LECTURE NOTES IN PHYSICS

U.-G. Meißner
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(Eds.)

# Lectures on Flavor Physics

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# Preface

This volume contains the written versions of some selected lectures delivered at the "41. Internationale Universitätswochen für Theoretische Physik" in Schladming, Austria. The 41st "Schladming Winter School" took place during the period February 22nd–28th, 2003. The theme of the School was "Flavor Physics".

Flavor physics is one of the hot topics in contemporary elementary particle physics, because it relates to fundamental questions like the origin of masses, the size and strength of CP violation, or the oscillations between various neutrino species. One thus explores the Standard Model as well as its possible extensions and related phenomena in astrophysics and cosmology. The lectures collected in this volume deal with important (theoretical) developments at various length scales. At low energies and for light quarks, one is able to analyze the strong interactions in terms of an effective field theory, as described in Jürg Gasser's lectures on light-quark dynamics. The interrelation between precisely calculable electroweak and the much more difficult strong interactions comes into play in precision QED observables, like the anomalous magnetic moment of the muon as discussed in the lectures by Marc Knecht. In fact, the presently available precision data might already give a glimpse at physics beyond the Standard Model.

The issue of CP violation in kaon and B-meson systems is addressed in the lectures by Andrzej Buras. Only with the advent of the B-factories, CP violation in the Standard Model could be measured beyond the kaon system, and one is now testing the unitarity of the CKM matrix and tries to understand the fundamental mechanism underlying CP violation. The richness of this field is underlined by the lectures of Matthias Neubert, who presents a new scheme to systematically calculate strong-interaction effects in certain B-decays and also dwells on the unitarity triangle. Last but not least, with the recent measurements of neutrino oscillations at Super-Kamiokande, SNO, and Kamland, the review on the foundations of the various forms of flavor transitions in vacuum and in media by Walter Grimus is very timely.

At the School, we had further lectures on the experimental status of B-decays (by D. Hitlin) as well as neutrino physics (by G. Drexlin), on lattice calculations of flavor-physics matrix elements (by G. Martinelli), and an introduction to supersymmetry (by H. Dreiner). They are not included here, but their essential content can be found from other sources in the literature.

Here, we should like to express our sincere gratitude to the lecturers for all their efforts in preparing and presenting their lectures. We are especially grateful to those colleagues who managed to find time to write up their lectures. We thank also the main sponsors of the School, the Austrian Federal Ministry for Education, Science, and Culture as well as the Government of Styria, for providing financial support. In addition, we acknowledge the contributions from the University of Graz and the valuable organisational and technical assistance by the town of Schladming, Ricoh Austria, and Hornig Graz. Furthermore, we are grateful to our secretaries, S. Fuchs and E. Monschein, a number of graduate students from our institute, and, last but not least, our colleagues from the organizing committee for their valuable assistance in preparing and running the school.

Bonn and Graz,
December 2003

*Ulf-G. Meißner*
*Willibald Plessas*

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**Neutrino Physics – Theory**

# Light-Quark Dynamics

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## 1 Introduction

At low energies $E \ll M_W$, the interactions of leptons and hadrons are described by QCD + QED up to corrections of order $(E/M_W)$. If we disregard the electromagnetic interactions, we are left with QCD that contains only a few parameters: the renormalization group invariant scale $\Lambda$ and the running quark masses $m_u, m_d, m_s \ldots$. The quark masses $m_u, m_d$ and $m_s$ are small on a typical hadronic scale like the mass of the rho or of the proton. It makes therefore sense to consider the limit where these masses are set equal to zero (chiral limit). The remaining quarks $c, b, \ldots$ are not light: although one may of course study the theoretical limit in which these masses also vanish, it does not seem to be possible to recover the actual mass values by an expansion around that limiting case. At low energies, a better approximation is obtained if the quarks $c, b, \ldots$ are instead treated as infinitely heavy. In this limit, the degrees of freedom associated with these quarks freeze and may be ignored in the effective low energy theory.

In the chiral limit, QCD contains therefore only one parameter, the scale $\Lambda$. The mass of the proton is a pure number multiplying $\Lambda$, and likewise for all the other states of the theory – the numbers $M_\rho/M_p, M_\Delta/M_p, \ldots$ are determined in a parameter free manner. In this sense, the chiral limit of QCD may be called a theory without any adjustable parameters: QCD is of course unable to predict the value of $M_p$ in GeV units, but it determines all dimensionless hadronic quantities in a parameter free manner. The elastic cross section for $pp$ scattering e.g. is some fixed function of the variables $s/M_p^2$ and $t/M_p^2$ , multiplying the square of the Compton wavelength of the proton.

