Jean-Jacques Laffont

Fundamentals of Public Economics

translated by

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Revised English-language edition

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Preface

This book is the first of a series of volumes intended to be used in a year-long course in economic theory designed for advanced undergraduate or graduate students. Each volume can be read independently from the others. In the introduction to the present book, I will review the background that I assume of the reader.

Avoiding whenever possible complicated mathematics, I have sought to make available to the student a treatise in microeconomic theory that takes into account the latest developments. The chapters end with optional (starred) sections that may be skipped in an initial reading and with lists of references and recommended readings that will allow the student to delve more deeply into the topics that have been discussed. These readings will fill in some of the gaps in my presentation and encourage the student to do further research. The volume ends with a series of exercises, which are preceded by a series of worked problems. These problems and exercises will help the student evaluate his or her understanding of the course.

The students of l'Ecole Nationale de la Statistique et de l'Administration Economique (ENSAE) and of the Master's Program in Econometrics at the Université des Sciences Sociales de Toulouse have greatly contributed to this work. I am also grateful to B. Belloc, M. Boyer, S. Barbera, C. Crampes, X. Freixas, L. A. Gérard-Varet, R. Guesnerie, A. Grimaud, F. Laisney, M. Moreaux, S. Moresi, P. Picard, and M. Salles for their helpful remarks on certain chapters. Finally, I wish to thank P. Champsaur ad G. Laroque for allowing me to use certain exercises they devised while teaching at ENSAE.

Mathematical Notation, Definitions, and Results

Notation

$\exists$	there exist(s)
$\forall$	for any
$\in$	belongs to
$\subset$	is a subset of
$\Leftrightarrow$	if and only if
$\Rightarrow$	implies

A function is said to be C^n if it is n times continuously differentiable.
Let $x \in \mathbf{R}^n$, $y \in \mathbf{R}^n$. Then

$$x \cdot y = \sum_{l=1}^{n} x_l y_l$$

denotes the inner (dot) product.

$\lvert A \rvert$	number of elements in the set A
$\propto$	proportional to, with a positive multiplicative constant

Definitions

1. A binary relation R^i defined on X^i is

reflexive $\Leftrightarrow \forall x^i \in X^i, \quad x^i R^i x^i$.

transitive $\Leftrightarrow \forall x^{i1}, x^{i2}, x^{i3} \in X^i$,

$$x^{i1} R^i x^{i2} \text{ and } x^{i2} R^i x^{i3} \Rightarrow x^{i1} R^i x^{i3}.$$

complete $\Leftrightarrow \forall x^{i1}, x^{i2} \in X^i, \quad x^{i1} R^i x^{i2}$ or $x^{i2} R^i x^{i1}$.

A preordering is a reflexive and transitive binary relation. Since we always consider a preordering to be complete, we say that a preordering is a complete, reflexive, and transitive binary relation.

2. A set $Y \subset \mathbf{R}^L$ is convex $\Leftrightarrow \forall x \in Y, \forall y \in Y, \forall \lambda \in [0, 1]$,

$\lambda x + (1 - \lambda) y \in Y$.

Fundamentals of Public Economics

Contents

Introduction

Before beginning this book, the reader should have a complete understanding of the two fundamental theorems of welfare economics derived from the "basic microeconomic model." These theorems are developed, for example, in the first five chapters of Edmond Malinvaud's *Lectures in Microeconomic Theory*. In this introduction, I review briefly their significance after presenting the notation of the basic model that will be used throughout the book. This volume is devoted to some fundamental problems of public economics, with the problems themselves presented in terms of welfare economics.

I.1 Notation

The economy consists of L economic goods indexed by $l = 1, \ldots, L$, I consumers indexed by $i = 1, \ldots, I$, and J firms indexed by $j = 1, \ldots, J$. The indices corresponding to economic agents will always be superscripts and those corresponding to goods will be subscripts.

