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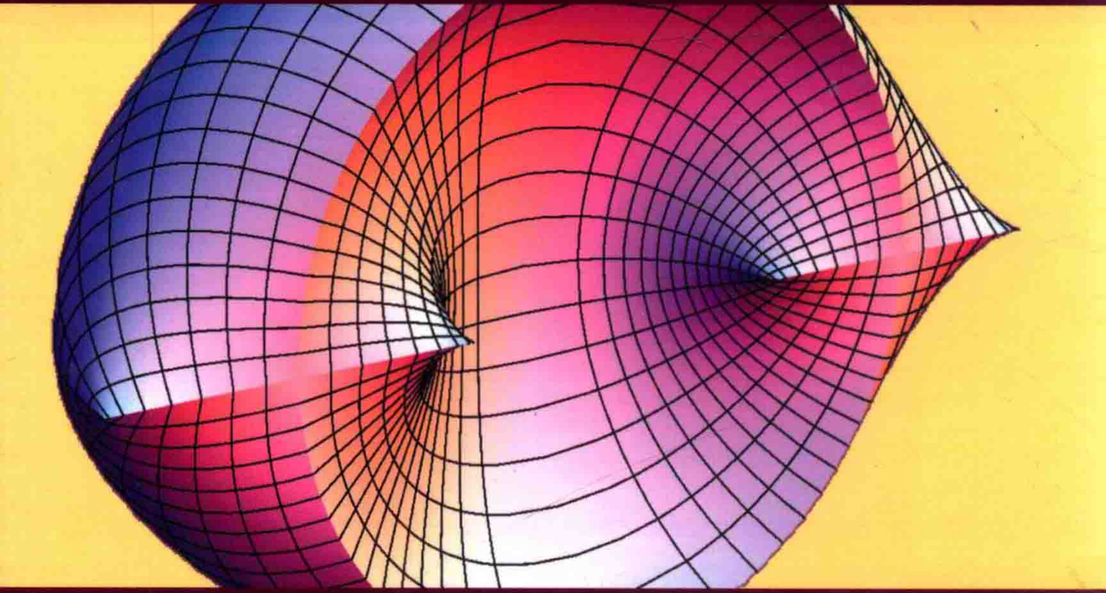

Nonlinear Physical Systems

Spectral Analysis, Stability and Bifurcations

Edited by
Oleg N. Kirillov
Dmitry E. Pelinovsky

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Nonlinear Physical Systems

Preface

The BIRS Workshop on *Spectral Analysis, Stability and Bifurcations in Modern Nonlinear Physical Systems*[1] brought together a unique combination of experts in modern dynamical systems, mathematical physics, partial differential equations (PDEs), numerical analysis, operator theory and applications.

One of the immediate outcomes of the meeting is this post-conference volume of papers from the participants of the workshops making its materials available to a wider audience. This book presents unique viewpoints of the participants on the history, current state of the art and prospects of research in their fields contributing to the progress of stability theory. In this book, we have compiled a collection of essays – mathematical, physical and mechanical. The contributions show connections between different approaches, applications and ideas. We believe that such a book could set the benchmarks and goals for the next generation of researchers and be a true event in modern stability theory. The other outcomes will be seen over a long period of time, when the ideas formulated and discussed during the workshop, as well as new collaborations made, will lead to new scientific publications and new research discoveries.

This book covers the problems of spectral analysis, stability and bifurcations arising from the nonlinear PDEs of modern physics. Bifurcations and stability of solitary waves, stability analysis in hydro- and magnetohydrodynamics and dissipation-induced instabilities will be treated with the use of the theory of Krein and Pontryagin space, index theory, the theory of multiparameter eigenvalue problems and modern asymptotic and perturbative approaches. All chapters contain mechanical and physical examples and combine both tutorial and advanced sections,

1 Took place at the Banff International Research Station for Mathematical Innovation and Discovery, Banff, Canada on 4–9 November 2012. For more information see http://www.birs.ca/events/2012/5-day-workshops/12w5073.

making them attractive both to professionals working in the field and non-specialists interested in knowing more about modern methods and trends in stability theory.

Chapter 1, written by Davide Bigoni and his colleagues, opens the book and presents the reader with sophisticated experiments with simple mechanical structures demonstrating buckling under tensile dead loading (without elements subject to compression at all) and flutter or oscillatory instability of a two-link pendulum that is caused by Coulomb friction. This new look at the classical mechanics is directly motivated by the successes of modern materials science.

