

SIXTH EDITION

Introductory

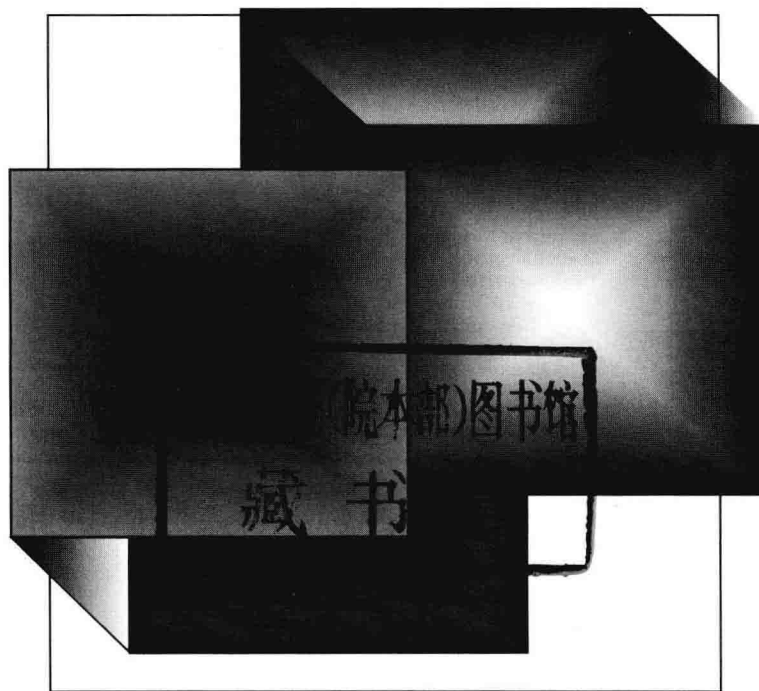
Linear Algebra
WITH APPLICATIONS



Bernard Kolman

Sixth Edition

INTRODUCTORY LINEAR ALGEBRA WITH APPLICATIONS



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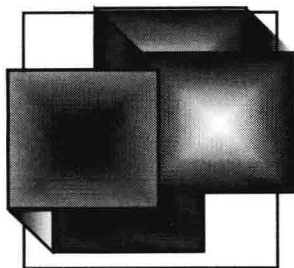
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*To the memory of Lillie
and to
Lisa and Stephen*



Preface

Material Covered

This book presents an introduction to linear algebra and to some of its significant applications and is designed for a course at the freshman or sophomore level. There is more than enough material for a semester or quarter course. By omitting certain sections, it is possible in a one-semester or quarter course to cover the essentials of linear algebra (including eigenvalues and eigenvectors), to show how the computer is used, and to have a little time left over for several applications. The book can also be used for a year course in linear algebra or for a second course emphasizing applications of linear algebra. A suggested pace for covering the basic material is given at the end of the Preface. The level and pace of the course can be readily changed by varying the amount of time spent on the theoretical material and on the applications. Calculus is not a prerequisite; examples and exercises using very basic calculus are included and these are labeled “Calculus Required.”

The emphasis is on the computational and geometrical aspects of the subject, keeping abstraction down to a minimum. Thus we sometimes omit proofs of difficult or less rewarding theorems, while amply illustrating them with examples. The proofs that are included are presented at a level appropriate for the student. We have also concentrated our attention on the essential areas of linear algebra; the book does not attempt to cover the subject exhaustively.

What Is New in the Sixth Edition

We have been very pleased by the widespread acceptance of the first five editions of this book. Encouraged by the success of the calculus reform movement that has been going on in this country during the last few years, a start has been made on developing ways to improve the teaching of linear algebra. The *Linear Algebra Curriculum Study Group* and others have made a number of recommendations for doing this. In preparing this edition, we have considered these recommendations as well as faculty and student suggestions. Although many changes have been made in this edition, our objective has remained the same as in the earlier editions: *to develop a textbook that will help the instructor to teach and the student to learn the basic ideas of linear algebra and to see some of its applications.* To achieve this objective, the following features have been developed in this edition:

- New Chapter 3 has been added on Vectors in R^2 and R^n .
- New Appendix B has been added on Further Directions.