It is unfortunately very difficult to really *calculate* masses, cross sections and decay amplitudes in this beautiful theory, because the lagrangian of QCD is formulated in terms of quark and gluon fields which do not create asymptotically observed particles. Several methods have therefore been devised in the past to cope with this problem in different regimes of the energy scale:

*i) Processes at high energies.* At high energies, the effective coupling constant $\alpha_{\text{QCD}}$ becomes small, and conventional perturbation theory in $\alpha_{\text{QCD}}$ is applicable.

*ii) Lattice calculations.* This is the only method known today which leads directly from the QCD lagrangian to the mass spectrum, decay matrix elements, scattering lengths etc. On the other hand, the CPU time needed for

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full fledged QCD calculations is enormous, and I believe that one may still have to wait a long time before this program achieves the accuracy one is aiming at in the framework of effective field theory.

*iii) Chiral perturbation theory* (ChPT). This method exploits the symmetry of the QCD lagrangian and its ground state: one solves in a perturbative manner the constraints imposed by chiral symmetry and unitarity by expanding the Green functions in powers of the external momenta and of the quark masses $m_u, m_d$ and $m_s$. To illustrate the idea, consider the process $\pi^+(p_1)\pi^-(p_2) \to \pi^0(p_3)\pi^0(p_4)$. Chiral symmetry implies that the corresponding scattering amplitude has the following form near threshold,

$$T = \frac{M_\pi^2 - s}{F_\pi^2} + O(p^4) \;\; ; \;\; s = (p_1 + p_2)^2 \;, \tag{1}$$

where $F_\pi = 92.4$ MeV is the pion decay constant, and $M_\pi$ denotes the pion mass. This result is due to Weinberg [1], who used current algebra and PCAC to analyse the Ward identities for the four-point functions of the axial currents. It displays the first order term in a systematic expansion of the scattering amplitude in powers of momenta and of quark masses. This term algebraically dominates the remainder, denoted by the symbol $O(p^4)$, for sufficiently small energies and thus provides an accurate parameterization of the full amplitude near threshold. As one goes away from threshold, the higher order terms come into play. We will see in the following that ChPT is a method that allows one to determine these corrections in a systematic manner.

ChPT is a particular example of an *effective field theory* (EFT). The method is in use since about 20 years, and it was therefore not possible to provide a detailed review in my lectures – for a recent comprehensive introduction to ChPT, I refer the reader to [2]. Instead, I discussed a few basic principles and applications, in the hope that students become interested in this fascinating topic and continue with their own studies and research projects.

The article is organized as follows. In Sect. 2, the flavor symmetries of QCD are discussed, and their Nambu-Goldstone realization explained. In Sect. 3, the Goldstone theorem is stated and illustrated with the free scalar field, with the linear sigma model ($L\sigma M$) and with QCD. In addition, the interaction of the Goldstone bosons at low energy is investigated. Section 4 contains a discussion of the effective field theory of the $L\sigma M$ and of QCD at low energy. In Sect. 5 are illustrated some calculations with these EFT, and Sect. 6 contains a detailed discussion of the elastic $\pi\pi$ scattering amplitude in this framework. In Sect. 7, it is shown how Roy equations may be used to determine low-energy constants that appear in the calculation of the $\pi\pi$ scattering amplitude. A short outlook on other topics is given in Sect. 8.

# 2 QCD with Two Flavours

In this section, I discuss the flavour symmetries of QCD.

## 2.1 Symmetry of the Lagrangian

The lagrangian of QCD is

$$\mathcal{L} = -\frac{1}{2g^2}\langle G_{\mu\nu}G^{\mu\nu}\rangle_c + \mathcal{L}_{ud} , \tag{2}$$

where

$$\begin{aligned}\mathcal{L}_{ud} &= \bar{u}\not{D}u + \bar{d}\not{D}d - m_u\bar{u}u - m_d\bar{d}d \\ &= (\bar{u}\ \bar{d})\begin{pmatrix}\not{D} - m_u & 0 \\ 0 & \not{D} - m_d\end{pmatrix}\begin{pmatrix}u \\ d\end{pmatrix} \ ; \ \not{D} = i\gamma^u(\partial_\mu - iG_\mu) .\end{aligned}$$

$G_{\mu\nu}$ denotes the field strength associated with the gluon field $G_\mu$, and $\langle A\rangle_c$ stands for the color trace of the matrix $A$.