The semi-classical n-dimensional quantum tunneling effect, through a hyperbolic fixed point, is treated by Jean-François Bony *et al.* in Chapter 2. The transfer operator which solves this microlocal Cauchy problem appears to be a Fourier integral operator which gives outgoing waves in terms of incoming waves. As an application, the longtime behavior of the Schrödinger group at barrier top is described in term of resonances with explicit generalized spectral projections. Another application is to obtain resonances free regions for homoclinic trapped sets.

A semi-classical limit of a quantum problem on angular momenta interacting in a magnetic field has led Richard Cushman and his colleagues to a curious one-parameter family of Hamiltonian systems in Chapter 3. Their system exhibits an S^1-equivariant sign exchange bifurcation in its linearization about an equilibrium point. The stability of this bifurcation under small S^1-invariant perturbations by linear Hamiltonian vector fields is shown in an instructive manner involving the method of versal deformations.

In Chapter 4, Olivier Doaré discusses the counter-intuitive destabilizing effect of damping in the problems of fluid–structure interaction. A model problem considered is a fluid–conveying pipe where the viscous damping is shown to destabilize the negative energy waves. The fluid-conveying pipe is a model problem for many fluid-elastic systems where a compliant structure interacts with a flow, such as flags, plates, shells, walls or wings. The model is of particular interest in the modern energy-harvesting applications.

Sergey Dobrokhotov and Anatoly Anikin discuss in Chapter 5 the splitting of the lowest eigenvalues of the multidimensional Schrödinger operator with the double-well potential. As a rule, the splitting formula is based on the instanton, which is a singular trajectory of the Newtonian system with inverted potential. However, a physically relevant form of the formula should involve, as the authors demonstrate, not the instanton but an appropriate unstable periodic trajectory (libration).

Periodic potentials and solitons are the subject of Chapter 6, written by Nir Dror and Boris Malomed. To stabilize the solitons in a two-dimensional Bose-Einstein condensate, a linear periodic potential is induced by means of the optical lattices, which are the interference patterns created by laser beams shone through the condensate. Such periodic potentials give rise to bandgaps in the corresponding linear spectrum, which, in combination with the self-focusing or self-defocusing nonlinearity, support various types of localized mode. The authors demonstrate that bound complexes built of the dipole solitons, in the form of bi-dipoles and four-dipole non-topological states, vortices and quadrupoles, are all stable if the underlying dipole is stable.

A steady Euler flow of an inviscid incompressible fluid is characterized as an extremum of the total kinetic energy with respect to perturbations constrained to an isovortical sheet. Yasuhide Fukumoto *et al.* analyze in Chapter 7 the criticality in the Hamiltonian to calculate the energy of three-dimensional waves on a steady vortical flow and to calculate the mean flow induced by nonlinear interaction of waves with themselves. The energy of waves on a rotating flow is expressible in terms of a derivative of the dispersion relation with respect to the frequency.

Pure imaginary eigenvalues in 1:1 semi-simple resonance (diabolical points in the physics language) typically occur in rotationally symmetrical non-dissipative models of physics and engineering. Its unfolding caused by symmetry-breaking and non-conservative perturbation is a reason for many instabilities such as the rotating polygon instability of swirling free surface flow. In Chapter 8, Igor Hoveijn and Oleg Kirillov map all possible singularities on the boundary of the stability domain of perturbed four-dimensional systems in 1:1 resonance and apply the result to the study of the enhancement of the modulation instability with dissipation.

Since the time of the celebrated Kelvin–Tait–Chetaev theorem, counts of unstable point spectra and other related counts that are referred to as index theorems have appeared across various distinct and unrelated fields due to their simple structure and importance for stability applications. Richard Kollár and Radomír Bosák give in Chapter 9 a unique and comprehensive survey of the index theorems motivated by very different physical, algebraic and control theory applications and also present a graphical Krein signature theory. The latter makes the proofs of index theorems for linearized Hamiltonians extremely elegant in the finite dimensional setting: a general result implying Vakhitov–Kolokolov criterion (or Grillakis–Shatah–Strauss criterion) as a corollary generalized to problems with arbitrary kernels, and a count of real eigenvalues for linearized Hamiltonian systems in canonical form.