- New sections have been added on
 - ◊ Dot product and Matrix Multiplication.
 - ◊ Introduction to Linear Transformations. Early introduction to this material in the new chapter on Vectors in R^2 and R^n .
 - ◊ Subspaces. Now treated separately from vector spaces.
 - ◊ Orthogonal Complements.
 - ◊ Inner Product Spaces (in new Appendix B).
 - ◊ Composite and Invertible Linear Transformations (in new Appendix B).
 - ◊ Electrical Circuits.
 - ◊ QR-factorization.
- Appendix A on complex numbers has been expanded and now consists of two sections.
- More geometric material has been added.
- Many new exercises (more than 200) at all levels have been added.
- Many of these additional exercises are qualitative, rather than quantitative, and their solutions require discussion rather than just computation.
- The section on least squares has been expanded.
- More material on computational techniques has been incorporated. In particular, the QR-factorization of a matrix is now discussed.
- The Cumulative Review of Part I has been expanded.
- Brief Previews of the longer applications in Chapters 7 and 8 are presented at appropriate places in the first six chapters.
- There is now a stronger emphasis on the geometrical presentation of basic ideas, which necessitates the use of a larger number of illustrative figures.
- Pedagogically improved MATLAB[®] (MATLAB is a registered trademark of The Mathworks, Inc.) M-files.
- These changes have resulted in a textbook that has more visualization, geometry, computation, and exercises whose solution calls for a verbal answer.

Exercises

The exercises in this book are grouped into three classes. The first class, **Exercises**, contains routine exercises. The second class, **Theoretical Exercises**, includes exercises that fill in gaps in some of the proofs and amplify material in the text. Some of these call for a verbal solution. In this technological age, it is especially important to be able to write with care and precision, and exercises of this type should help to sharpen that skill. These exercises can also be used to raise the level of the course and to challenge the more capable and interested student. The third class, **MATLAB Exercises**, consists of exercises developed by David R. Hill and labeled by the prefix ML (for MATLAB). These exercises are designed to be solved by MATLAB or another appropriate computer software package. Answers to all odd-numbered numerical and ML exercises appear in the

back of the book. There is also a comprehensive review of Part I, the basic linear algebra material, consisting of 75 true–false questions (with answers in the back of the book). An **Answer Manual**, containing answers to all even-numbered exercises and solutions to all theoretical exercises, is available (to instructors only) at no cost from the publisher.

Presentation

We have learned from experience that at the sophomore level, abstract ideas must be introduced quite gradually and must be based on some firm foundations. Thus we begin the study of linear algebra with the treatment of matrices as mere arrays of numbers that arise naturally in the solution of systems of linear equations, a problem already familiar to the student. Much attention has been devoted from one edition to the next to refining and improving the pedagogical aspects of the exposition. The abstract ideas are carefully balanced by the considerable emphasis on the geometrical and computational aspects of the subject.

Material Covered

Part I, consisting of Chapters 1 through 6, presents the basic linear algebra material. **Chapter 1** deals with matrices and their properties; Section 1.3, new to this edition, introduces matrix multiplication by using the dot product operation for n -vectors. Methods for solving systems of linear equations are discussed in this chapter. **Chapter 2** presents the basic properties of determinants and some of their applications. **Chapter 3**, new to this edition, deals with vectors in R^2 and R^n . In this chapter we give an early introduction to linear transformations and explore some of the many geometric ideas that arise naturally. In **Chapter 4** we come to a more abstract notion, that of a vector space. Section 4.9, new to this edition, discusses orthogonal complements; it is part of the added geometric emphasis in this edition. **Chapter 5**, on eigenvalues and eigenvectors, settles the diagonalization problem for symmetric matrices. **Chapter 6** covers linear transformations and matrices. Short applications to a wide variety of areas have been included in the first six chapters. More extensive applications comprise Part II of the book. Brief Previews of these applications are given at appropriate places in the first six chapters.

Part II consists of Chapters 7 and 8. **Chapter 7** contains an introduction to linear programming, an extremely important application of linear algebra. **Chapter 8** presents several other diverse applications of linear algebra: graph theory, electrical circuits (new to this edition), Markov chains, least squares (expanded in this edition), linear economic models, differential equations, the Fibonacci sequence, quadratic forms, conic sections, quadric surfaces, and the theory of games. The applications in Chapters 1 through 8 deal with problems in mathematics, physics, biology, the social sciences, management, business, and economics.

In Part III, which consists of **Chapter 9** on numerical linear algebra, we discuss rather briefly numerical methods commonly used in linear algebra to solve linear systems of equations and to find eigenvalues and eigenvectors of matrices. Section 9.4, on QR-factorization, is new to this edition. These methods

are widely used in conjunction with computers. However, the examples and exercises presented here can all be solved with handheld calculators.