It is useful to introduce left- and right-handed spinors,

$$u_L = \frac{1}{2}(1-\gamma_5)u \ , \ u_R = \frac{1}{2}(1+\gamma_5)u \ ,$$

$$\mathcal{L}_{ud} = (\bar{u}_L\ \bar{d}_L)\begin{pmatrix}\not{D} & 0 \\ 0 & \not{D}\end{pmatrix}\begin{pmatrix}u_L \\ d_L\end{pmatrix} - (\bar{u}_L\ \bar{d}_L)\begin{pmatrix}m_u & 0 \\ 0 & m_d\end{pmatrix}\begin{pmatrix}u_R \\ d_R\end{pmatrix} + L \leftrightarrow R .$$

QCD makes sense for any value of the quark masses. For $m_u = m_d = 0$, the lagrangian (2) is invariant under $U(2)$ rotations of the left- and right-handed fields,

$$\begin{pmatrix}u_I \\ d_I\end{pmatrix} \Rightarrow V_I\begin{pmatrix}u_I \\ d_I\end{pmatrix} \ ; \ V_I \in U(2) \ , \ I = L, R . \tag{3}$$

In other words, gluon interactions do not change the helicity of the quarks, see Fig. 1. On the other hand, the terms proportional to the quark masses are not invariant under the transformations (3), see Fig. 2.

According to the theorem of E. Noether, there is one conserved current for each continuous parameter in the symmetry group. As the group $U(2)$ has four real parameters, one expects eight conserved currents. However, due to quantum effects, one of these currents is not conserved, as a result of which there are only seven conserved currents in the limit of vanishing quark masses,

$$\begin{aligned}L^a_\mu &= \bar{q}_L\gamma_\mu\frac{\tau^a}{2}q_L \ , \ R^a_\mu = \bar{q}_R\gamma_\mu\frac{\tau^a}{2}q_R \ ; \ a = 1,2,3 \\ V_\mu &= \bar{q}\gamma_\mu q \ ; \ q = \begin{pmatrix}u \\ d\end{pmatrix} .\end{aligned} \tag{4}$$

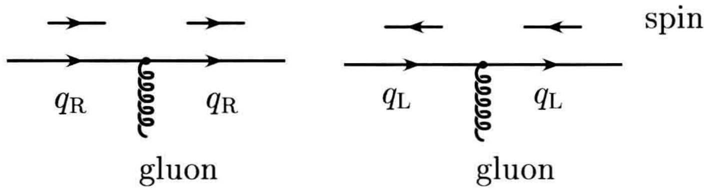


**Fig. 1.** Gluon interactions do not change the helicity of the quarks

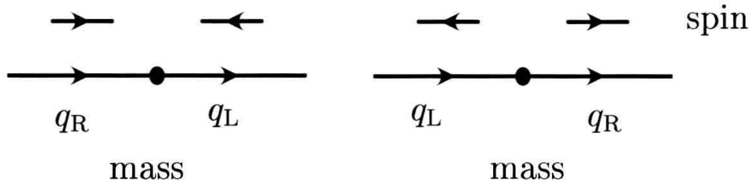


**Fig. 2.** The mass terms change the helicity of the quarks

The world at

$$m_u = m_d = 0$$

is called *the chiral limit of QCD*, and the above statements are summarized as: *In the chiral limit, $\mathcal{L}_{\rm QCD}$ is symmetric under global $SU(2)_L \times SU(2)_R \times U(1)_V$ transformations. The corresponding 7 Noether currents (4) are conserved.*

### 2.2 Symmetry of the Ground State

It is useful to introduce in addition the vector and axial currents

$$\begin{aligned} V^{\mu a} &= \bar{q}\gamma^\mu \frac{\tau^a}{2} q = L^{\mu a} + R^{\mu a} \ , \\ A^{\mu a} &= \bar{q}\gamma^\mu \gamma_5 \frac{\tau^a}{2} q = R^{\mu a} - L^{\mu a} \ ; \ a = 1,2,3 \ . \end{aligned}$$