Let X^i be the consumption set for consumer i; this set is often taken to be the positive orthant $\mathbf{R}^L_+$.[1] The quantity consumed of good 1 by consumer i is represented by x^i_l, and $x^i = (x^i_1, \ldots, x^i_L) \in X^i$ characterizes the consumption bundle of consumer i. Consumer i's preferences are represented either by a preordering (that is, by a complete, reflexive, transitive binary relation)[2] denoted by R^i, or by a utility function denoted by $U^i(.)$. Then $x^{i1} R^i x^{i2}$ means: consumer i either prefers the bundle of goods x^{i1} to the bundle of goods x^{i2} or is indifferent between them. Substituting P^i for R^i indicates strict preference. The preordering R^i is represented by a utility function $U^i(.)$ if and only if

$$x^{i1} R^i x^{i2} \Leftrightarrow U^i(x^{i1}) \geqslant U^i(x^{i2})$$

$$x^{i1} P^i x^{i2} \Leftrightarrow U^i(x^{i1}) > U^i(x^{i2}).$$

Consumer i's initial endowment is denoted by $w^i \in \mathbf{R}^L_+$. Let $y^j = (y^j_1, \ldots, y^j_L)$ be the production vector for producer j. We usually follow the convention that outputs (products) have a positive sign and inputs (factors) a negative sign. Essentially, this convention permits us to write the profit of firm j as the inner (dot) product

1. $\mathbf{R}^L_+ = \{x : x \in \mathbf{R}^L,\ x_l \geqslant 0,\ l = 1, \ldots, L\}$.
2. See the mathematical definitions, p. ix.

$$p \cdot y^j = \sum_{l=1}^{L} p_l y_l^j$$

where $p \in \mathbf{R}_+^L$ specifies the price vector.[3] The technology of firm j is represented either by the production set $Y^j \subset \mathbf{R}^L$, or by the production function $f^j(y^j) = 0$. When the firms are privately owned, θ^{ij} indicates the share of firm j owned by consumer i, $j = 1, \ldots, J$, $i = 1, \ldots, I$.

I.2 The Fundamental Theorems of Welfare Economics

A *private property competitive equilibrium* is characterized by a price vector $p^* \in \mathbf{R}_+^L$ and an allocation $(x^{*1}, \ldots, x^{*I}; y^{*1}, \ldots, y^{*J})$ such that

(i) y^{*j} maximizes profit $p^* \cdot y^j$ in the production set Y^j, that is,

$$p^* \cdot y^{*j} \geqslant p^* \cdot y^j \quad \text{for any } y^j \text{ in } Y^j \qquad j = 1, \ldots, J;$$

(ii) x^{*i} maximizes utility $U^i(x^i)$ in the budget set given by

$$B^i = \left\{ x^i : x^i \in X^i \text{ and } p^* \cdot x^i \leqslant p^* \cdot w^i + \sum_{j=1}^{J} \theta^{ij} p^* \cdot y^{*j} \right\} \qquad i = 1, \ldots, I;$$

(iii) supply equals demand on all markets:

$$\sum_{i=1}^{I} x^{*i} = \sum_{j=1}^{J} y^{*j} + \sum_{i=1}^{I} w^i.$$

The assumption of competitive behavior indicates that each agent takes prices as given; we say that he exhibits parametric behavior with respect to prices. We justify this assumption by modeling the economy with a large number of economic agents so that each agent is "negligible."

An allocation $(x^1, \ldots, x^I; y^1, \ldots, y^J)$ is said to be *feasible* if and only if

(i) $x^i \in X^i$ for $i = 1, \ldots, I$,

(ii) $y^j \in Y^j$ for $j = 1, \ldots, J$,

(iii) $\displaystyle\sum_{i=1}^{I} x^i \leqslant \sum_{j=1}^{J} y^j + \sum_{i=1}^{I} w^i.$

3. When the consumer supplies labor, the labor component of his consumption bundle also has a negative sign. Then the consumption set cannot be characterized by $\mathbf{R}_+^L$.

A *Pareto optimum* is a feasible allocation $(x^{*1}, \ldots, x^{*I}; y^{*1}, \ldots, y^{*J})$ such that there exists no other feasible allocation $(\tilde{x}^1, \ldots, \tilde{x}^I; \tilde{y}^1, \ldots, \tilde{y}^J)$ that would give at least as much utility to all consumers and more utility to at least one consumer, such that

$$U^i(\tilde{x}^i) \geqslant U^i(x^{*i}) \qquad i = 1, \ldots, I,$$

and there exist i' such that

$$U^{i'}(\tilde{x}^{i'}) > U^{i'}(x^{*i'}).$$

In this book, we will make strong assumptions on the preferences and on technology in order to facilitate the presentation of specific problems fundamental to public economics. In this spirit, the two fundamental theorems of welfare economics are presented with convenient assumptions.