Part IV, which consists of **Chapter 10**, provides a brief introduction to MATLAB (which stands for MATRIX LABORATORY), a very useful software package for linear algebra computation, described below.

Chapters 7, 8, and 9 are almost entirely independent, the one exception being Section 8.11, The Theory of Games, which requires Chapter 6 as preliminary study. The applications can either be covered after completing the entire linear algebra material in the course or they can be taken up as soon as the material required for a particular application has been developed. The chart at the end of the Preface giving the prerequisites for each of the applications and the brief Previews will be helpful in deciding which applications to cover and when to cover them. Some of the sections, in particular those in Chapters 8 and 9, can also be used as independent student projects. Classroom experience with the latter approach has met with favorable student reaction. Thus the instructor can be quite selective both in the choice of material and in the method of study of the applications.

Appendix A, on complex numbers, introduces in a brief but thorough manner complex numbers and their use in linear algebra. **Appendix B** presents two more advanced topics in linear algebra: inner product spaces and composite and invertible linear transformations.

End of Chapter Material

Every chapter contains a summary of **Key Ideas for Review**, a set of Supplementary Exercises (answers to all odd-numbered numerical exercises appear in the back of the book), and a Chapter Test (all answers appear in the back of the book).

MATLAB Software

Although the ML exercises can be solved using a number of software packages, in our judgment MATLAB is the most suitable package for this purpose. MATLAB is a versatile and powerful software package whose cornerstone is its linear algebra capabilities. MATLAB incorporates professionally developed quality computer routines for linear algebra computation. The code employed by MATLAB is written in the C language however. Many of the routines/functions are written in the MATLAB language, and are upgraded as new versions of MATLAB are released. MATLAB is available for Microsoft Windows, Macintosh computers, and for Unix and VMS workstations. MATLAB is available from The Math Works, Inc., 24 Prime Park Way, Natick, MA 01760, [(508) 653-1415], e-mail: info@mathworks.com. The Student version is available from Prentice Hall at a reasonable cost. This Student Edition of MATLAB also includes a version of Maple, thereby providing a symbolic computational capability.

Chapter 10 of this edition consists of a brief introduction to MATLAB's capabilities for solving linear algebra problems. Although programs can be written within MATLAB to implement many mathematical algorithms, *it should be noted that the reader of this book is not asked to write programs. The user is*

merely asked to use MATLAB (or any other comparable software package) to solve specific numerical problems.

Approximately 18 programs, called scripts or M-files, have been developed in MATLAB and are available to users of this book through The Math Works, Inc. The reader can download the supplemental M-files from The MathWorks anonymous FTP site at <ftp.mathworks.com> in the directory `/pub/books/kolman/`. These M-files are designed to transform many of MATLAB's capabilities into courseware. This is done by providing pedagogy that allows the student to interact with MATLAB, thereby letting the student think through all the steps in the solution of a problem and relegating MATLAB to act as a powerful calculator to relieve the drudgery of tedious computation. Indeed, this is the ideal role for MATLAB (or any other similar package) in a beginning linear algebra course, for in this course, more than in many others, the tedium of lengthy computations makes it almost impossible to solve a modest-size problem. Thus, by introducing pedagogy and reigning in the power of MATLAB, these M-files provide a working partnership between the student and the computer. Moreover, the introduction to a powerful tool such as MATLAB early in the student's college career opens the way for other software support in higher-level courses, especially in science and engineering.

A MATLAB Workbook

An excellent supplement to this book is *Linear Algebra Labs with MATLAB*, 2nd ed., by David R. Hill and David E. Zitarelli, Prentice Hall, 1996. This workbook provides labs and applications of the material that are designed to be solved using MATLAB. The primary goal of the workbook is to provide experiences that will aid in the understanding of the basic ideas of linear algebra. The workbook can be obtained at a reduced cost when purchased together with this book. This package can be ordered as ISBN 0-13-848409-0.

Student Solutions Manual

The *Student Solutions Manual*, prepared by Nina Edelman, Temple University, and Dennis R. Kletzing, Stetson University, contains solutions to all odd-numbered exercises, both numerical and theoretical.