The corresponding 6 axial and vector charges $Q^a_{A,V}$ are conserved and commute with the hamiltonian $H_0 = H_{\rm QCD}|_{m_u=m_d=0}$,

$$[H_0, Q^a_V] = [H_0, Q^a_A] = 0 \ ; \ a = 1,2,3 \ . \tag{5}$$

Consider now eigenstates of $H_0$,

$$H_0|\psi\rangle = E|\psi\rangle \ .$$

Then the states $Q^a_A|\psi\rangle$ and $Q^a_V|\psi\rangle$ have the same energy $E$, but carry opposite parity. On the other hand, there is no trace [3] of such a symmetry in nature. The resolution of the paradox has been provided by Nambu and

Lasinio back in 1960 [4]: whereas the vacuum is annihilated by the vector charges, it is not invariant under the action of the axial charges,

$$Q_V^a|0\rangle = 0 \ , \ Q_A^a|0\rangle \neq 0 \ . \tag{6}$$

There are two important consequences of this assumption:

i) The spectrum of $H_0$ contains three massless, pseudoscalar particles (Goldstone bosons (GB); Goldstone [5]). We will see more of this in the following section.
ii) The axial charges $Q_A^a$, acting on any state in the Hilbert space, generate Goldstone bosons,

$$Q_A^a|\psi\rangle = |\psi, G_1, \ldots, G_N, \ldots\rangle \ .$$

These are not one-particle states, and are therefore not listed in PDG, and there is therefore no contradiction anymore.

A theory with (5), (6) is called *spontaneously broken*: the symmetry of the hamiltonian is not the same as the symmetry of the ground state.

Where are the three massless, pseudoscalar states? The three pions $\pi^\pm, \pi^0$ are the lightest hadrons. They are not massless, because the quark masses are not zero in the real world [6]:

$$\begin{aligned} m_u &\simeq 5\,\text{MeV} \ , \\ m_d &\simeq 9\,\text{MeV} \ . \end{aligned} \tag{7}$$

In the following, we assume that the flavour symmetry of QCD is spontaneously broken to the diagonal subgroup,

$$\boxed{SU(2)_L \times SU(2)_R \to SU(2)_V}$$

and work out the consequences for the *interactions* between the Goldstone bosons.

Remark: Vafa and Witten [7] have shown that – modulo highly plausible assumptions – the vector symmetry $SU(2)_V$ is not spontaneously broken.

## 2.3 A Remark on Isospin Symmetry

Even if the quarks are not massless, the QCD lagrangian has a residual symmetry at $m_u = m_d$: it is invariant under the transformations (3) with $V_R = V_L \in SU(2)$. This symmetry is called *isospin symmetry.* We know from textbooks that isospin symmetry violations in the strong interactions are small[1]. On the other hand, according to (7), one has

$$m_d/m_u \simeq 1.8 \ . \tag{8}$$

[1] There are two sources of isospin violations: those due to electromagnetic interactions, and those due to the difference in the up and down quark masses

How can then isospin be a good symmetry if the quark masses differ so much? Consider the neutral and the charged pions: is it so that their masses differ by

$$(M_{\pi^+}^2 - M_{\pi^0}^2)/M_{\pi^0}^2 \simeq \frac{m_d - m_u}{m_d + m_u} \simeq 0.3 \ ?$$

The answer is no: one has

$$\begin{aligned} M_{\pi^+}^2 &= (m_u + m_d)B + \cdots \ , \\ M_{\pi^0}^2 &= (m_u + m_d)B + \cdots \ , \end{aligned}$$

where the ellipses denote higher order terms in the quark mass expansion. The neutral and the charged pion have the same leading term, the quark mass difference shows up only in the quadratic piece,

$$M_{\pi^+}^2 - M_{\pi^0}^2 = O[(m_u - m_d)^2] \ .$$

The perturbation due to the quark masses can be written as

$$\begin{aligned} m_u \bar{u}u + m_d \bar{d}d &= \frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) \\ &+ \frac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d) \ . \end{aligned}$$

Isospin is a good symmetry, not because $(m_d - m_u)/(m_d + m_u)$ is small, but because the matrix elements of the operator $\frac{1}{2}(m_d - m_u)(\bar{u}u - \bar{d}d)$ are small with respect to the hadron masses. The bulk part in the pion mass difference is generated by electromagnetic interactions.