THEOREM 1 If $U^i(.)$ is strictly increasing with respect to each of its arguments for $i = 1, \ldots, I$, a private property competitive equilibrium (if it exists) is a Pareto optimum.

To be able to prove the existence of a private property competitive equilibrium, we must make much stronger assumptions. However, the Pareto optimality property of the competitive equilibrium (if it exists) is quite general and can be grasped intuitively. Theorem 1 indicates that equilibrium price signals are sufficient to coordinate decentralized economic activities in a satisfactory way according to the Pareto criterion. By his individual maximization behavior, each economic agent responds to prices by equating his marginal rates of substitution (for consumers) and transformation (for firms) to these prices. Since all agents face the same prices, all the marginal rates are equated to each other in the equilibrium. Combined with market equilibria, these equalities characterize Pareto optima in a convex environment.[4]

Although this intuition is useful in interpreting the role of prices, it does not help us understand the optimality of the competitive equilibrium (when it exists) in nonconvex environments. For this purpose, a very simple argument by contradiction will suffice. If the competitive equilibrium is dominated by a feasible allocation (refer to the definition of a Pareto optimum), then the value of the consumption bundle for consumer i' in this

4. By a convex environment, I mean nonincreasing returns for firms and convex preferences for consumers.

new allocation, at the competitive equilibrium prices, is greater than the value of his endowment (otherwise he would have chosen this allocation in the competitive equilibrium). For every other consumer, the value of his consumption bundle in the new allocation, at the competitive equilibrium prices, must be at least as large as the value of his endowment. Consequently the new allocation cannot be a feasible allocation.

The second theorem, although just as fundamental, is more difficult to understand intuitively.

THEOREM 2 If $U^i(.)$ is continuous, quasi-concave[5] and strictly increasing on the consumption set $X^i = \mathbf{R}^L_+$ with $w^i_l > 0, l = 1, \ldots, L, i = 1, \ldots, I$, and if Y^j is convex, $j = 1, \ldots, J$, for any given Pareto-optimal allocation $(x^{*1}, \ldots, x^{*I}; y^{*1}, \ldots, y^{*J})$, there exists a price vector $p^* \in \mathbf{R}^L_+$ such that

(i) x^{*i} maximizes $U^i(x^i)$ in the set

$$\{x^i : x^i \in \mathbf{R}^L_+,\ p^* \cdot x^i \leqslant p^* \cdot x^{*i}\} \qquad i = 1, \ldots, I,$$

(ii) y^{*j} maximizes $p^* \cdot y^j$ in Y^j, $j = 1, \ldots, J$.

Thus, under the convexity assumptions, Pareto-optimal allocations may be decentralized in the following sense. If we give each consumer an income[6] of $R^{*i} = p^* \cdot x^{*i}$ and if we announce to all economic agents the price vector p^*, profit maximization by firm j, $j = 1, \ldots, J$, and utility maximization by consumer i subject to the budget constraint $p^* \cdot x^i \leqslant R^{*i}$, $i = 1, \ldots, I$, lead to consumption and production plans that are compatible and that coincide with the chosen Pareto-optimal allocation. This result is fundamental to understanding decentralized planning and can be interpreted in a private property economy as follows. Pareto optimality of the private property competitive equilibrium is satisfactory with respect to the efficiency criterion but it may lead to undesirable income distributions. The second theorem states: whichever Pareto optimum we wish to decentralize (therefore, whichever Pareto optimum corresponds to the justice criterion taken), it is possible to decentralize this allocation as a competitive equilibrium so long as the income of the agents is chosen appropriately, that is, in a private property economy so long as the appropriate lump-sum transfers are made.