Chapter 10 provides an example of counting unstable eigenvalues in the problems of vortex dynamics presented by Paolo Luzzatto-Fegiz and Charles H.K. Williamson. They demonstrate that the turning points in impulse of the vortex array correspond to a change in the number of unstable modes. Furthermore, whether the isovortical rearrangements involve the introduction or removal of an unstable mode can be inferred from the shape of a fold in the phase velocity–impulse plot.

In Chapter 11, the fluid dynamical theme is continued by Sherwin Maslowe who provides a general and comprehensive survey of the finite amplitude theory and discusses in detail the critical layer analyses that indicate, in particular, important resolution requirements for computational schemes.

A main motivation for studying Hamiltonian systems is their universality. In Chapter 12, Philip Morrison and George Hagstrom show how infinite-dimensional noncanonical Hamiltonian systems enlarge this universality class. Any specific system within the classes of systems considered may possess steady-state bifurcations, positive and negative energy modes and Krein's theorem for the Hamiltonian Hopf bifurcations. An analogous situation transpires for the continuous steady-state and Hamiltonian Hopf bifurcations. However, continuous spectra are difficult to deal with mathematically and functional analysis is essential. For example, we can interpret the continuous Hamiltonian Hopf bifurcation as the Hamiltonian Hopf bifurcation with the second mode coming from the continuous spectrum. Chapter 12 sets the stage for the explicit treatment of bifurcations with the continuous spectrum that is considered in Chapter 13.

A hybrid fluid-kinetic model of plasma physics considered by Philip Morrison and his coauthors in Chapter 14 combines a magnetohydrodynamics (MHD) part for a description of bulk fluid components and a Vlasov kinetic theory part that describes an energetic plasma component. In the considered model, a Hamiltonian structure is found that allows the authors to implement the energy-Casimir method for an explicit derivation of sufficient stability conditions.

Semigroups (or dynamical systems) of contractions in Hilbert space with non-self-adjoint generators considered by Francis Nier in Chapter 15 are motivated by the linearization of incompressible 2D-Navier-Stokes equation in the vortex formulation around Oseen vortices and by the Feller semigroup associated with the Langevin dynamics, which solves the Kramers–Fokker–Planck equation. The accurate estimates for the exponential decay of such semigroups with parameter-dependent non-self-adjoint generators obtained by the author substantially involve the theory of pseudo-spectrum.

The theory of pseudo-spectrum reappears in Chapter 16 where Michael Overton gives a broad survey of recent achievements in stability optimization for polynomials and matrices. The optimization problems discussed in this chapter typically lead to optimizers that are polynomials with multiple roots or matrices with non-derogatory multiple eigenvalues. The higher their multiplicity, the more these multiple roots or eigenvalues are sensitive to small perturbations; furthermore, computing these minimizers numerically is difficult. Instead of optimizing eigenvalues it is proposed to consider optimization of the pseudo-spectral radius and pseudo-spectral abscissa, which is computationally less difficult than for the spectral radius and spectral abscissa.

In Chapter 17, Dmitry Pelinovsky returns to the index theory and proves the index theorem in a rather general setting motivated by the problems of stability of nonlinear waves in KdV-type evolution equations. The directions leading to further extensions of this result are pointed out.

In the final Chapter 18, Zensho Yoshida and Philip Morrison describe several facets of noncanonical Hamiltonian systems, namely, the Poisson operator (field tensor) of a noncanonical Hamiltonian system has a non-trivial kernel (and thus, a cokernel) that foliates the phase space (Poisson manifold), imposing topological constraints on the dynamics. When we can "integrate" the kernel of the Poisson operator to construct Casimir elements, the Casimir leafs foliate the Poisson manifold and, then, the effective energy is the energy-Casimir functional. The theory is applied to the tearing-mode instability, where a tearing mode is regarded as an equilibrium point on a helical-flux Casimir leaf. As long as the helical-flux is constrained, the tearing mode cannot grow. However, it is shown that a singular perturbation that allows the system to change the helical flux can cause a tearing mode to grow if it has an excess energy with respect to a fiducial energy of the Beltrami equilibrium at the bifurcation point.

Oleg N. KIRILLOV
Dmitry E. PELINOVSKY
October 2013

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