Suggested Pace for Basic Material, Part I

Chapter 1	7 lectures
Chapter 2	3 lectures
Chapter 3	3 lectures (omitting Sections 3.4 to 3.6)
Chapter 4	10 lectures
Chapter 5	5 lectures
Chapter 6	4 lectures
<hr/>	
	32 lectures

Prerequisites for Applications

Chapter 7	Section 1.5
Section 8.1	Section 1.4
Section 8.2	Section 1.5
Section 8.3	Section 1.5
Section 8.4	Sections 1.5, 1.6, 3.2, and 4.9
Section 8.5	Chapter 1
Section 8.6	Section 5.1 and Calculus
Section 8.7	Section 5.1
Section 8.8	Section 5.2
Section 8.9	Section 8.8
Section 8.10	Section 8.8
Section 8.11	Chapter 7
Section 9.1	None
Section 9.2	Section 1.5
Section 9.3	Sections 1.6 and 9.2
Section 9.4	Section 4.8
Section 9.5	Chapter 5

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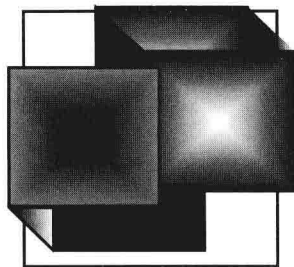
We thank Dennis R. Kletzing, Stetson University, who typeset the entire manuscript, the *Student Solutions Manual*, and the *Answer Manual*. He found a number of errors in the manuscript and cheerfully performed miracles under a very tight schedule. It was a pleasure working with him.

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B. K. and D. H.



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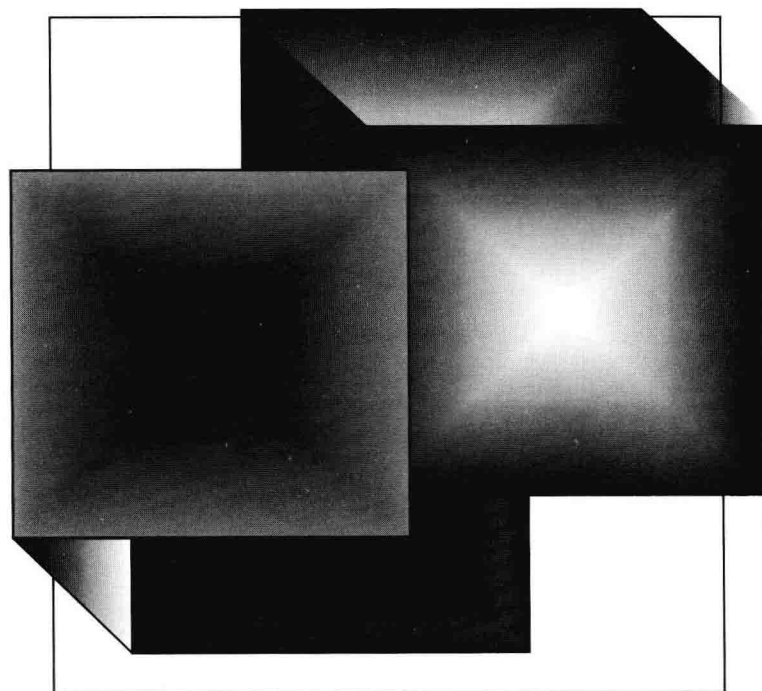
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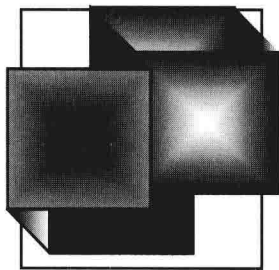
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PART I

INTRODUCTORY LINEAR ALGEBRA





Linear Equations and Matrices

1.1 ▼ Linear Systems

A good many problems in the natural and social sciences as well as in engineering and the physical sciences deal with equations relating two sets of variables. An equation of the type

$$ax = b,$$

expressing the variable b in terms of the variable x and the constant a , is called a **linear equation**. The word *linear* is used here because the graph of the equation above is a straight line. Similarly, the equation

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b, \quad (1)$$

expressing b in terms of the variables x_1, x_2, \dots, x_n and the known constants a_1, a_2, \dots, a_n , is called a **linear equation**. In many applications we are given b and the constants a_1, a_2, \dots, a_n and must find numbers x_1, x_2, \dots, x_n , called **unknowns**, satisfying (1).

A **solution** to a linear equation (1) is a sequence of n numbers s_1, s_2, \dots, s_n , which has the property that (1) is satisfied when $x_1 = s_1, x_2 = s_2, \dots, x_n = s_n$ are substituted in (1).

Thus $x_1 = 2, x_2 = 3$, and $x_3 = -4$ is a solution to the linear equation

$$6x_1 - 3x_2 + 4x_3 = -13,$$

because

$$6(2) - 3(3) + 4(-4) = -13.$$

This is not the only solution to the given linear equation, since $x_1 = 3, x_2 = 1$, and $x_3 = -7$ is another solution.