ_{ij}^* during the hold period will change from cycle to cycle for kinematic hardening and the total time integral is obtained from the sum of the cycles.

In order to evaluate the bound, an elastic-plastic solution is required. For the purposes of analysis, the pipe is represented by an uniaxial model. The equi-biaxial behavior under thermal loading is modeled by using an elastic modulus $E/(1 - \nu)$ and a plastic modulus $2E_p$ where ν is Poisson's ratio and E, E_p are the elastic and plastic moduli in an uniaxial test.

In the uniaxial model the total strain, ε, was taken constant through the thickness and represents the mean hoop strain in the pipe. An applied stress represents the hoop stress arising from the pressure. The elastic-plastic analysis was performed using a small computer program. The stress distribution was described by values at a number of points (41) through the thickness. Given the change in thermal strain, $\Delta\theta$ (assumed quadratic through the thickness) and an estimate of the change in total strain, $\Delta\varepsilon$, the change in elastic-plus-plastic strain $(\Delta\varepsilon - \Delta\theta)$ was obtained at each point. From this the change in stress at each point was found and an iterative procedure was used to determine that value of $\Delta\varepsilon$ for which the mean change in stress balances any change in the applied pressure. The numerical method was similar to that described in Section 3.1 of Ref. 19 except that no creep analysis was required. The relationship between stress change and elastic-plus-plastic strain change is given on p. 984 of Ref. 19.

Performing an elastic-plastic analysis at each change in pressure and/or temperature yielded the stress history σ^* and the mean hoop strain (or U^*). Creep strains required for the bound only occur during the dwell period when the elastic-plastic stress σ^* is constant with time at each point (equal to σ_h^*, say). For any one cycle $\int \dot{D}\, dt$ is then simply the product of $n\sigma_h^*/(n + 1)$ and the creep strain due to $n\sigma_h^*/(n + 1)$ in the hold time. Creep strains have been obtained directly from the data presented by Corum [7] for which a stress index $n = 2.8$ is appropriate for the times and stress range of interest. The value of σ_h^* changes from cycle to cycle and the total time integral required has been obtained from the sum of the cycles using a time-hardening rule. Integration through the thickness using Simpson's rule yields the volume integral.

Optimization of the bound was performed by repeating the elastic-plastic solution for a number of different values of constant additional pressure R. The bound was found to be not particularly sensitive to R varying by less than 20% for R between 10% and 50% of the applied pressure. The optimum value of R was 150 psi which is about 20% of the peak applied pressure.

The results of the upper bound calculation are shown along with the experimental results in Fig. 9. The agreement is quite good.