5. See the mathematical definitions, p. ix.
6. R is used to denote an income as well as a preordering.

I.3 The Purpose of this Volume

Taken together, the two theorems make up the theoretical foundation of liberal thought; we therefore need to examine the range of application and the robustness of these results. This volume considers the following particular problems.

(1) The basic model ignores very important issues such as externalities (for example, pollution), the multitude of public goods that are not exchanged in market transactions, and fundamental nonconvexities, in particular production under increasing returns. We study the degree to which these complexities invalidate the fundamental theorems of welfare economics in a private property economy. Then, we look at instruments of public intervention designed to restore the Pareto efficiency of the competitive mechanism or to achieve Pareto optimality through planning.

(2) Our search for criteria to evaluate public intervention having allocational or distributional goals will lead us to reflect on the theory of the government as an aggregator of individual preferences as well as on the difficulties faced by any collective decision-maker in obtaining information in a world where information is decentralized.

(3) For most of this book, we adopt a dichotomous approach in the spirit of theorems 1 and 2. We treat allocational and distributional problems separately by invoking the feasibility of lump-sum transfers, which are very powerful instruments of political economics. However, the need to treat both problems simultaneously in many situations emerges as a major conclusion to this volume, one which is analyzed in detail in chapter 7.

Since they require a more complex benchmark than the basic microeconomic model provides, many problems of public economics are not considered in this book. Public economics problems that arise from an absence of future markets (for example, the choice of a social discount rate) would be topics for a book that concerned itself with the extension of the basic microeconomic model to an intertemporal framework. Problems that arise from an absence of contingent claims markets (for example, consumer quality protection) would be treated in a book on the economics of uncertainty and information. Finally, the main part of our analysis is undertaken in a Walrasian world; the theory of public economics in a disequilibrium situation remains for future work. (Refer, however, to sections 7.6*–7.8*.)

1 Externalities

The theory of external effects or externalities is basic to environmental economics. In this chapter, we present the essential results necessary for addressing questions such as: In what respect is pollution an economic problem? Should it be eliminated completely, or, on the contrary, is there an optimal level of pollution? What should we understand by the slogan, "polluters should pay"? What are the advantages and disadvantages of various economic policy solutions such as taxes, subsidies, the creation of markets in pollution rights, or awarding quotas of rights to pollute?

1.1 The Nature of Externalities

We start from a classic definition of externality, namely, any indirect effect that either a production or a consumption activity has on a utility function, a consumption set, or a production set. By "indirect," we mean both that the effect is created by an economic agent other that the one who is affected and that the effect is not transmitted through prices.[1] This definition indicates that the basic notion of externality depends on the definition of economic agents and the existence of markets that coordinate transactions among these agents.

For example, consider two firms that pollute each other's environment; each one imposes a negative external effect on the other. If both firms merge, the external effects simply become technical relationships within one firm: hence, the externalities are internalized. If a market in pollution rights is created between the firms, firm j must buy from firm j' a pollution right just as it would buy any other product from it: hence, the externalities are incorporated into market transactions.

In a barter economy, that is to say, an economy without markets, any exchange may be decomposed into two externalities. Quantity q_l^j of good l accepted by agent j in exchange for agent i's quantity q_k^i of good k creates an externality for agent i. Agent i's utility depends on his own trade offer q_k^i but also on that of the other agent, q_l^j. Alternatively, a market could be created for each component of each agent's activity vector as it relates to each of the other agents. In that case, no externalities would exist. If all economic agents are grouped into a single unit, no externalities can exist. However, the problem of organizing economic activity within this unit

1. External effects of this sort are categorized as technological (or nonpecuniary). For a discussion of pecuniary externalities, the reader is referred to section 1.8*.

would be quite complex.[2] The more the economic agents are subdivided into groups, the more externalities are generated in the economy.

Starting with an economic system, that is, given a set of economic agents and existing markets, externalities may or may not come into play. The existence, and eventually the justification, of these external effects may be understood only after an explanation of the size of economic units and a determination of the number of markets is given. Unfortunately, the conventional wisdom about these two problems is quite limited. The impossibility of excluding users of a public good, technological nonconvexities and fixed entry costs on markets, transaction costs, the availability of information, and the cost of transmitting and acquiring information appear to be fundamental determinants of the size of economic units and of the number of markets. There is no general equilibrium analysis that, starting from an elementary definition of agents and their objectives, yields endogenously the definition of an observed economic system (in the sense mentioned above).