## 3 Goldstone Bosons

In this section, I discuss the Goldstone theorem, illustrate it with several examples and consider the interaction of Goldstone bosons at low energy.

### 3.1 The Goldstone Theorem

We consider a quantum field theory which has the following properties:

i) There is a conserved current (i.e., an object that transforms as a four-vector under proper Lorentz transformations),

$$A_\mu(x) \ ; \ \partial^\mu A_\mu = 0.$$

ii) There is an operator $\Phi(x)$ such that

$$\langle 0|[Q, \Phi]|0\rangle \neq 0 \ ; \ Q = \int d^3x A_0(x^0, \vec{x}). \tag{9}$$

Then the Goldstone theorem [5] applies:

1. There exists a massless particle in the theory,

$$|\pi(\mathbf{p})\rangle \ , \ p^2 = 0 \ .$$

2. The current $A_\mu$ couples to the massless state,

$$\langle 0|A_\mu(0)|\pi(\mathbf{p})\rangle = ip_\mu F \neq 0 \ .$$

From the condition (9), it is seen that the charge $Q$ does not annihilate the vacuum.

### 3.2 The Free Scalar Field

We begin with a very simple example, the free, massless scalar field. The lagrangian is given by

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi \ .$$

For the current $A_\mu$, we take

$$A_\mu = \partial_\mu \phi \ .$$

This current is conserved, because $\phi$ is a free field. Consider now $\Phi = \phi$. From the canonical commutation relations, it follows that the condition (9) is satisfied. Therefore, the Goldstone theorem applies. Indeed, we can easily check directly:

- $\phi$ generates massless states ,

and

- $\langle 0|A_\mu(0)|\pi\rangle = -ip_\mu \neq 0$ .

### 3.3 The Linear Sigma Model

We consider the linear sigma model ($L\sigma M$ ), because it allows one to illustrate many features of effective field theories. At the same time, it serves as a model with spontaneous symmetry breaking. The lagrangian is

$$\mathcal{L}_\sigma = \frac{1}{2}\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} - \frac{g}{4}(\vec{\phi}^2 - v^2)^2 \ , \tag{10}$$

where $\vec{\phi} = (\phi^0, \phi^1, \phi^2, \phi^3)$ denotes four real fields, and $\vec{\phi}^2 = \phi^k \phi^k$ [repeated indices are summed over in the absence of a summation symbol]. In the following, we assume that

$$v^2 > 0 \ ,$$

and discuss

- the symmetry properties of $\mathcal{L}_\sigma$
- spontaneous symmetry breakdown
- Goldstone bosons
- quantization
- Goldstone boson scattering

**Symmetry Properties.** Here, we consider the classical theory and observe that $\mathcal{L}_\sigma$ is invariant under four-dimensional rotations of the vector $\vec{\phi}$,

$$\phi^i \to R^{ik}\phi^k \ , \ R \in O(4) \ .$$

The matrices $R$ can be parametrized in terms of six real parameters. Let us consider infinitesimal rotations

$$R = 1 + \varepsilon + O(\varepsilon^2) \ .$$

Because $R$ is an orthogonal matrix, $\varepsilon$ is antisymmetric, $\varepsilon + \varepsilon^T = 0$. Every real and antisymmetric four by four matrix can be expanded in terms of six generators,

$$\varepsilon = \sum_{i=1}^{3} \left( c_i \varepsilon^i_V + d_i \varepsilon^i_A \right) ,$$

where $c_i, d_i$ are 6 real parameters. The generators

$$\epsilon^1_A = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \epsilon^2_A = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \epsilon^3_A = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\epsilon^1_V = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \epsilon^2_V = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \quad \epsilon^3_V = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

satisfy the commutation relations

$$[\varepsilon^a_V, \varepsilon^b_V] = \varepsilon^{abc}\varepsilon^c_V \ ,$$
$$[\varepsilon^a_V, \varepsilon^b_A] = \varepsilon^{abc}\varepsilon^c_A \ ,$$
$$[\varepsilon^a_A, \varepsilon^b_A] = \varepsilon^{abc}\varepsilon^c_V \ ,$$

with $\varepsilon^{123} = 1$, cycl. The linear combinations

$$Q^a_L = \frac{1}{2}(\varepsilon^a_V - \varepsilon^a_A) \ , \ Q^a_R = \frac{1}{2}(\varepsilon^a_V + \varepsilon^a_A) \ ,$$