The Arrow-Debreu general equilibrium model does not attempt to treat as variable the size of agents and the number of markets, just as it does not attempt to explain the agents' tastes. Rather, given agents and markets, the model analyzes the functioning of the economy.[3] In this chapter, we remain constrained by this model although we try to keep these general considerations in mind, especially when we derive public policy prescriptions.

We illustrate externalities formally with the help of three examples.

A firm's polluting of a river and thus decreasing the possibilities for swimming is an example of the first type of externality, namely the external effect of a production activity on a consumption set. Without pollution, the consumption set allows any swimming consumption up to 24 hours on a daily basis and any vitamin consumption (see figure 1.1). On the other hand, the presence of pollution, y, decreases the physiologically feasible daily consumption of swimming to a level $\alpha(y)$ less than 24 hours. However, this level increases with the consumption of vitamins.

More generally, if $X^i \subset \mathbf{R}^L$ is the consumption set in the absence of externalities, the consumption set with externalities becomes a correspondence that associates to each economic environment of consumer i

2. From this problem stems the eventual need for planning methods adapted to technological interdependencies when information is decentralized (the reader is referred to Aoki 1971 and chapter 6 of Laffont 1977).

3. However, the reader is referred to the last paragraph of section 1.5.

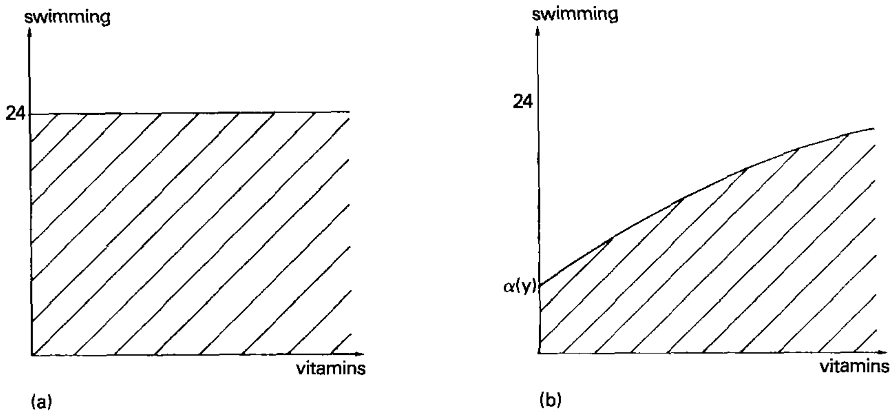

Figure 1.1
Consumption sets: (a) Without externalities; (b) with externalities.

(that is to say, the production levels of the firms and the consumption levels of the other consumers $(y^1, \ldots, y^J, x^1, \ldots, x^{i-1}, x^{i+1}, \ldots, x^I)$), a set of physiologically feasible consumption vectors

$$X^i(y^1, \ldots, y^J, x^1, \ldots, x^{i-1}, x^{i+1}, \ldots, x^I).$$

The noise emanating from the stereo system of one's neighbor is a typical example of a second type of externality, namely, a consumption externality that is formalized by considering that the utility function of a concerned consumer i depends on agent j's music consumption x^j_m, that is,

$$U^i(x^i, x^j_m).$$

More generally, all the environmental variables may affect $U^i(.)$.

Meade's famous example (1952) of the beekeeper and the orchard is a good illustration of the third type of externality, namely a mutual production externality. Since his bees pollinate the flowers, the beekeeper (firm 1) affects the production possibilities of the orchard in a positive way. Conversely, by providing flowers from which honey can be gathered, the orchard (firm 2) promotes the production of honey (see figure 1.2). More generally, the technology of firm j represented by a production set or by a production function depends on its entire environment $(y^1, \ldots, y^{j-1}, y^{j+1}, \ldots, y^J, x^1, \ldots, x^I)$, that is,

$$Y^j(y^1, \ldots, y^{j-1}, y^{j+1}, \ldots, y^J, x^1, \ldots, x^I),$$

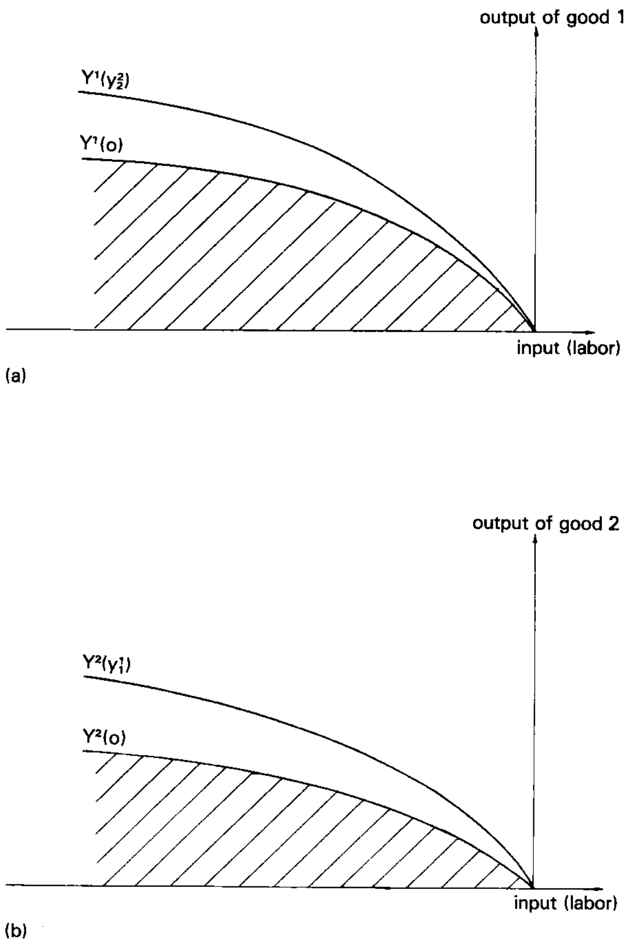

Figure 1.2
Production sets with and without externalities: (a) of the beekeeper; (b) of the orchard.

so that

$$f^j(y^1, \ldots, y^{j-1}, y^j, y^{j+1}, \ldots, y^J, x^1, \ldots, x^I) = 0.$$

1.2 Optimal Allocation of Resources

Consider an economy with two goods, two firms, and a single consumer. There are two externalities affecting firm 2, one generated by the consumer's consumption of good 1, x_1, and the other by the production of good 1 by firm 1, y_1^1. We can think of the example of the pollution of a river by a city and a firm that affects a water-using firm located downstream. The technologies are characterized by

$y_1^1 = f^1(y_2^1)$ $\quad$ f^1 is differentiable and concave[4]

$y_2^2 = f^2(y_1^2, y_1^1, x_1)$ $\quad$ f^2 is differentiable and concave[5].

The consumer has a utility function $U(x_1, x_2)$ that is differentiable, increasing, and strictly quasi-concave, and an initial endowment (w_1, w_2). The Pareto optimum for this economy is obtained by solving the problem

$$\text{Max } U(x_1, x_2)$$

$$\begin{aligned} \text{subject to} \quad & y_1^1 + y_1^2 + w_1 - x_1 \geqslant 0 && \lambda_1 \\ & y_2^1 + y_2^2 + w_2 - x_2 \geqslant 0 && \lambda_2 \\ & -y_1^1 + f^1(y_2^1) \geqslant 0 && \mu_1 \\ & -y_2^2 + f^2(y_1^2, y_1^1, x_1) \geqslant 0 && \mu_2, \end{aligned}$$

where $(\lambda_1, \lambda_2, \mu_1, \mu_2)$ are the Kuhn-Tucker multipliers associated with the constraints. In much of this book (with some exceptions), we will simplify the optimization problems by making assumptions that lead to interior optima; in particular, we will frequently ignore inequalities and boundary solutions.

The first-order conditions for the above optimization problem (which, given our assumptions of concavity, are also sufficient) can be written as:

4. See the mathematical definitions, p. ix.
5. $df^1/dy_2^1 \leqslant 0$, $\partial f^2/\partial y_1^2 \leqslant 0$ because inputs are of negative sign.